N$\rightarrow\Delta$ charge quadrupole form factor and proton structure effects in atomic hydrogen

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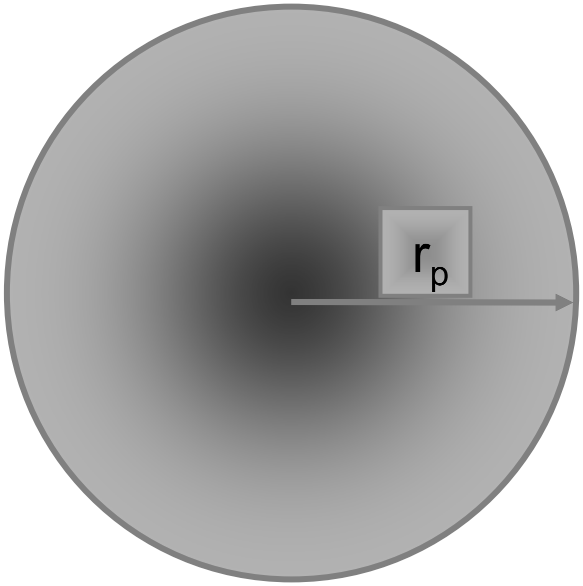
1. Introduction
2. Electromagnetic N$\rightarrow\Delta$(1232) excitation
3. Implications for hydrogen atom hyperfine splitting
4. Summary

PSAS 2008, Windsor, Canada, 23 July 2008

1. Introduction

Size of proton

finite radial extension of proton charge distribution



$$\rho = \rho(r)$$

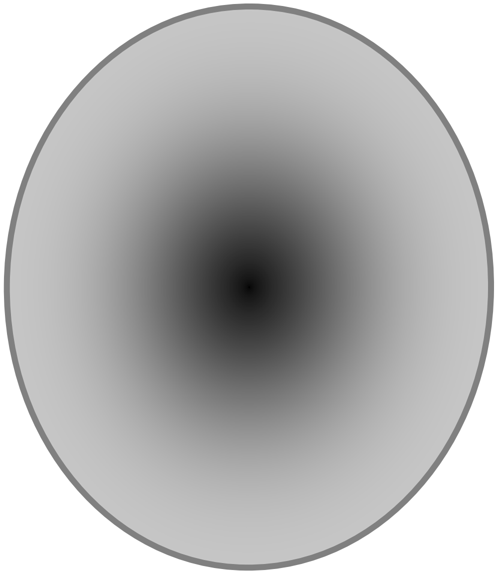
Measurement of proton charge radius

$$r_p(\mathrm{exp}) = 0.862(12)\ \mathrm{fm}$$

Simon et al., Z. Naturf. 35a (1980) 1

Nucleon shape

nonspherical
charge distribution of proton



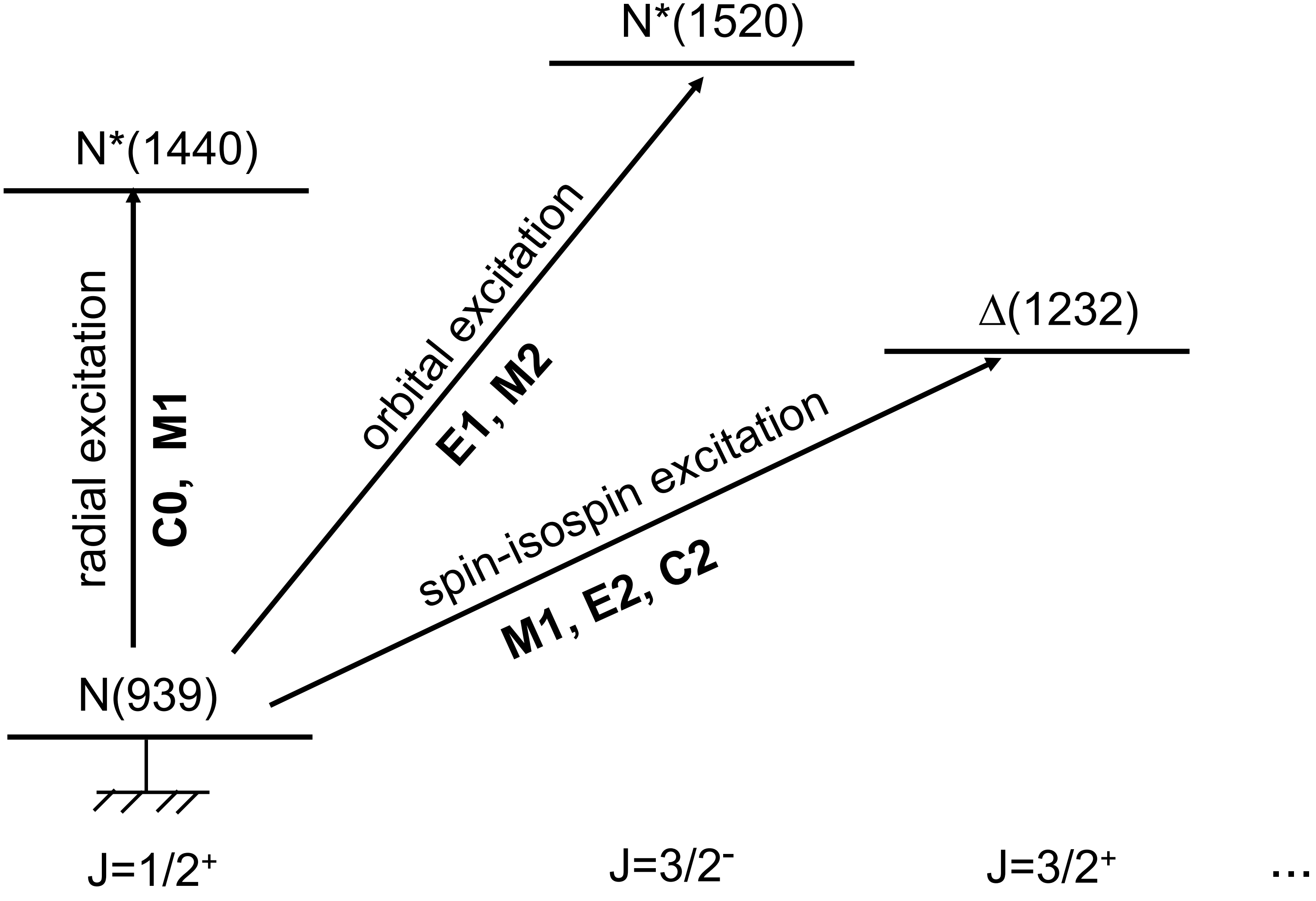
$$\rho = \rho(\vec{r}) = \rho(r, \theta, \varphi)$$

Extraction of N→Δ transition quadrupole moment from data

$$Q_{N\rightarrow\Delta}\,(\mathrm{exp}) = -0.0846(33)\ \mathrm{fm}^2$$

Tiator et al., EPJ A17 (2003) 357

Nucleon excitation spectrum



N*(1520)

N*(1440)

radial excitation

C0, M1

orbital excitation

E1, M2

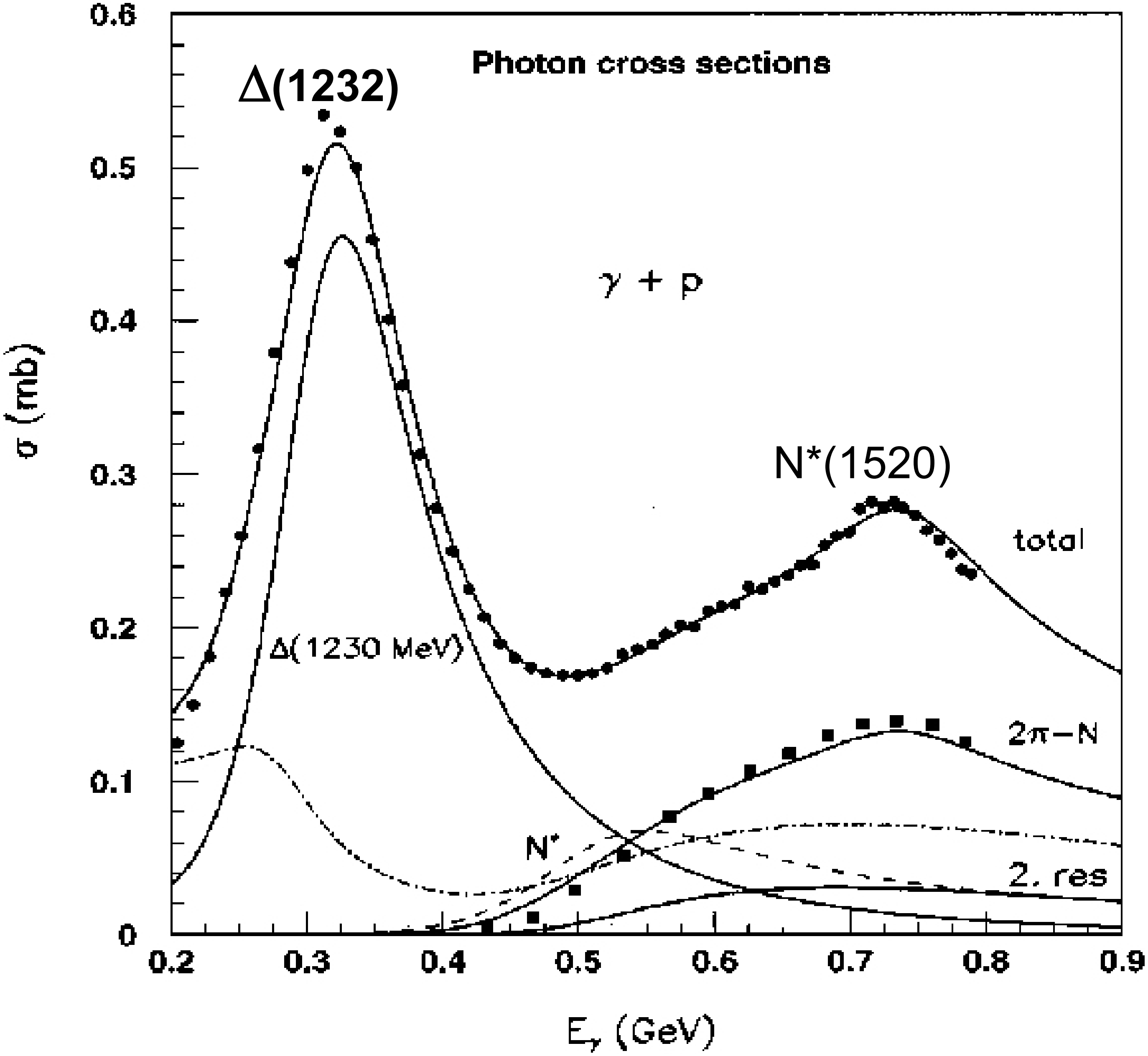
Δ(1232)

spin-isospin excitation

M1, E2, C2

N(939)

