Coulomb problem for vector particles (W)

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Colomb center – small charged black hole,
new heavy charged particle
(dark matter candidate)

S=1 particle W in Coulomb field: fall to the center

Tamm; Corben,Schwinger (1940),… : infinite charge of W near Coulomb center.

Density of charge for W

$$\rho_W(r) = -C_1\, r^{2\gamma-4} + C_2\, \delta(\mathbf{r})$$

$$C_2 = \infty, \quad \int_0^r r^{2\gamma-4}\, d^3r = \infty$$

We suggest solution:

QED: Fermion vacuum polarization creates impenetrable potential barrier $U_{vac}=\tau/r^4$ near Coulomb center (specific for S=1 wave equation) .

Coefficient $\tau=Z^2\alpha^3\,\beta/m^2$ may be arbitrary small.

No fall, Coulomb problem for S=1 solved.

Renormalizable models: SU(2), "standard" SU(2)xU(1) .

The renormalizabilty of a theory does not guarantee a reasonable behavior at small distances for non-perturbative problems, such as bound states (all orders in $Z\,\alpha$, infinite sum of ladder diagrams).

To prevent collaps the theory needs additional fermions or scalars which switch the ultraviolet behaviour from asymptotic freedom (antiscreening) to screening.

Wave equation for S=0

$$-\nabla_\mu \nabla^\mu \phi = m^2 \phi, \quad \nabla_\mu = \partial_\mu + ieA_\mu$$

$$\hbar = c = 1, \qquad A_0 = -\frac{Ze}{r}$$

$$\left(\varepsilon + \frac{Ze^2}{r} \right)^2 \phi = \left(-\Delta + m^2 \right) \phi$$

$$-\nabla_{\mu}\nabla^{\mu}\psi+\frac{ie}{2}\sigma_{\mu\nu}F^{\mu\nu}\,\psi=m^{2}\psi$$

S=1/2, Dirac equation

S=1, equation of motion (from Standard model)

$$L_W = -\frac{1}{2}\left(\nabla_\mu W_\nu - \nabla_\nu W_\mu\right)^* \left(\nabla_\mu W_\nu - \nabla_\nu W_\mu\right)$$

$$+ m^2 W_\mu^* W^\mu + i\, e\, F^{\mu\nu} W_\mu^* W_\nu$$

$$\left(\nabla^2 + m^2\right) W^\mu + 2ie\, F^{\mu\nu}\, W_\nu - \nabla^\mu \nabla^\nu\, W_\nu = 0$$

$$m^2 \nabla_\mu W^\mu + ie\, j_\mu W^\mu = 0$$

$$j^\mu = \partial_\nu F^{\nu\mu}$$

S=1, the role of external current j

$$\left(\nabla^2 + m^2\right) W^\mu + 2ie\, F^{\mu\nu}\, W_\nu - \nabla^\mu \nabla^\nu\, W_\nu = 0$$

$$m^2 \nabla_\mu W^\mu + ie\, j_\mu W^\mu = 0$$

$$\left(\nabla^2 + m^2\right) W^\mu + 2ieF^{\mu\nu} W_\nu + \frac{ie}{m^2} \nabla^\mu (j_\nu W^\nu) = 0$$

Magnetic g-factor g=2

S=1, equation in the vacuum (outside the Coulomb center)

If $\quad j^{\mu} = 0 \quad$ then

$$\left(\nabla^{2} + m^{2}\right) W^{\mu} + 2 i e F^{\mu\nu} W_{\nu} = 0$$

Discrete energy spectrum: Zommerfeld-type formula

$$\varepsilon = m\left(1+\frac{(Z\alpha)^2}{(\gamma+n-j-1/2)^2}\right)^{-1/2}, \quad j=0,1\ldots$$

$$\gamma=\left((j+1/2)^2-Z^2\alpha^2\right)^{1/2}$$

$1S_1$, $2P_0$, $\{2S_1$, $2P_1\}$, $2P_2$, $3P_0$, $\{3S_1$, $3P_1$, $3D_1\}$, $\{3P_2$, $3D_2\}$, $3D_3$

$j=0$ states $2P_0, 3P_0\ldots$ disappear for $Z\alpha > 1/2$

However, relativistic corrections are smaller than 10% even for $Z\alpha = 1/2$

Simple spectrum for magnetic g-gactor $g=2$ only (Standard Model value for W).

Proca theory with $g=1$ fails.

Test: direct calculation of relativistic corrections

Very complicated expressions for corrections to kinetic energy , spin - orbit, electric quadrupole and ΔU terms. However, a "miracle" happens: a very simple expression for the total sum

$$\delta\varepsilon = m\frac{(Z\alpha)^4}{n^3}\left(\frac{3}{8n}-\frac{1}{2j+1}\right),\ \ j=0,1\ldots \to$$

if both the magnetic dipole and electric quadrupole moments coincide with the Standard Model values.

Wave function:Hedgehog
S=1, j=0

$$\mathbf{W} = \mathrm{v}(\mathrm{r})\,\mathbf{n}, \qquad \mathbf{n} = \mathbf{r}/r$$

$$r = \frac{Z\alpha}{\varepsilon}x, \qquad \mathrm{v}(\mathrm{r}) = \frac{1+x}{x}\phi,$$

$$-\kappa^2\phi = \left(-\frac{d^2}{dx^2} - \frac{2(Z\alpha)^2}{x} - \frac{(Z\alpha)^2}{x^2} + \frac{2}{(x+1)^2}\right)\phi,$$

$$\kappa^2 = (Z\alpha)^2\frac{m^2-\varepsilon^2}{\varepsilon^2} > 0$$

Catastrophic behaviour of charge density at the origin for the hedgehog, j=0.

$$\rho_W(r) = -C_1 r^{2\gamma-4} + C_2\,\delta(\mathbf{r})$$

$$C_2 = \infty$$

$$\int_0^r r^{2\gamma-4}\, d^3r = \infty$$

Summary so far

- r>0: Sommerfeld spectrum for the Coulomb problem?
- But the boson falls on the Coulomb center (if the size of the center is small enough).
- There is no problem in non-relativistic limit: usual Coulomb wave functions, singular term $C_1 \sim (Z\alpha)^9$.
- Include contact term containing Coulomb center density ρ.

Finite nuclear size R, $U_{eff} \sim -1/R^4$, large (infinite for R=0) number of states "live" inside nucleus, energy $E<-mc^2$,

W^+W^- pair creation for arbitrary small charge Z. The problem remains for R=0.