$J=1/2^+$ $J=3/2^-$ $J=3/2^+$ …


Photon cross sections
Δ(1232)
γ + p
N*(1520)
total
Δ(1230 MeV)
2π−N
N*
2. res
σ (mb)
E$_γ$ (GeV)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9

Properties of the nucleon

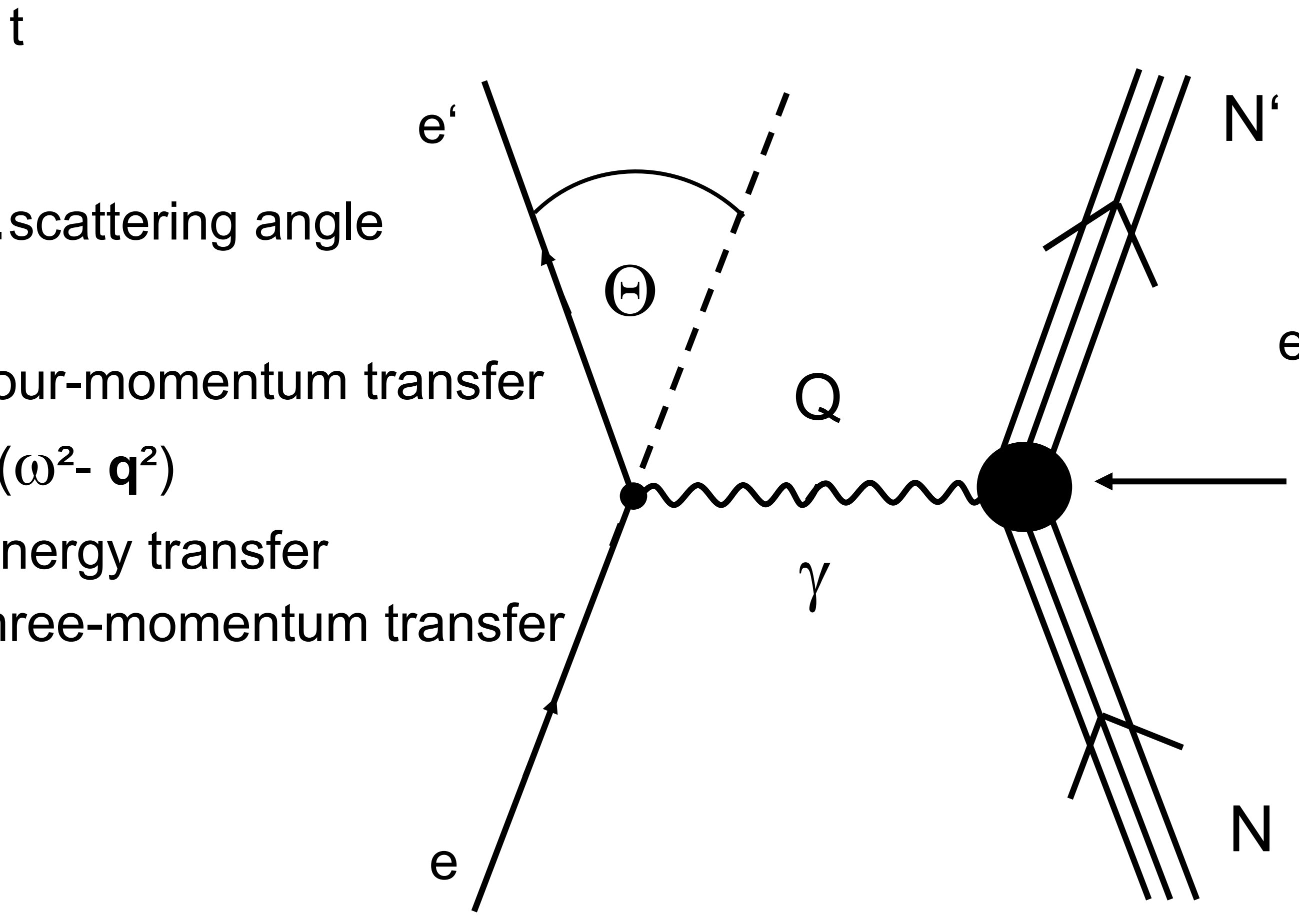
- finite spatial extension **(size**)
- nonspherical charge distribution (**shape**)
- excited states (**spectrum**)

size **shape** **spectrum**

How are these properties related?

2. Electromagnetic N→Δ excitation

Elastic electron-nucleon scattering



Θ…scattering angle

Q... four-momentum transfer

$Q^2 = -(\omega^2 - \mathbf{q}^2)$

ω…energy transfer

q... three-momentum transfer

elastic form factors

$$G^N_{C,M}(Q^2)$$

$G^N_C(Q^2)$ charge

$G^N_M(Q^2)$ magnetic

e... electron

γ... photon

N... nucleon (p,n)

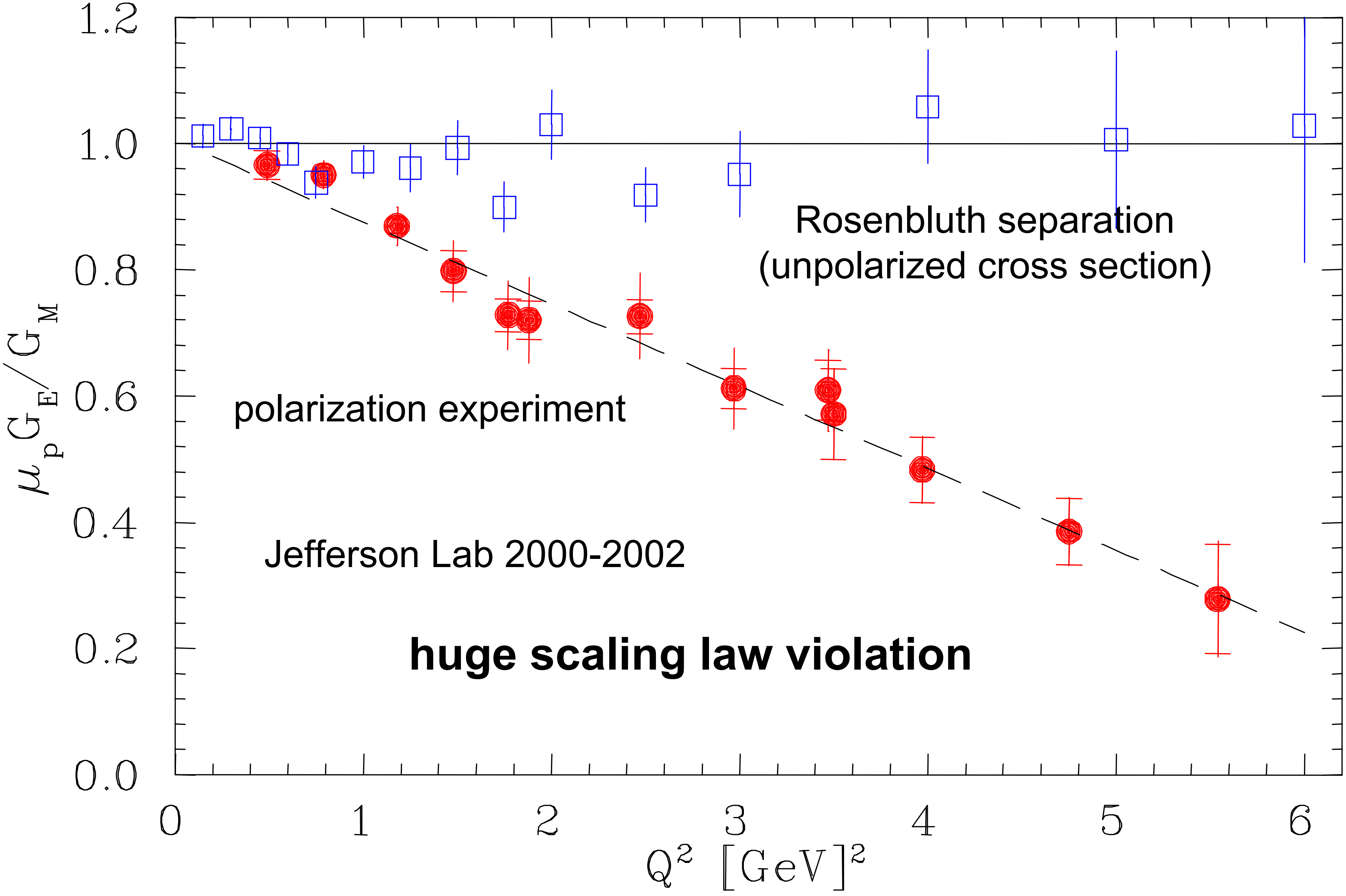
Importance of elastic form factors

Fourier transforms of charge and current distributions ρ(**r**)

e.g. $$G_C^p(q^2) = \rho(q) = \int d^3r \exp(i\vec{q}\cdot\vec{r})\,\rho(\vec{r})$$

inverse transform $$\rho(r) = \frac{1}{(2\pi)^3}\int d^3q \exp(-i\vec{q}\cdot\vec{r})\,\rho(q)$$

Polarized electron scattering



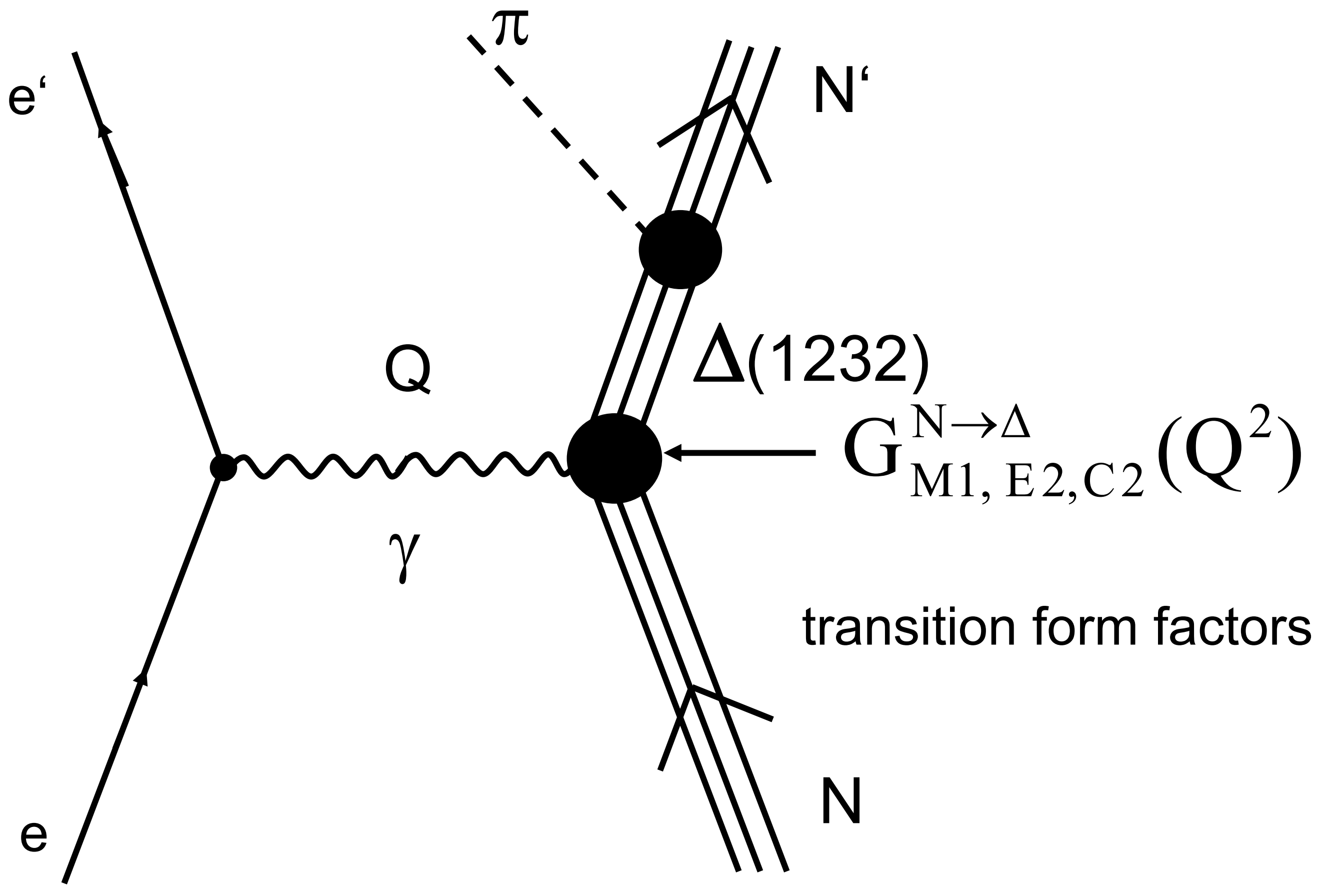
Rosenbluth separation
(unpolarized cross section)

polarization experiment

Jefferson Lab 2000-2002

huge scaling law violation

Inelastic electron-nucleon scattering



Transition form factors provide additional information on nucleon ground state structure

Normalization of inelastic form factors

$$G_{M1}^{p\to\Delta^+}(0) = \mu^{p\to\Delta^+}$$ transition magnetic moment

$$G_{C2}^{p\to\Delta^+}(0) = Q^{p\to\Delta^+}$$ transition quadrupole moment

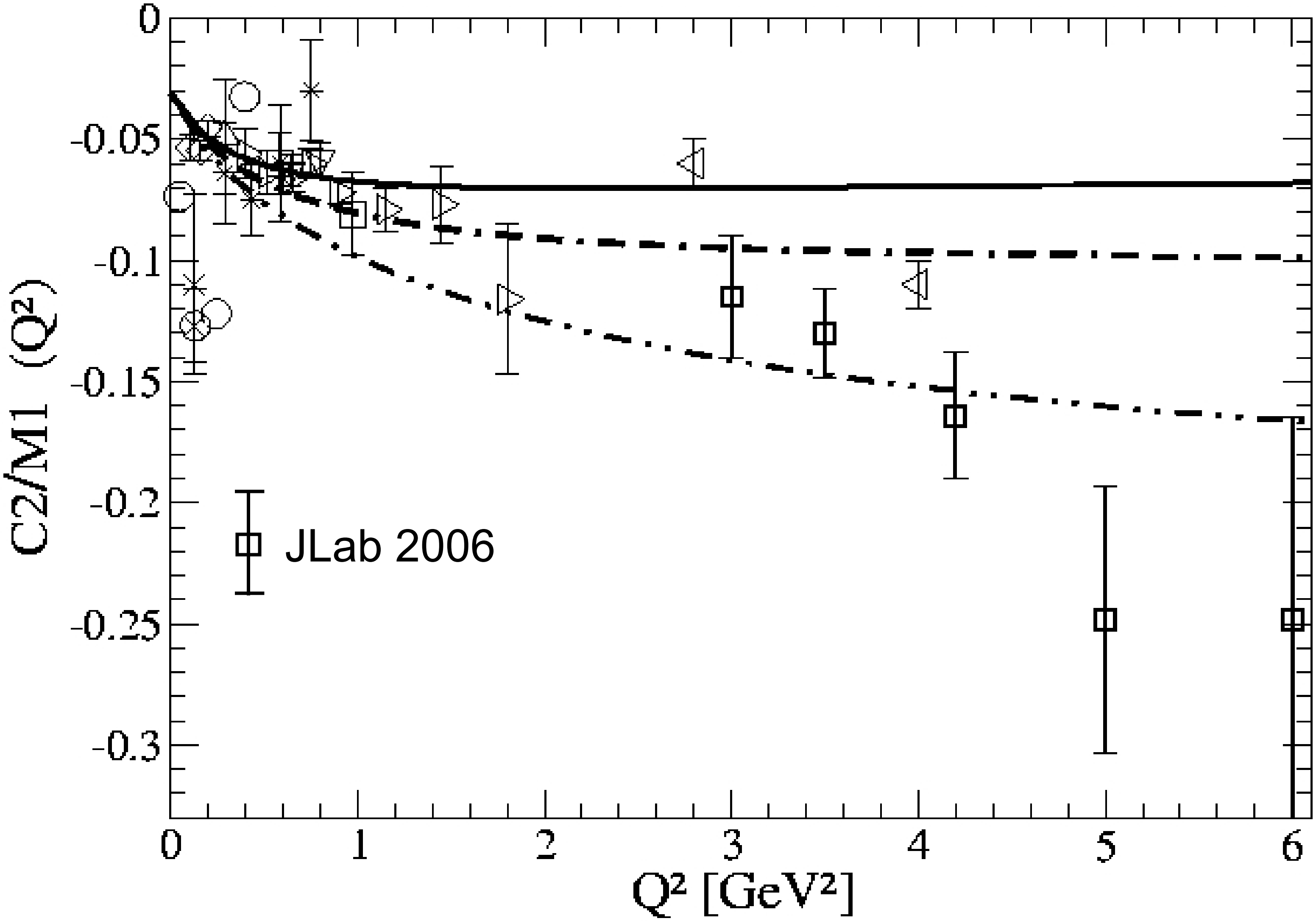
usual definition of multipole moments
as in classical electrodynamics

Experimentally, C2/M1 ratio can be determined.

Definition of C2/M1 ratio
in terms of N→Δ transition form factors:

$$\frac{C2}{M1}(Q^2) = \frac{|\vec{q}|\,M_N}{6}\,\frac{G_{C2}^{p\to\Delta^+}(Q^2)}{G_{M1}^{p\to\Delta^+}(Q^2)}$$

Electro-pionproduction: e+N→e‘+N+π in Delta(1232) resonance region



N and N→Δ form factor relations

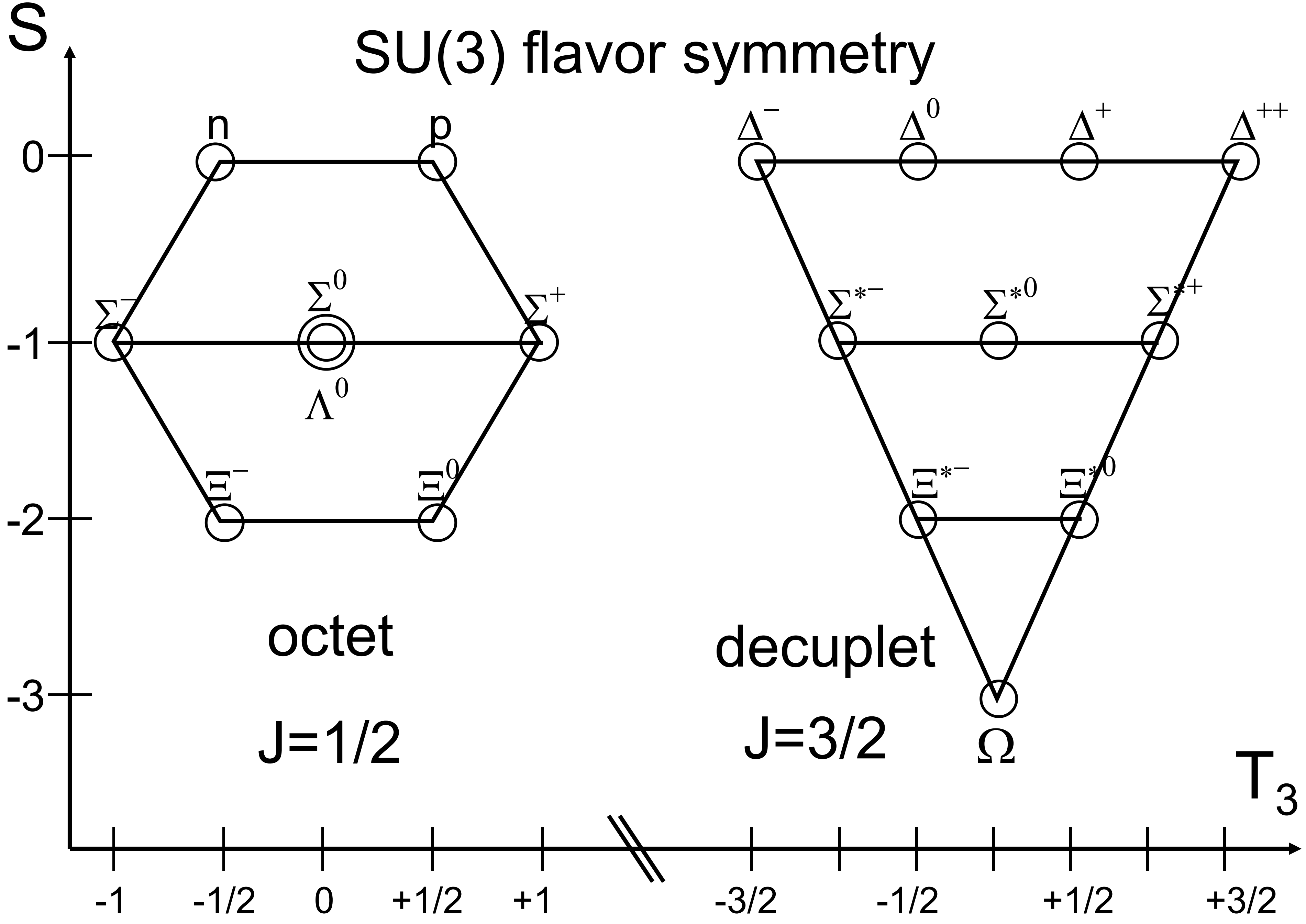
Strong interaction symmetries

Strong interactions
are
approximately invariant
under

- **SU(2) isospin,**
- **SU(3) flavor,**
- **SU(6) spin-flavor**

symmetry transformations.