S=1, the role of external current

$$\left(\nabla^2 + m^2\right) W^\mu + 2ie\, F^{\mu\nu}\, W_\nu - \nabla^\mu \nabla^\nu\, W_\nu = 0$$

$$m^2 \nabla_\mu W^\mu + ie\, j_\mu W^\mu = 0$$

$$\left(\nabla^2 + m^2\right) W^\mu + 2ieF^{\mu\nu} W_\nu + \frac{ie}{m^2} \nabla^\mu (j_\nu W^\nu) = 0.$$

Υ – potential

in spherically symmetrical, static case

$$eA_\mu \to eA'_\mu = eA_\mu + \Upsilon_\mu$$

$$\Upsilon_\mu = \frac{e}{m^2} j_\mu$$

$$U \to U' = U + \Upsilon$$

$$\Upsilon = \frac{e\rho}{m^2} = -\frac{1}{m^2}\Delta U\,, \quad E < -mc^2$$

Vacuum polarization as a savior

$$U(r) = -\left(1 + S(r)\right)\frac{Z\alpha}{r}$$

$$S(r) = -\alpha\,\beta\,\ln(mr), \quad r \to 0$$

$$\beta = 1.42(1) \quad Gell - Mann - Low\ function$$

$$\Upsilon = -\frac{1}{m^2}\Delta U = \frac{Z\alpha^2\beta}{m^2 r^3}$$

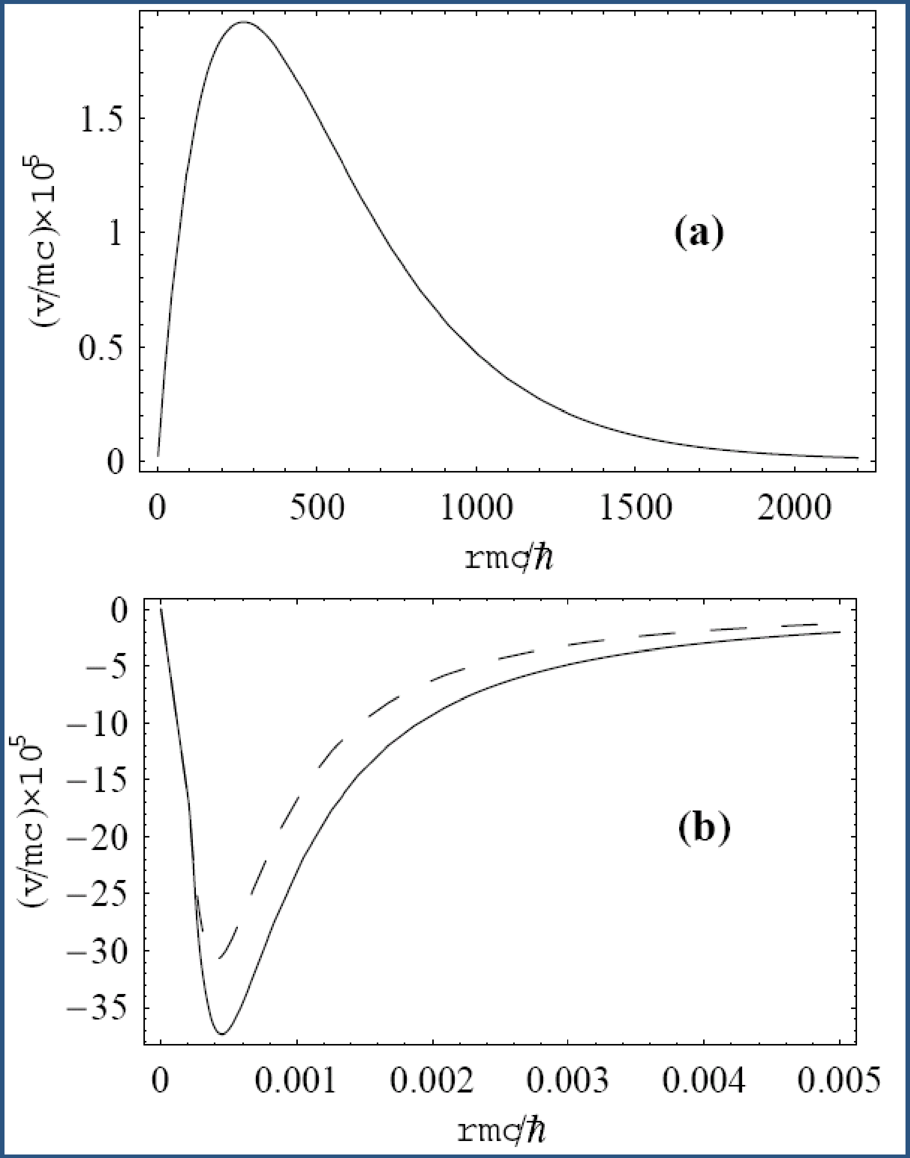
Wave functions, density of charge