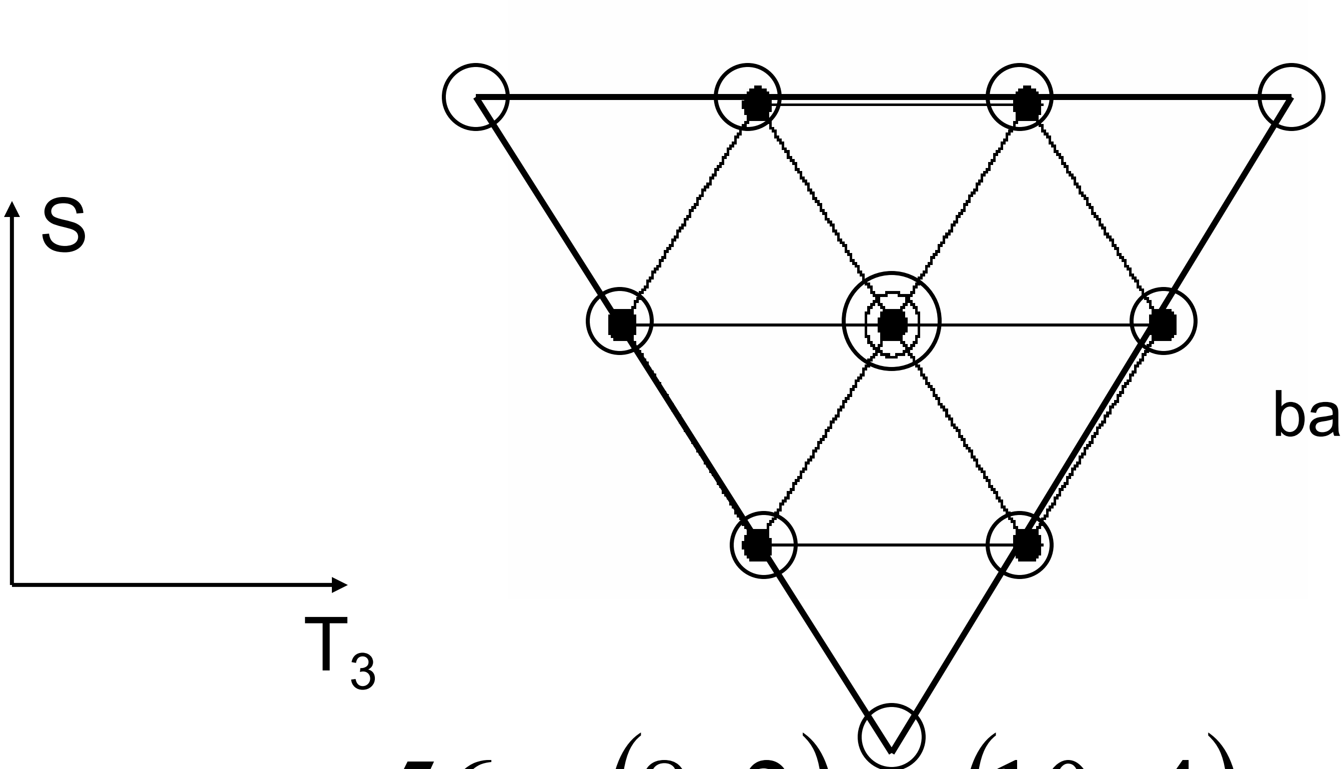
SU(3) flavor symmetry



SU(6) spin-flavor symmetry

combines SU(3) multiplets
with
different spin and flavor
to
SU(6) spin-flavor supermultiplets.

SU(6) spin-flavor supermultiplet



baryon ground state
supermultiplet

$$56=\left(\underset{\substack{\uparrow\\ \text{flavor}}}{8},\ \underset{\substack{\uparrow\\ \text{spin}}}{2}\right)+\left(\underset{\substack{\uparrow\\ \text{flavor}}}{10},\ \underset{\substack{\uparrow\\ \text{spin}}}{4}\right)$$

SU(6) mass formula

$$\mathbf{M} = \mathrm{M}_0\ \mathbf{1} + \mathrm{M}_1\ \mathbf{Y} + \mathrm{M}_2 \left(\mathbf{T}(\mathbf{T}+\mathbf{1}) - \frac{\mathbf{Y}^2}{4} \right) + \mathrm{M}_3\ \underset{\uparrow}{\mathbf{J}(\mathbf{J}+\mathbf{1})}$$

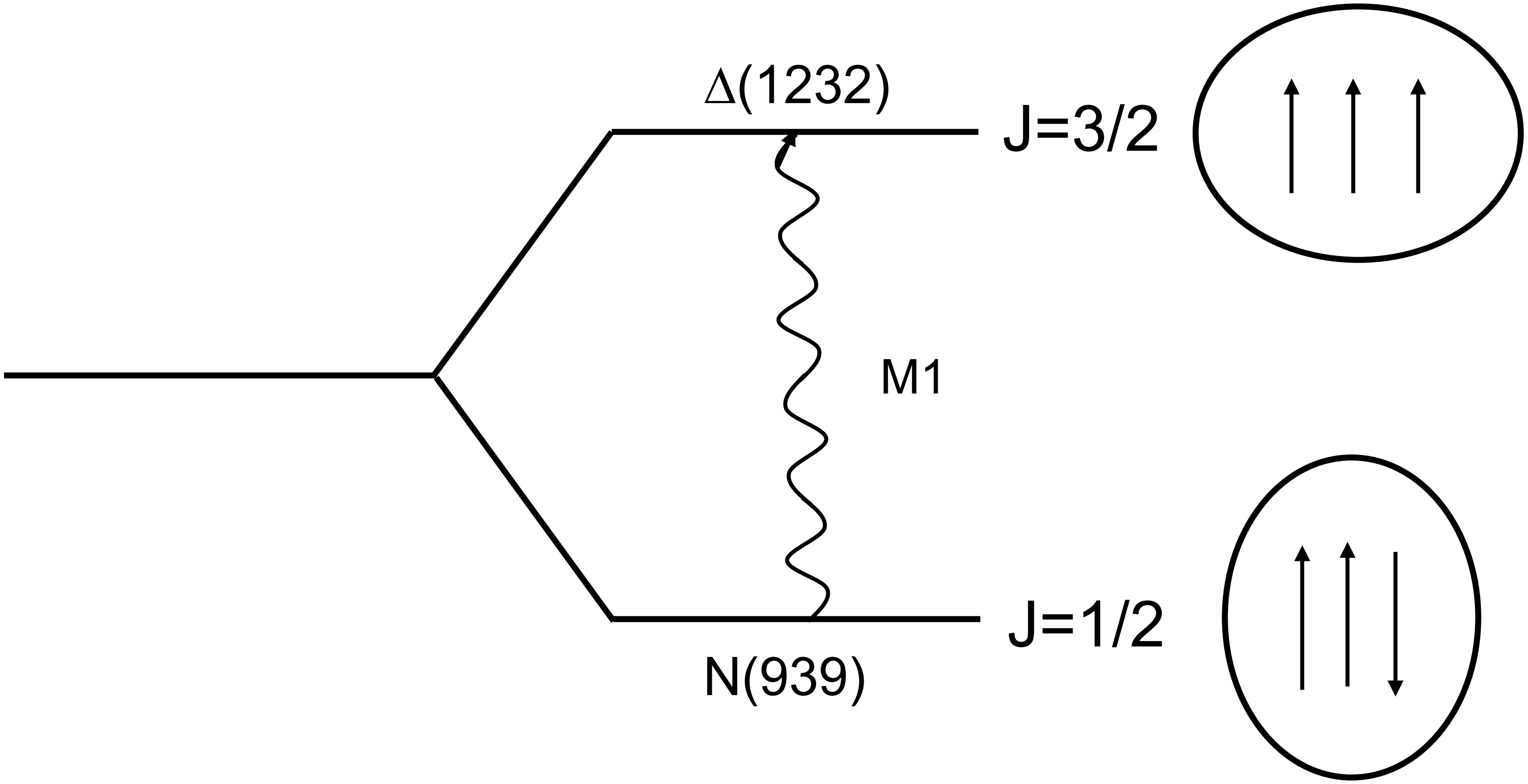
SU(6) symmetry breaking term

$$\sim \vec{\sigma}_i \cdot \vec{\sigma}_j$$

Relations between octet and decuplet baryon masses

e.g. $$\mathrm{M}_{\Delta^+} - \mathrm{M}_{\Delta^0} = \mathrm{M}_p - \mathrm{M}_n$$

Delta-Nucleon mass splitting



two-body spin-spin
interaction
between quarks
is more repulsive
in Δ(1232)

$$M_{\Delta} - M_{N} \sim \left\langle J = \frac{3}{2} \middle| \sum_{i<j}^{3} \sigma_i \cdot \sigma_j \middle| J = \frac{3}{2} \right\rangle - \left\langle J = \frac{1}{2} \middle| \sum_{i<j}^{3} \sigma_i \cdot \sigma_j \middle| J = \frac{1}{2} \right\rangle \sim 293\,\mathrm{MeV}$$

Multipole expansion in spin-flavor space

two-body charge density $\rho_{[2]}$

$$\rho_{[2]} = -B\sum_{i\neq j}^{3} e_i\left[2\,\underbrace{\vec{\sigma}_i\cdot\vec{\sigma}_j}_{\text{spin scalar}} - \underbrace{\left(3\,\sigma_{iz}\,\sigma_{jz} - \vec{\sigma}_i\cdot\vec{\sigma}_j\right)}_{\text{spin tensor}}\right]$$

most general structure of $\rho_{[2]}$ in spin-flavor space

prefactors in spin scalar (+2) and spin tensor (-1) determined by group algebra

$$\rho_{[2]} = -B \sum_{i \neq j}^{3} e_i \left[2 \underbrace{\vec{\sigma}_i \cdot \vec{\sigma}_j}_{\text{spin scalar}} - \underbrace{\left(3\, \sigma_{iz}\, \sigma_{jz} - \vec{\sigma}_i \cdot \vec{\sigma}_j \right)}_{\text{spin tensor}} \right]$$

neutron charge radius $$r_n^{\,2} = \left\langle 56_n \middle| \rho_{[2]} \middle| 56_n \right\rangle = 4B$$

N→Δ transition
quadrupole moment $$Q_{p\to\Delta^+} = \left\langle 56_{\Delta^+} \middle| \rho_{[2]} \middle| 56_p \right\rangle = 2\sqrt{2}\,B$$

$$\boxed{Q_{p\to\Delta^+} = \frac{1}{\sqrt{2}}\, r_n^2}$$

Buchmann et al., PRC 55 (1997) 448

N $\rightarrow\Delta$ quadrupole moment

Extraction of p$\rightarrow$ Δ^+(1232) transition quadrupole moment from electron-proton and photon-proton scattering data

experminent

$$Q_{p\rightarrow\Delta^+(1232)}(\mathrm{exp}) = -0.108(9)\ \mathrm{fm}^2$$

Blanpied et al., PRC 64 (2001) 025203

$$Q_{p\rightarrow\Delta^+(1232)}(\mathrm{exp}) = -0.0846(33)\ \mathrm{fm}^2$$

Tiator et al., EPJ A17 (2003) 357

theory

$$Q_{p\rightarrow\Delta^+(1232)} = \frac{1}{\sqrt{2}}\ \underset{\uparrow}{r_n}{}^2 = -0.0821(20)\ \mathrm{fm}^2$$

Buchmann et al., PRC 55 (1997) 448

neutron charge radius

Relations between octet and decuplet electromagnetic form factors

$$G_{M1}^{p\to\Delta^+}(Q^2) = -\sqrt{2}\; G_M^n(Q^2)$$

$$\mu^{p\to\Delta^+} = -\sqrt{2}\; \mu^n$$

magnetic form factors
Beg, Lee, Pais, 1964

$$G_{C2}^{p\to\Delta^+}(Q^2) = -\frac{3\sqrt{2}}{Q^2}\; G_C^n(Q^2)$$

$$Q^{p\to\Delta^+} = \frac{1}{\sqrt{2}}\; r_n^2$$

charge form factors
Buchmann, 2000
Buchmann, Hernandez, Faessler, 1997

Definition of C2/M1 ratio

$$\frac{C2}{M1}(Q^2) = \frac{\left|\vec{q}\right| M_N}{6} \frac{G_{C2}^{p\to\Delta^+}(Q^2)}{G_{M1}^{p\to\Delta^+}(Q^2)}$$

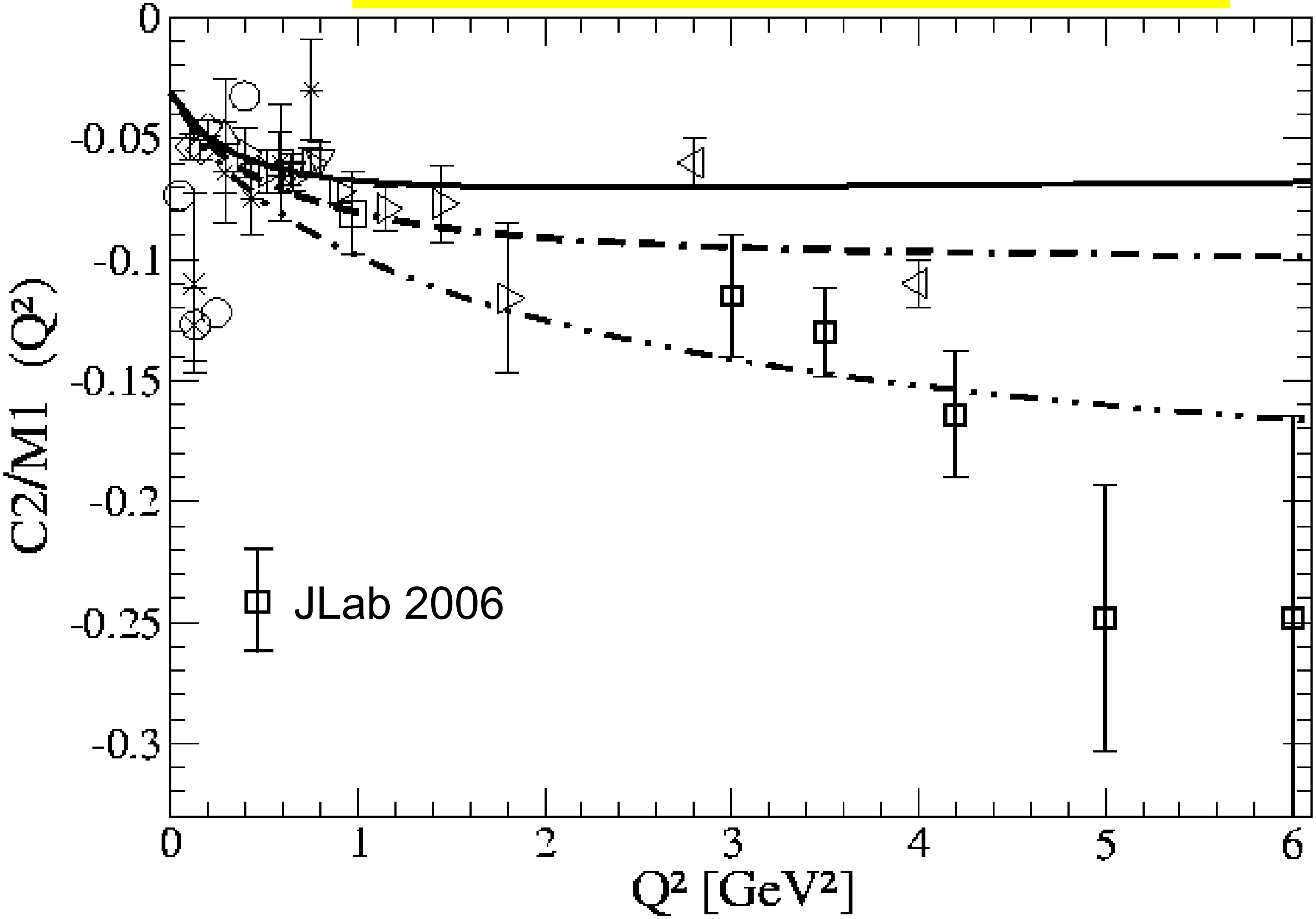
Insert form factor relations

$$\frac{C2}{M1}(Q^2) = \frac{\left|\vec{q}\right|}{Q} \frac{M_N}{2Q} \frac{G_C^n(Q^2)}{\underset{\uparrow}{G_M^n(Q^2)}}$$