$$j = 0 \quad \Rightarrow \quad \mathbf{W} = \mathrm{v}(r)\,\mathbf{n}$$

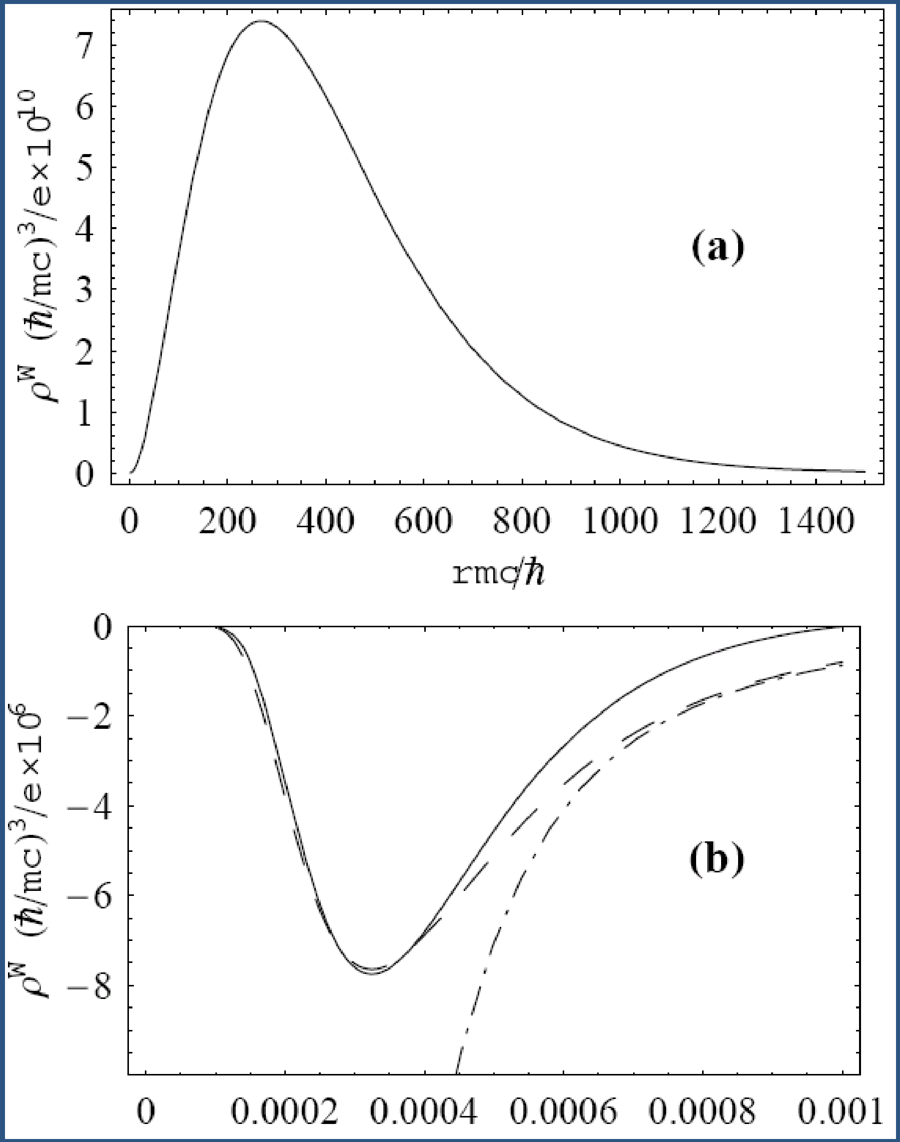
$$\mathrm{v}(r) = \frac{a}{m\,r^2}\exp\left(-\frac{Z\alpha\left(\alpha\beta\right)^{1/2}}{m\,r}\right), \quad r \to 0$$

$$\rho_W(r) = -\frac{4a^2 Z\alpha\, e}{m\,r^5}\exp\left(-\frac{Z\alpha\left(\alpha\beta\right)^{1/2}}{m\,r}\right), \quad r \to 0$$

Wave function , *j=0*



Charge density , *j=0*



Suggested solution:

Fermion vacuum polarization creates impenetrable potential barrier $U_{vac}=\tau/r^4$ near Coulomb center (specific for S=1 wave equation which contains term with Δ (U+Uehling)) .

Coefficient $\tau=Z^2\alpha^3\ \beta/m^2$ may be arbitrary small.

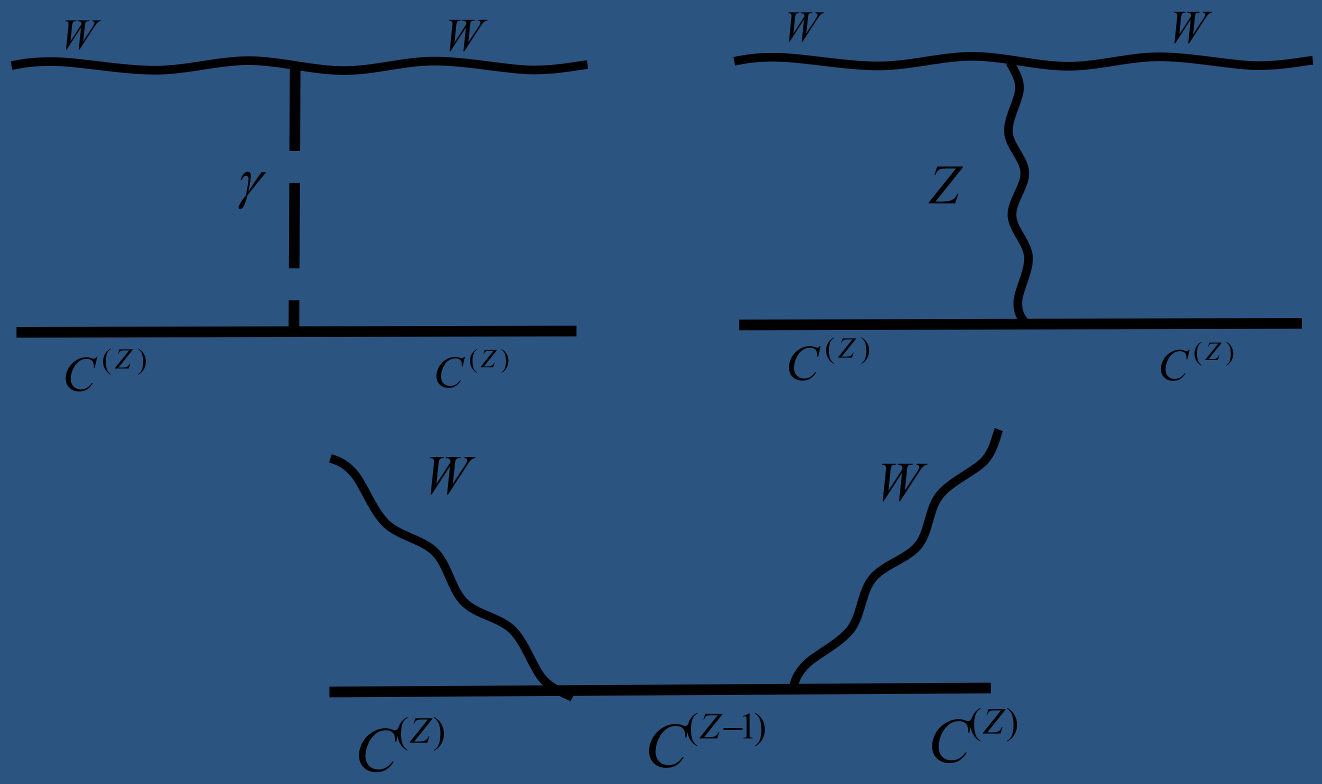
No fall, Coulomb problem for S=1 solved!

Questions:

1. No "good theory" without Fermions?
2. Does renormalizabilty of a theory guarantee a consistent solution for bound states ("non-perturbative" problem, all orders in Z α)?

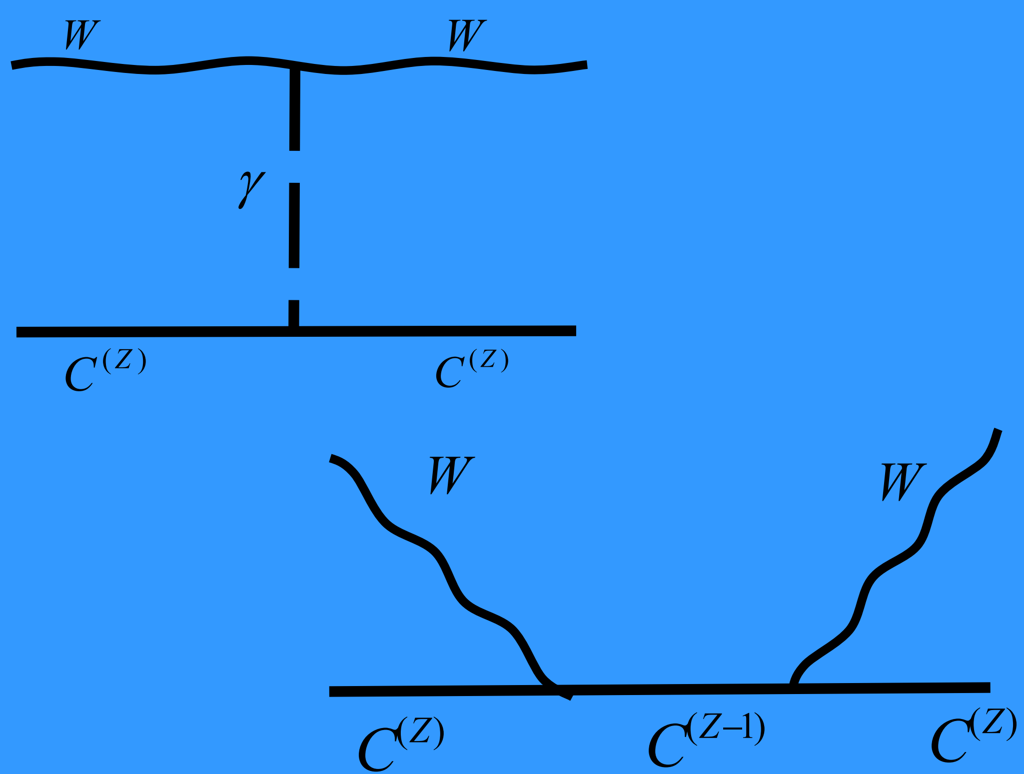
Renormalizable theory SU(2)xU(1)

- Doublet of heavy fermions $C_{(Z)}$, $C_{(Z-1)}$ of equal mass , charge $Q=Y/2+T_3$
- 3 lowest order diagrams for bound state



Start from simpler gauge theory SU(2)

- Doublet of heavy fermions $C_{(1/2)}$, $C_{(-1/2)}$ of equal mass, charge $Q=T_3$
- 2 lowest order diagrams for bound state



Lowest order scattering

- For longitudinal polarization of W-bosons "Compton" diagram has "forbidden" increase with energy which violates unitarity. This forbidden contribution is cancelled exactly by the similar contribution of photon exchange diagram.

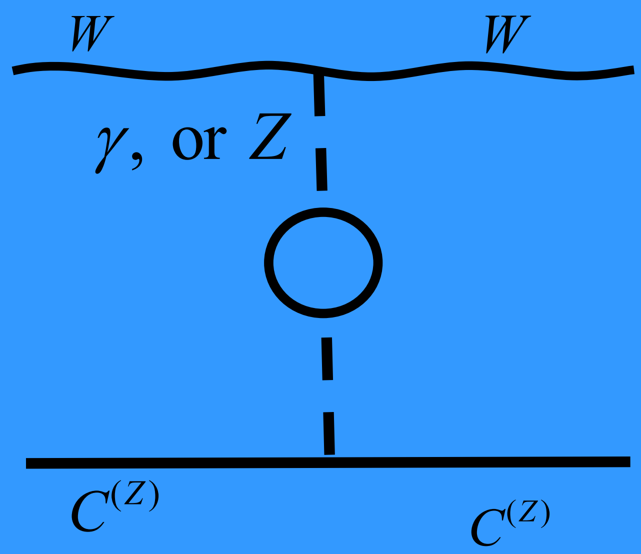
This is the consequence of the renormalizability of the theory.

Bound state problem

The "Compton diagram" gives a contact effective repulsive interaction ~ρ which acts inside nucleus only. It may eliminate the states "living" inside the finite nucleus. However, it does not save the W wave function from collaps to r~1/M where M is large (infinite) mass of the nucleus.

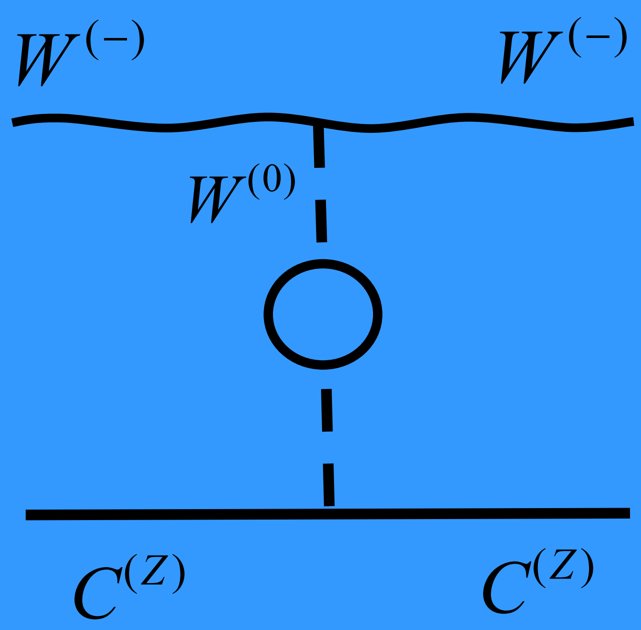
Higher orders:
most singular diagrams-vacuum polarization,

Vvac.pol ~ln(m r)/r
They determine “to fall or not to fall”.



Very small distances

- Mass of Z-boson may be neglected. It is convinient to use B,W_0 instead of γ,Z.
- W interacts with W_0 only. "Asymptotic freedom" in SU(2). Anti-screening.



SU(2) vacuum polarization gives attraction: fall!

$$U(r) = -\left(1 + S(r)\right)\frac{Z\alpha}{r}$$

$$S(r) = -\alpha\,\beta\,\ln(mr), \quad r \to 0$$

$$\beta < 0 \text{ asymptotic freedom}$$

$$\Upsilon = -\frac{1}{m^2}\Delta U = -\frac{Z\alpha^2\,|\,\beta\,|}{m^2 r^3}$$

Surprise !? Contact interaction term ΔU for S=1 gives the following results:

1. Electrodynamics, W bound to Coulomb center: Fermion vacuum polarization creates impenetrable potential barrier $U_{vac}=\tau/r^4$.

No fall, Coulomb problem for S=1 solved.

Screening, negative vacuum charge, repulsion of W^-.