C2/M1 expressed via neutron elastic form factors

A. J. Buchmann, Phys. Rev. Lett. 93 (2004) 212301

data: electro-pionproduction
curves: elastic neutron form factors



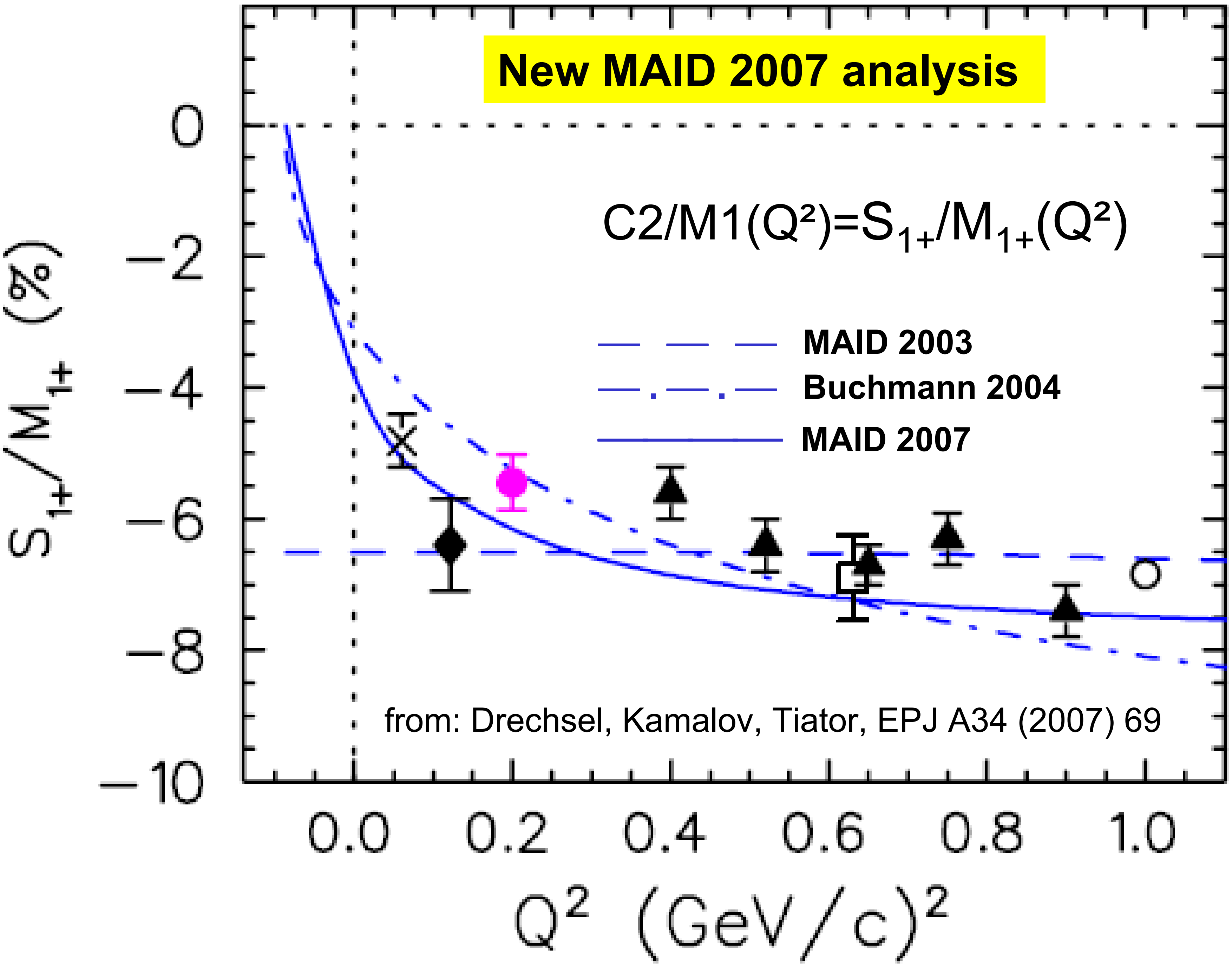
d=2.80

d=1.75

d=0.80

Maid 2007
reanalysis

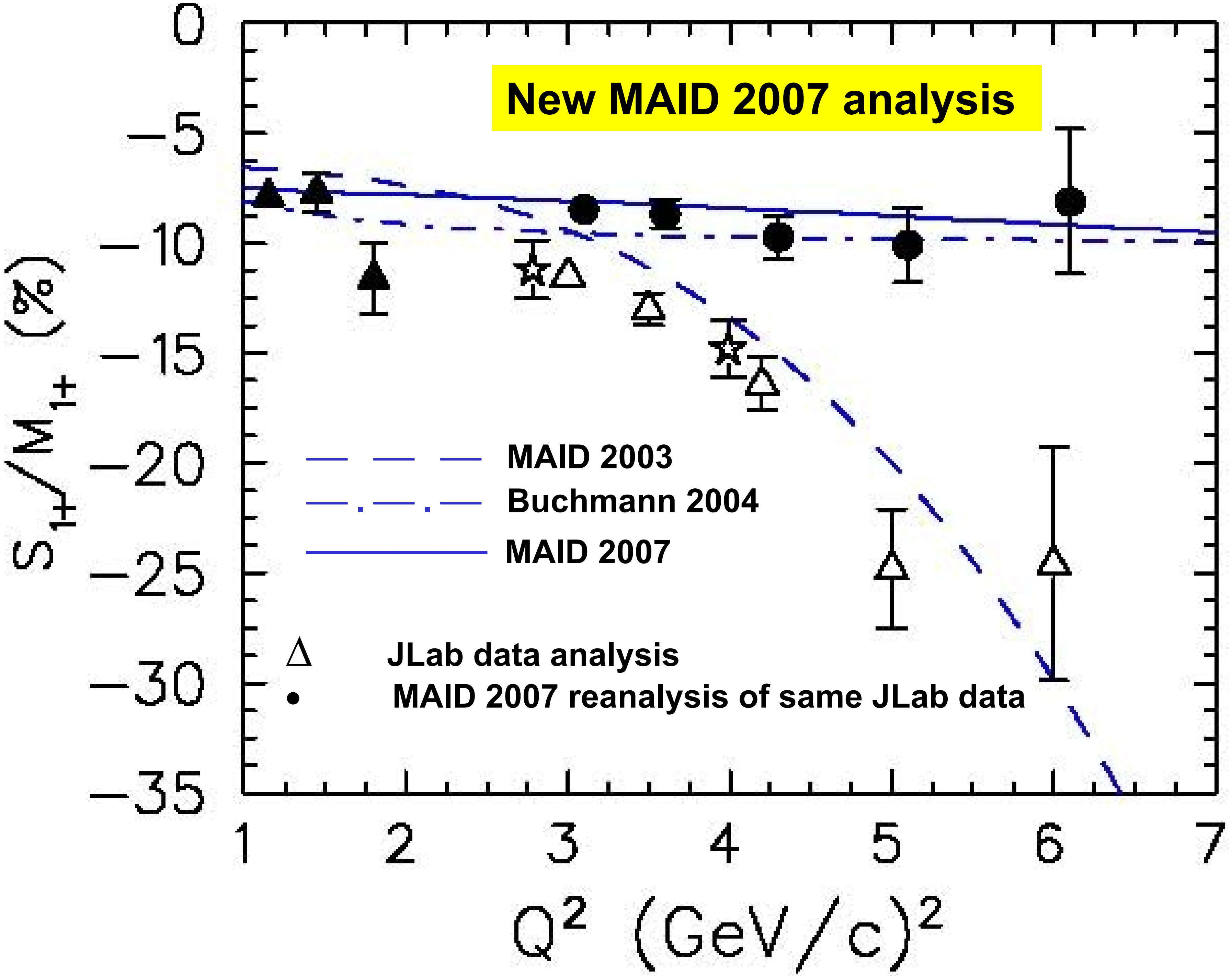
from: A.J. Buchmann, Phys. Rev. Lett. 93, 212301 (2004).



New MAID 2007 analysis

$$C2/M1(Q^2)=S_{1+}/M_{1+}(Q^2)$$

from: Drechsel, Kamalov, Tiator, EPJ A34 (2007) 69


New MAID 2007 analysis
0
−5
−10
−15
−20
−25
−30
−35
S_{1+}/M_{1+} (%)
1
2
3
4
5
6
7
Q^2 $(GeV/c)^2$
MAID 2003
Buchmann 2004
MAID 2007
Δ JLab data analysis
● MAID 2007 reanalysis of same JLab data

Intrinsic quadrupole form factor of nucleon

How can one interpret these results?

to learn something about the geometric shape

of the proton and Δ(1232), one has to determine their

intrinsic quadrupole moments Q_0

Definition of **intrinsic** quadrupole moment

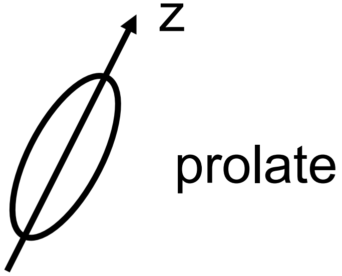
$$Q_0 = \int dr^3 \, \rho(\vec{r}) \, (3z^2 - r^2)$$

defined in body fixed frame

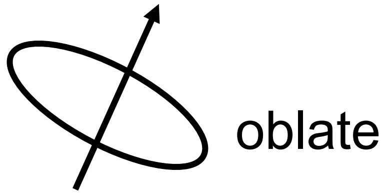
Intrinsic quadrupole moment of baryon B

$$Q_B = \int dr^3 \, \rho_B(\vec{r}) \; (3z^2 - r^2)$$

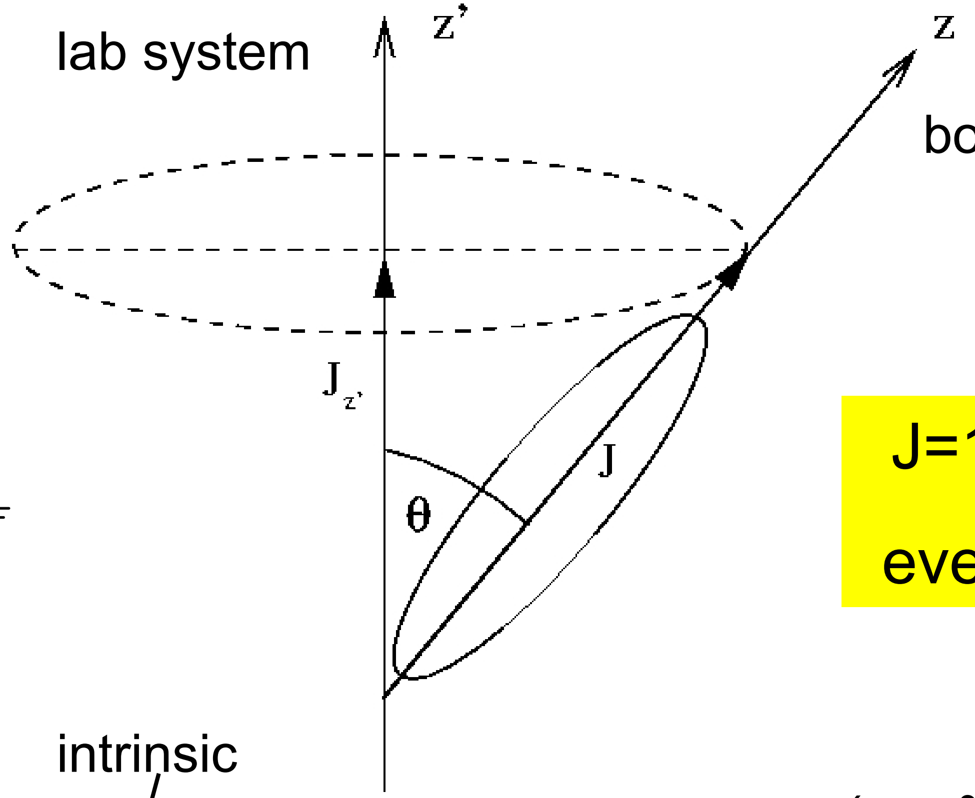
If ρ_B concentrated along z-axis, $3z^2$- term dominates → $Q_B > 0$



If ρ_B concentrated in x-y plane, r^2 -term dominates → $Q_B < 0$



Intrinsic (Q_0) vs. spectroscopic (Q) quadrupole moment



$$\cos\Theta = \frac{J_z}{\sqrt{J^2}}$$

J=1/2 → Q=0

even if $Q_0 \neq 0$.

spectroscopic

intrinsic

$$Q = P_2(\cos\Theta)\ Q_0 = \frac{1}{2}\left(3\cos^2\Theta - 1\right)Q_0 = \left(\frac{3J_z^2 - J(J+1)}{2J(J+1)}\right)Q_0$$

projection factor

Nucleon model calculations of Q_0

Calculation of Q_0 in three different nucleon models

- quark model
- pion-nucleon model
- collective model

All three models lead to qualitatively the same result for Q_0:

Neutron charge radius determines the sign und size of the **intrinsic** N und Δ quadrupole moments.