2. Renormalizable non-Abelian gauge theories SU(2), Standard model SU(2)xU(1), … heavy fermion doublet, asymptotic freedom: attraction $U_{vac}=-\tau/r^4$.

W^- falls to the center (r<1/M)!

Anti-screening, positive "vacuum charge", very singular attraction.

The renormalizabilty is not sufficient to have "Bohr's atom". Addional light fermions (or scalars) may save the problem, switch the ultraviolet behavior from asymptotic freedom to Landau pole.

Difference with perturbation theory results (in scattering problems)

- Perturbation theory: in high energy limit W-boson mass may be neglected. The results are the same as for unbroken SU(2) symmetry

(without Higgs vacuum value).

- All orders in $Z\alpha$: there are no Coulomb bound states for zero W-boson mass.

No smooth transition to the unbroken symmetry (m_W =0). Spontaneous symmetry breaking creates bound states.

It is not enough to have renormalizable theory.

Charge density of W^+ may be negative! PRL2007

Standing wave W=sin(pr)

Density $\rho = \sin^2(pr) - (p^2/m^2)\cos(2pr)$

p>m negative areas

Near Coulomb center

$\rho = - C\, r^{-(4-2\gamma)} < 0$, negative (infinite charge without vacuum polarization barrier!)

S=1 particle has electric quadrupole moment

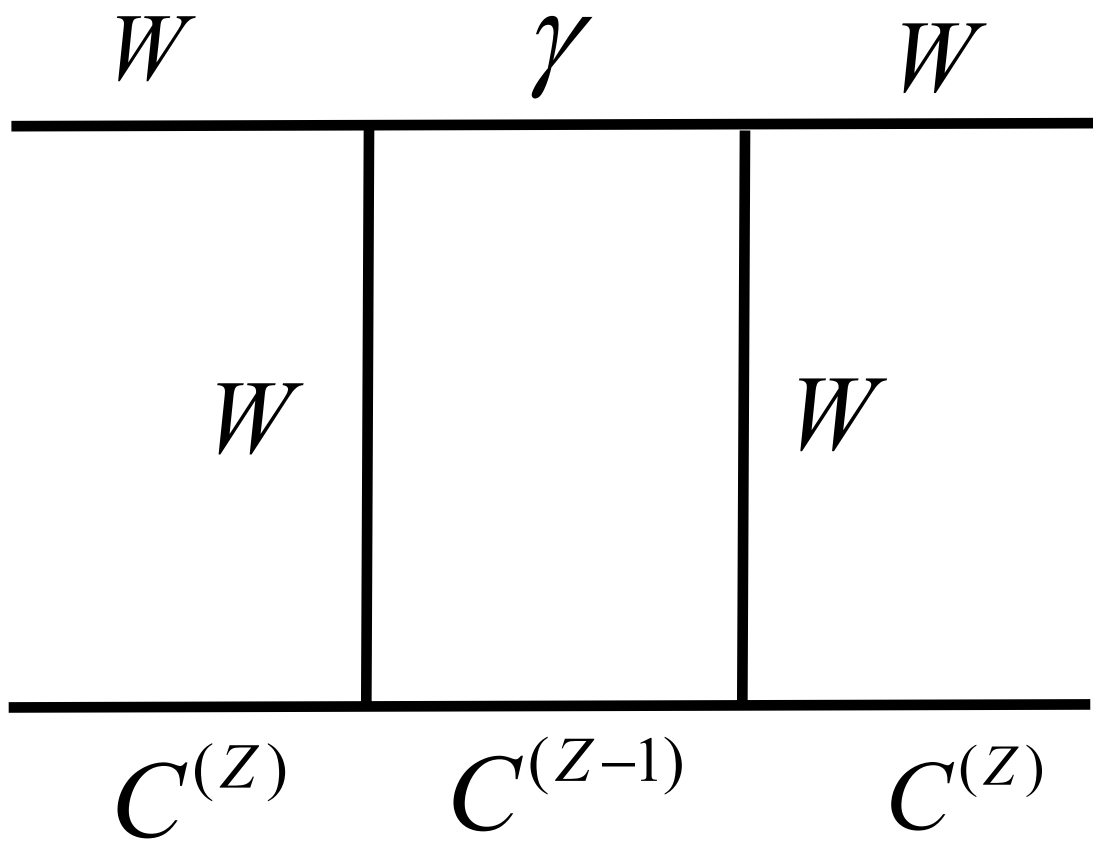
- $\rho_c = \sin^2(pr)$ – usual density of charge
- Charge density of electric quadrupole contains second derivative of ρ_c

  $\rho_q = -(p^2/m^2)\cos(2pr)$

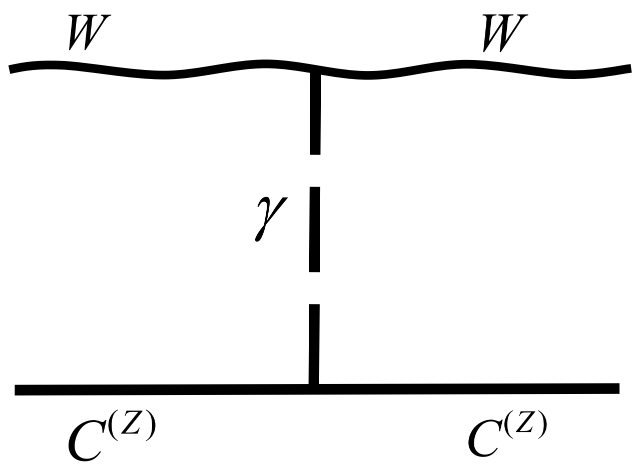
  High momentum p – quadropole density dominates

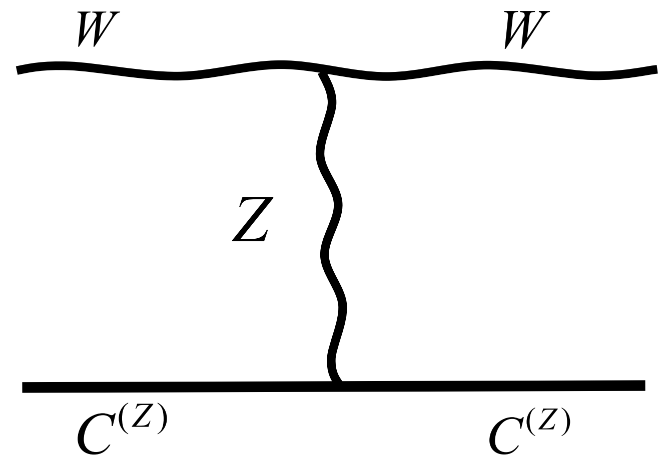
  Near Coulomb center - high momentum

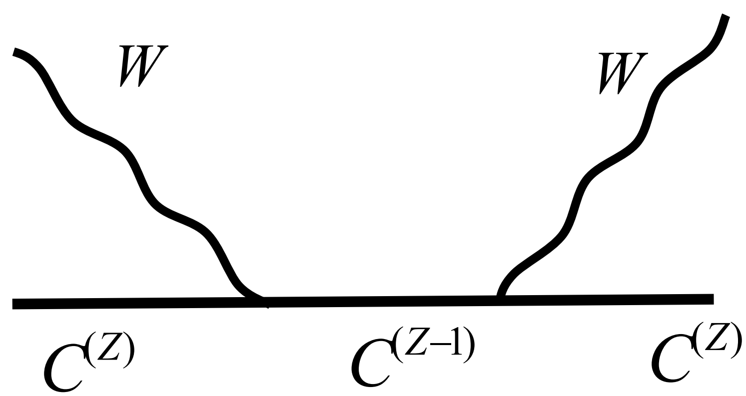
Diagrams



γ - exchange, Z - exchange, W - scattering







- **Introduction – brief history of Coulomb problem:**
  - **Nonrelativistic case**
  - **Relativistic cases S=0, S=1/2, S=1.**
  - **Mystery; the W-boson (S=1) falls on the Coulomb center (1940 !)**
- **Solution of the mystery:**
  - **Vacuum polarization**
  - **Repulsion at the origin**
- **Conclusions:**
  - **Coulomb problem for S=1 is well formulated,**
  - **Renormalizability of the Standard Model is reassured**

Nonrelativistic case

$$E\psi = H\psi$$

$$H = -\frac{1}{2m}\Delta - \frac{Z\alpha}{r}$$

$$E_n = -\frac{mZ^2\alpha^2}{n^2}, \;\; n = 1,2...$$

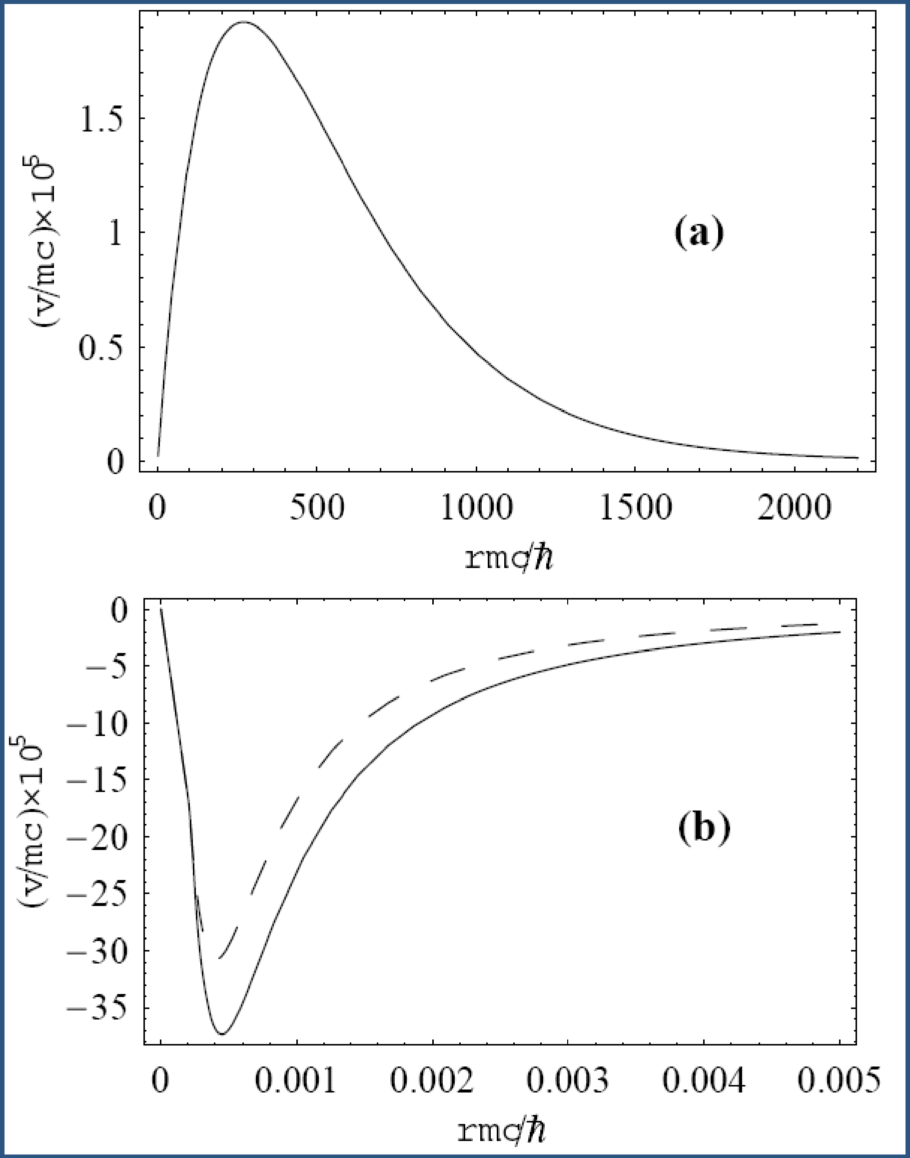
Wave functions, density of charge

$$j = 0 \quad \Rightarrow \quad \mathbf{W} = \mathrm{v}(r)\,\mathbf{n}$$

$$\mathrm{v}(r) = \frac{a}{m\,r^2}\exp\left(-\frac{Z\alpha\left(\alpha\beta\right)^{1/2}}{m\,r}\right), \quad r \to 0$$

$$\rho_W(r) = -\frac{4a^2 Z\alpha\, e}{m\,r^5}\exp\left(-\frac{Z\alpha\left(\alpha\beta\right)^{1/2}}{m\,r}\right), \quad r \to 0$$

Wave function , *j=0*



Zommerfeld formula

$$\varepsilon = m\left(1+\frac{(Z\alpha)^2}{(\gamma+n-j-1/2)^2}\right)^{-1/2}, \quad j = 0,1\ldots$$

$$\gamma = \left((j+1/2)^2 - Z^2\alpha^2\right)^{1/2}$$

$$i\gamma_\mu \nabla^\mu \psi = m\psi$$

$$-\nabla_\mu \nabla^\mu \psi + \frac{ie}{2}\sigma_{\mu\nu} F^{\mu\nu}\psi = m^2\psi$$