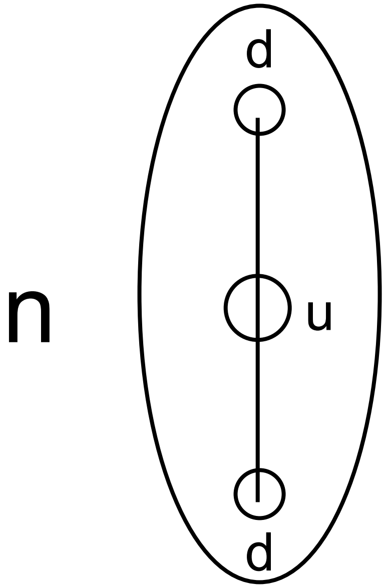
Buchmann and Henley, Phys. Rev. C63, 015202 (2001)

Intrinsic quadrupole moment Q_0 in quark model

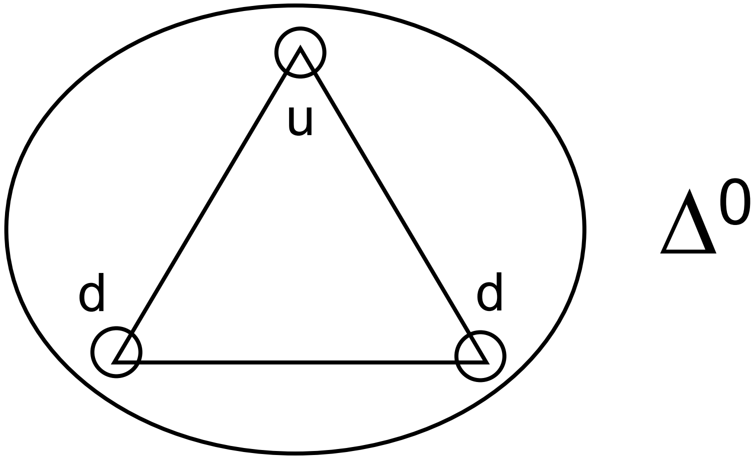
$$Q_0\,(N) = -r_n^2 > 0$$

$$Q_0\,(\Delta) = \; r_n^2 < 0$$

Buchmann and Henley,
Phys. Rev. C63, 015202 (2001)

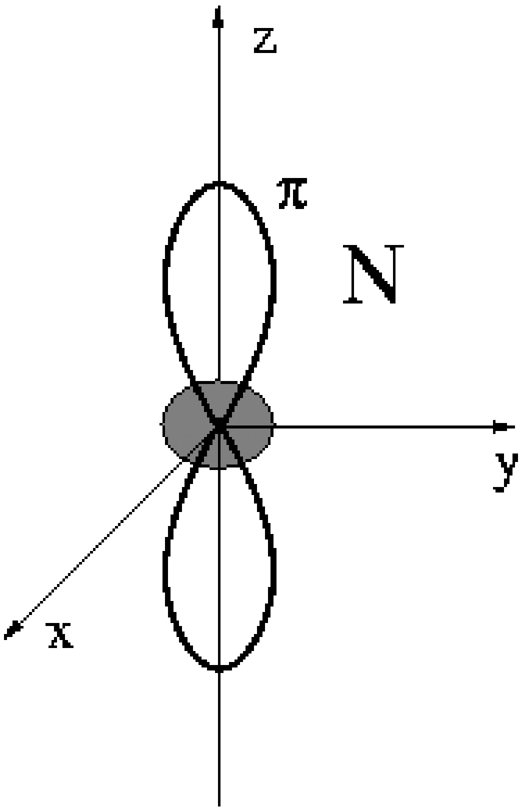


N(939) is **prolate**



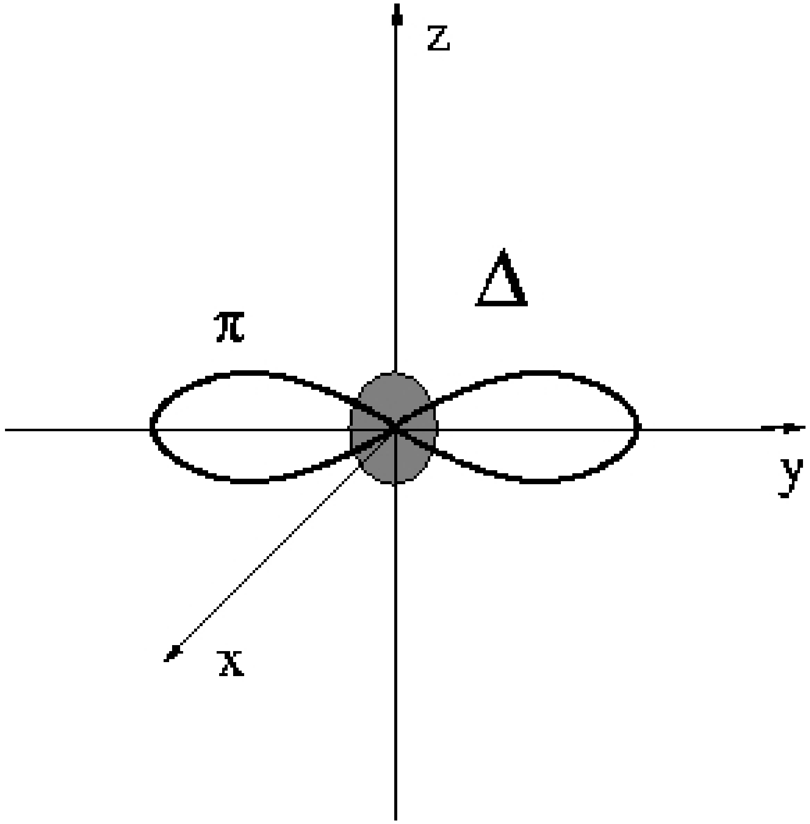
Δ(1232) is **oblate**.

Interpretation in pion-nucleon model



Q_0 (N) > 0

prolate



Q_0 (Δ) < 0

oblate

A. J. Buchmann and E. M. Henley, Phys. Rev. C63, 015202 (2001)

Intrinsic charge quadrupole form factor

There is now considerable evidence that the proton charge density $\rho^{p}(\vec{r})$ is not spherically symmetric

$$\rho^{p}(\vec{r}) = \rho^{p}(r, \theta, \varphi)$$

Expand $\rho^{p}(\vec{r})$ into multipoles

$$\rho^{p}(\vec{r}) = \underbrace{\rho_0(r)\ Y_0^0(\hat{r})}_{\text{monopole}} + \underbrace{\rho_2(r)\ Y_0^2(\hat{r})}_{\text{quadrupole}} + \cdots$$

How can one get information on $\rho_2(r)$?

Decomposition of nucleon charge form factors

$$G_C^p(Q^2) = \underbrace{G_0^p(Q^2)}_{\text{monopole}} - \frac{1}{6}\,Q^2\,\underbrace{G_2^p(Q^2)}_{\text{quadrupole}}$$

$$G_C^n(Q^2) = G_0^n(Q^2) + \frac{1}{6}\,Q^2\,G_2^n(Q^2) \qquad (1)$$

ansatz for intrinsic quadrupole form factor

$$G_2^p(Q^2) = G_2^n(Q^2) = -\sqrt{2}\;G_{C2}^{p\to\Delta^+}(Q^2) = \frac{6}{Q^2}\,G_C^n(Q^2) \qquad (2)$$

normalization monopole

$$G_0^p(0) = 1$$

normalization quadrupole

$$G_2^p(0) = G_2^n(0) = Q_0^p = -r_n^2$$

Decomposition of nucleon charge form factors

Using ansatz in Eq.(2) we get

$$G_C^p(Q^2) = \underbrace{G_0^p(Q^2)}_{\text{spherical}} - \underbrace{G_C^n(Q^2)}_{\text{deformed}} = G_C^{IS}(Q^2) - G_C^n(Q^2) \quad (3)$$

$$G_C^n(Q^2) = \frac{1}{6} \quad Q^2 \quad \underbrace{G_2^n(Q^2)}_{\text{intrinsic quadrupole}} \qquad (4)$$

- spherical part in $G_C^p(Q^2)$ is given by isoscalar charge form factor
- spherical part in $G_C^n(Q^2)$ is zero
- deformation part is given by neutron charge form factor

Proton elastic form factor ratio $\mu_p \frac{G_C^p(Q^2)}{G_M^p(Q^2)}$

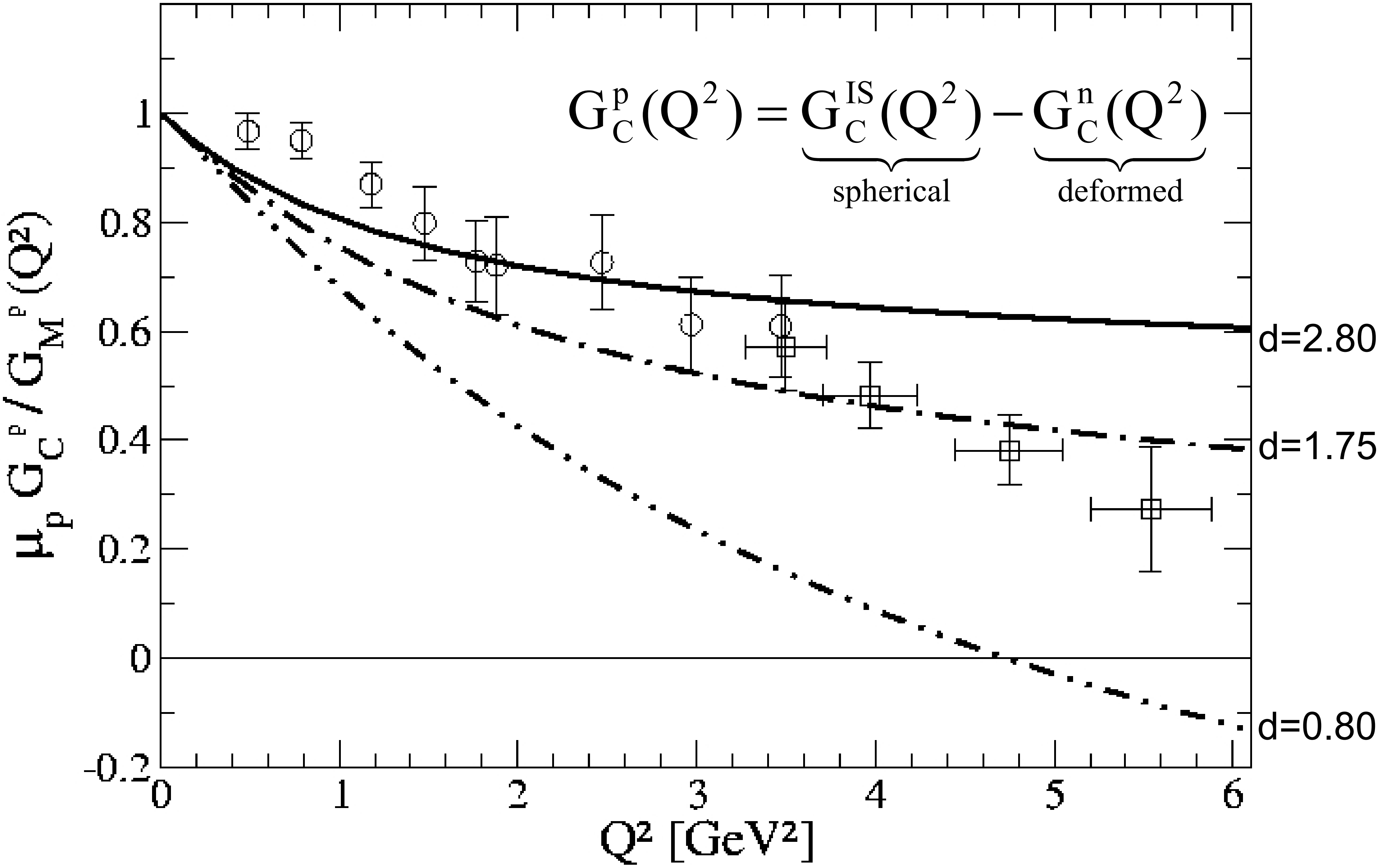
$$G_C^p(Q^2) = G_C^{IS}(Q^2) - G_C^n(Q^2)$$