S=1/2, Dirac equation

Zommerfeld formula

$$\varepsilon = m\left(1+\frac{(Z\alpha)^2}{(\gamma+n-j-1/2)^2}\right)^{-1/2}, \quad j=\frac{1}{2},\frac{3}{2}\ldots$$

$$\gamma = \left((j+1/2)^2 - Z^2\alpha^2\right)^{1/2}$$

$$L_W = -\frac{1}{2}\left(\nabla_\mu W_\nu - \nabla_\nu W_\mu\right)^* \left(\nabla_\mu W_\nu - \nabla_\nu W_\mu\right)$$

$$+ m^2 W_\mu^* W^\mu + i e F^{\mu\nu} W_\mu^* W_\nu$$

Maxwell (1864 ?)
Proca (1936)
Corben-Schwinger (1949)

A. Proca, Compt.Rend. **202**, 1490 (1936).
H. F. W. Massey and H. C. Corben, Proc.Camb.Phi.Soc. **35**, 463 (1939).
J. R. Oppenheimer, H. Snyder and R. Serber, Phys.Rev. **57**, 75 (1940).
I. E. Tamm. Phys. Rev. **58**, 952 (1940); Doklady USSR Academy of Science **8-9**, 551 (1940).
H. C. Corben and J. Schwinger, Phys.Rev. **58**, 953 (1940).
J. Schwinger Rev. Mod. Phys. **36**, 609 (1964).

S=1, equation of motion

$$L_W = -\frac{1}{2}\left(\nabla_\mu W_\nu - \nabla_\nu W_\mu\right)^* \left(\nabla_\mu W_\nu - \nabla_\nu W_\mu\right)$$

$$+ m^2 W_\mu^* W^\mu + i\,e\,F^{\mu\nu} W_\mu^* W_\nu$$

$$\left(\nabla^2 + m^2\right) W^\mu + 2ie\,F^{\mu\nu}\,W_\nu - \nabla^\mu \nabla^\nu\,W_\nu = 0$$

$$m^2 \nabla_\mu W^\mu + ie\,j_\mu W^\mu = 0 \qquad j^\mu = \partial_\nu F^{\nu\mu}$$

S=1, the role of external current

$$\left(\nabla^2 + m^2\right) W^\mu + 2ie\, F^{\mu\nu}\, W_\nu - \nabla^\mu \nabla^\nu\, W_\nu = 0$$

$$m^2 \nabla_\mu W^\mu + ie\, j_\mu W^\mu = 0$$

$$\left(\nabla^2 + m^2\right) W^\mu + 2ieF^{\mu\nu} W_\nu + \frac{ie}{m^2} \nabla^\mu (j_\nu W^\nu) = 0$$

S=1, equation in the vacuum

If $\quad j^{\mu} = 0 \quad$ then

$$\left(\nabla^2 + m^2\right) W^{\mu} + 2 i e F^{\mu\nu} W_{\nu} = 0$$

Zommerfeld formula

$$\varepsilon = m\left(1+\frac{(Z\alpha)^2}{(\gamma+n-j-1/2)^2}\right)^{-1/2}, \quad j = 0,1,\ldots$$

$$\gamma = \left((j+1/2)^2 - Z^2\alpha^2\right)^{1/2}$$

Solution for the hedgehog

$$-\kappa^2\phi = \left(-\frac{d^2}{dx^2} - \frac{2(Z\alpha)^2}{x} - \frac{(Z\alpha)^2}{x^2} + \frac{2}{(x+1)^2} \right)\phi$$

$$\phi = \left(\frac{d}{dx} + (\gamma + 1/2)\frac{x+1}{x} - \frac{1}{x+1} \right)\tilde{\phi}$$

$$-\kappa^2\,\tilde{\phi} = \left[-\frac{d^2}{dx^2} - \frac{2(Z\alpha)^2}{x} + \left(\gamma + \frac{3}{2}\right)\left(\gamma + \frac{1}{2}\right)\frac{1}{x^2} \right]\tilde{\phi}$$

What we gonna do ?

$$\left(\nabla^2 + m^2\right)W^{\mu} + 2ieF^{\mu\nu}W_{\nu} + \frac{ie}{m^2}\nabla^{\mu}\left(j_{\nu}W^{\nu}\right) = 0$$

$$\nabla_{\mu}W^{\mu} + \frac{ie}{m^2}j_{\mu}W^{\mu} = 0$$

Υ – potential

$$\nabla_{\mu} W^{\mu} + \frac{i\,e}{m^{2}} j_{\mu} W^{\mu} = 0$$

$$\nabla_{\mu} = \partial_{\mu} + ieA_{\mu}$$

$$eA_{\mu} \rightarrow eA'_{\mu} = eA_{\mu} + \Upsilon_{\mu}$$

$$\Upsilon_{\mu} = \frac{e}{m^{2}} j_{\mu}$$

Υ – potential

in spherically symmetrical, static case

$$eA_\mu \to eA'_\mu = eA_\mu + \Upsilon_\mu$$

$$\Upsilon_\mu = \frac{e}{m^2} j_\mu$$

$$U \to U' = U + \Upsilon$$

$$\Upsilon = \frac{e\rho}{m^2} = -\frac{1}{m^2}\Delta U$$

Υ – potential is large at small distances

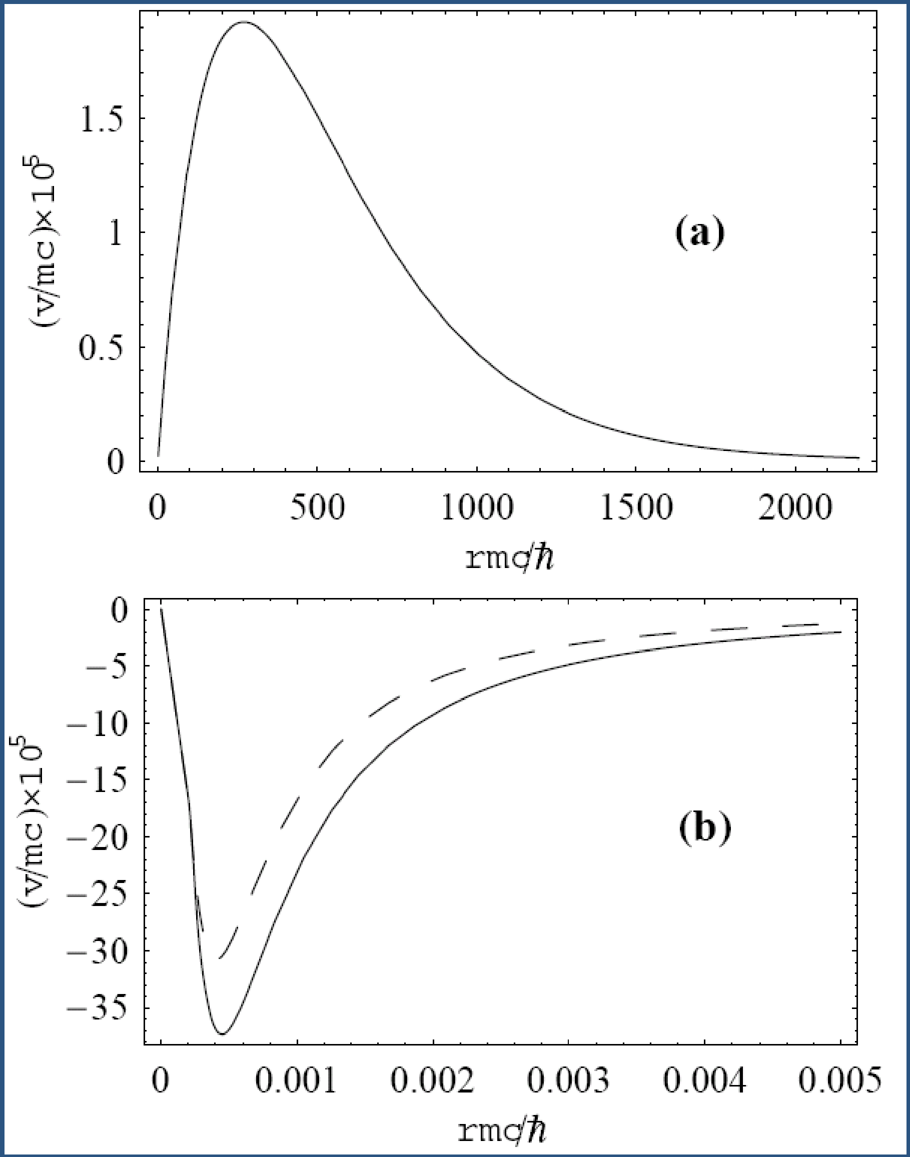
$$U \to U' = U + \Upsilon$$

$$\Upsilon = \frac{e\rho}{m^2} = -\frac{1}{m^2}\Delta U$$

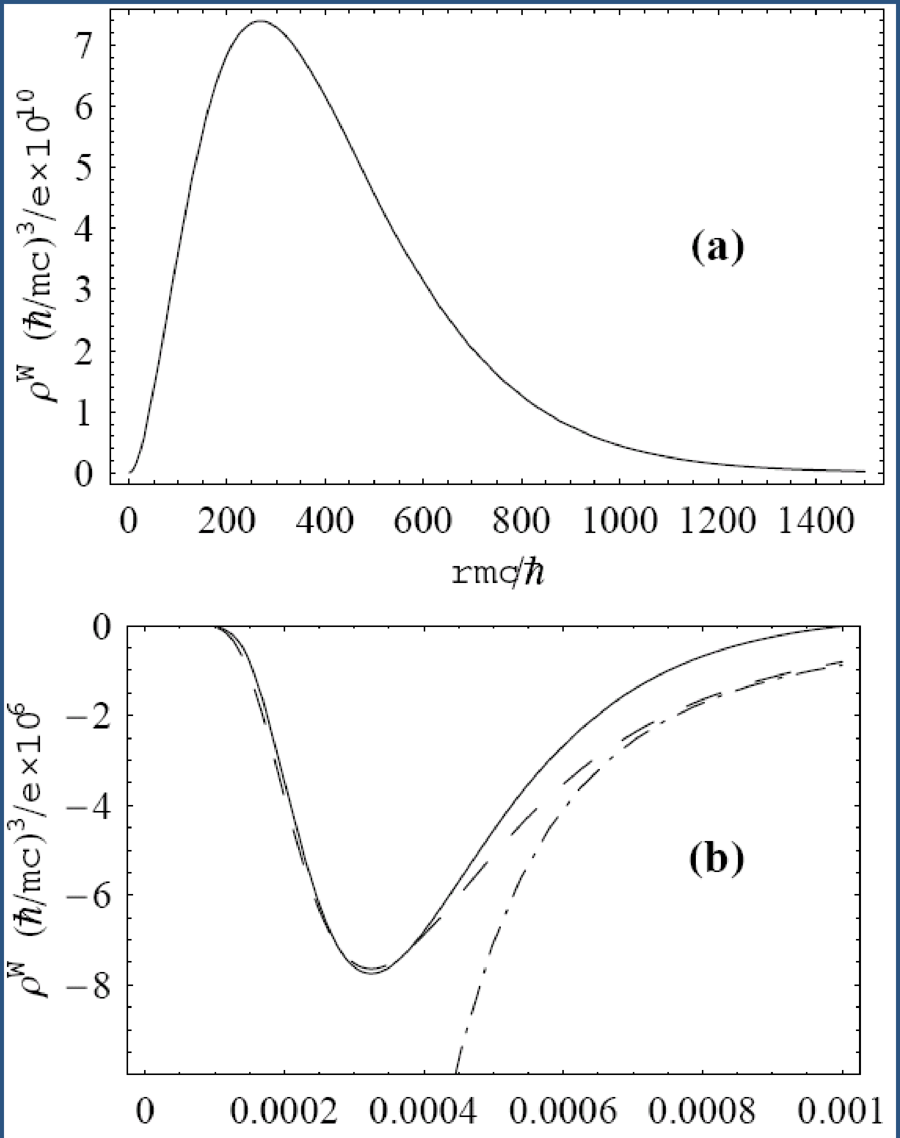
$$U \sim r^{\beta}$$

$$\frac{\Upsilon}{U} \sim \frac{1}{m^2 r^2} \gg 1 \quad \text{when} \quad mr \ll 1$$

Wave function , *j=0*



Charge density , *j=0*



Publications

- **hep-th/0512140**
  - Title: **Coulomb problem for vector bosons**
    Authors: **M.Yu.Kuchiev**, **V.V.Flambaum**
    Published: Phys.Rev.D
- **hep-th/0511149**
  - Title: **Coulomb problem for vector bosons versus Standard Model**
    Authors: **M. Yu. Kuchiev**, **V. V. Flambaum**
    Published: Modern Physics Letters

Conclusions

- Coulomb problem for S=1 is well defined for the first time:
  - Consider the Corben-Schwinger equation (which follows from the Standard Model)
  - Take into account the vacuum polarization
- The vacuum polarization, which is a *loop correction,* is found to play a defining role in the problem.

- Acknowledgments:

- The support of the Australian Research Council is appreciated