$$\mu_p \frac{G_C^p(Q^2)}{G_M^p(Q^2)} = 1 \; - \; 1.91 \; \frac{a\,\tau}{1+d\,\tau}$$

using simple parametrizations

$$G_C^{IS}(Q^2) = G_M^p(Q^2)\big/\mu_p = G_M^n(Q^2)\big/\mu_n = G_D(Q^2)$$ dipole

$$G_C^n(Q^2) = -\frac{a\,\tau}{1+d\,\tau}\, G_M^n(Q^2)$$ Galster


$G_C^p(Q^2) = \underbrace{G_C^{IS}(Q^2)}_{\text{spherical}} - \underbrace{G_C^n(Q^2)}_{\text{deformed}}$
$\mu_p G_C^p / G_M^p (Q^2)$
Q² [GeV²]
d=2.80
d=1.75
d=0.80
1
0.8
0.6
0.4
0.2
0
-0.2
0
1
2
3
4
5
6

Proton elastic form factor ratio

The observed decrease of $R = \mu_p \frac{G_C^p(Q^2)}{G_M^p(Q^2)}$ with increasing Q^2

can be understood with the help of the decomposition

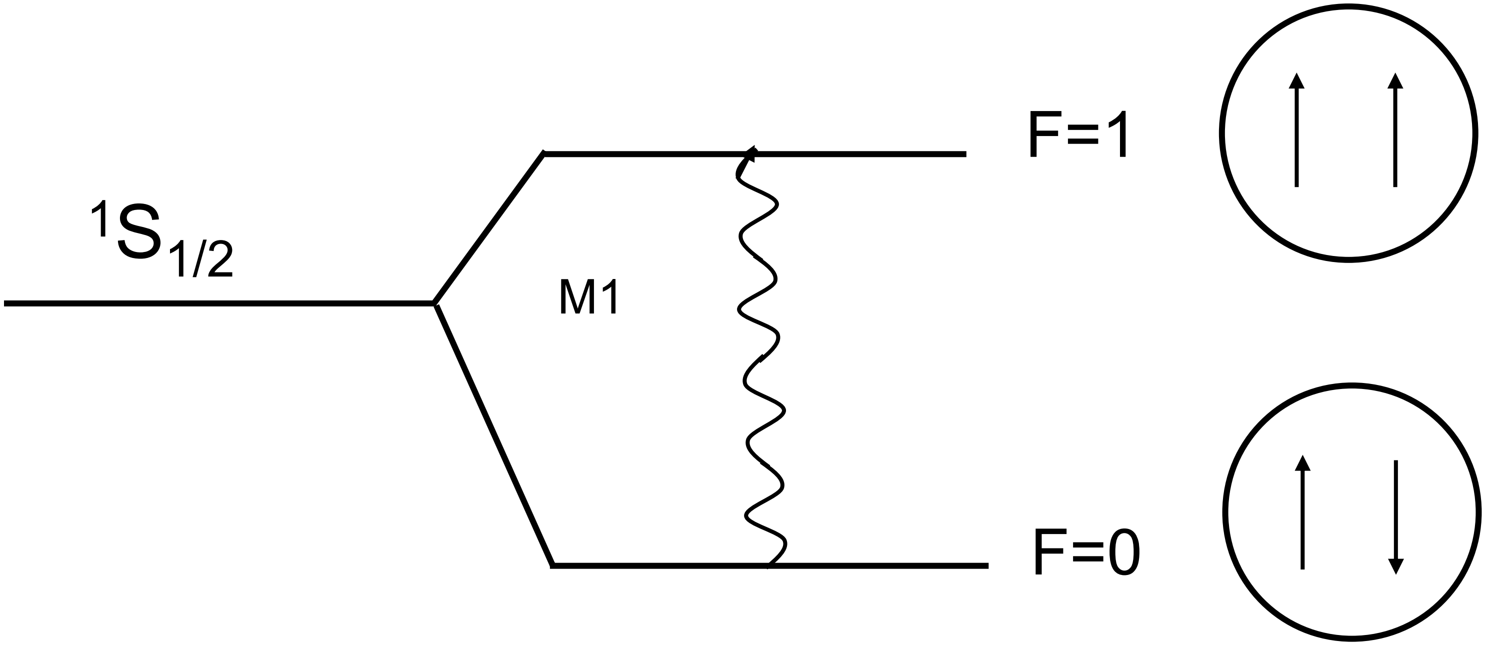
$$G_C^p(Q^2) = \underbrace{G_C^{IS}(Q^2)}_{\text{spherical}} - \underbrace{G_C^n(Q^2)}_{\text{deformed}}$$

The decrease of R comes from the intrinsic quadrupole form factor $G_2^p(Q^2)$.

Our theory relates the latter to the neutron charge form factor $G_C^n(Q^2)$.

3. Implications for hydrogen atom hyperfine splitting

Hydrogen ground state hyperfine splitting



$$H = -\frac{2}{3}\,\vec{\mu}_p \cdot \vec{\mu}_e\,\delta^3(\vec{r}_p - \vec{r}_e)$$

Fermi formula

$$E_F = \langle \Psi_e | H | \Psi_e \rangle_{F=1} - \langle \Psi_e | H | \Psi_e \rangle_{F=0}$$

$$= \frac{8}{3}\ \mu_p \cdot \mu_e\ |\Psi_e(r_p)|^2$$

point nucleon

$$r_p = 0$$

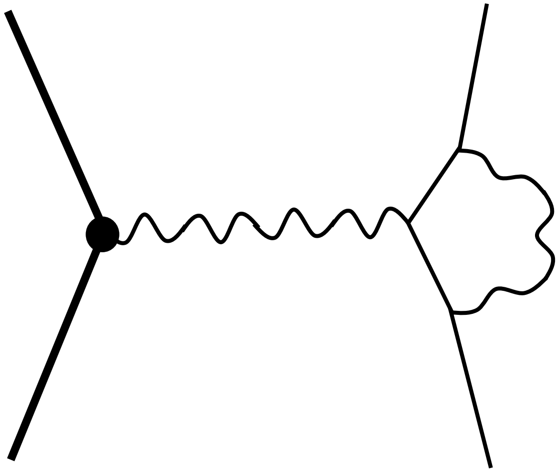
$$= \frac{8}{3}\alpha^4\ \frac{m_e{}^2 M_p{}^2}{(m_e + M_p)^3}\frac{\mu_p}{\mu_N}$$

$$= 5.8678509 \cdot 10^{-6}\ \text{eV}$$

$$= 1418.8401\ \text{MHz}$$

QED corrections

largest correction: electron anomalous magnetic moment due to **electron vertex correction**



$$\mu_e = \mu_B\,(1 + \frac{\alpha}{2\pi} - 0.328\left(\frac{\alpha}{2\pi}\right)^2 + \cdots) = 1.0011596\ \mu_B$$

this and other QED corrections leads to

$$\Delta E^{\mathrm{HFS}}_{\mathrm{QED}} = E_F(1+\delta_{\mathrm{QED}}) = 1,420,452.04 \quad \mathrm{kHz}$$

M. Eides et al., Phys. Rep. 342 (2001) 63

Experimental value

$$\Delta E_{\text{exp}}^{\text{HFS}} = 1\underset{\substack{\uparrow\\ \text{GHz}}}{,}420\underset{\substack{\uparrow\\ \text{MHz}}}{,}405\underset{\substack{\uparrow\\ \text{kHz}}}{,}\ 751.7667 \pm 0.0009\ \text{Hz}$$

measured up to 13 significant digits

L. Essen et al., Nature 229 (1971) 110

Difference between theory and experiment

$$D = \Delta E^{\mathrm{HFS}}_{\mathrm{theory(QED)}} - \Delta E^{\mathrm{HFS}}_{\mathrm{exp}} = 46.46\ \mathrm{kHz} = 32.75\ \ \mathrm{ppm}$$

add recoil contribution $\delta_{\mathrm{recoil}} = 5.85\ \ \mathrm{ppm}$

$$\longrightarrow \quad D = +38.60\ \ \mathrm{ppm}$$

finite nucleon size leads to a ***reduction*** of the theoretical value

Proton size correction (estimate)

$$\Psi_e(r) = N\, e^{-r/a_B} = N\,(1 - r/a_B + \ldots)$$

$$\Delta E^{\mathrm{HFS}}_{\mathrm{proton\ size}} = E_F \left(1 - 2\frac{r_p}{a_B} \right)$$

$$-2\ \frac{r_p}{a_B} \approx -2\ \frac{10^{-5}\ \dot{\mathrm{A}}}{0.5\ \dot{\mathrm{A}}} = -40 \cdot 10^{-6} = -40\ \mathrm{ppm}$$

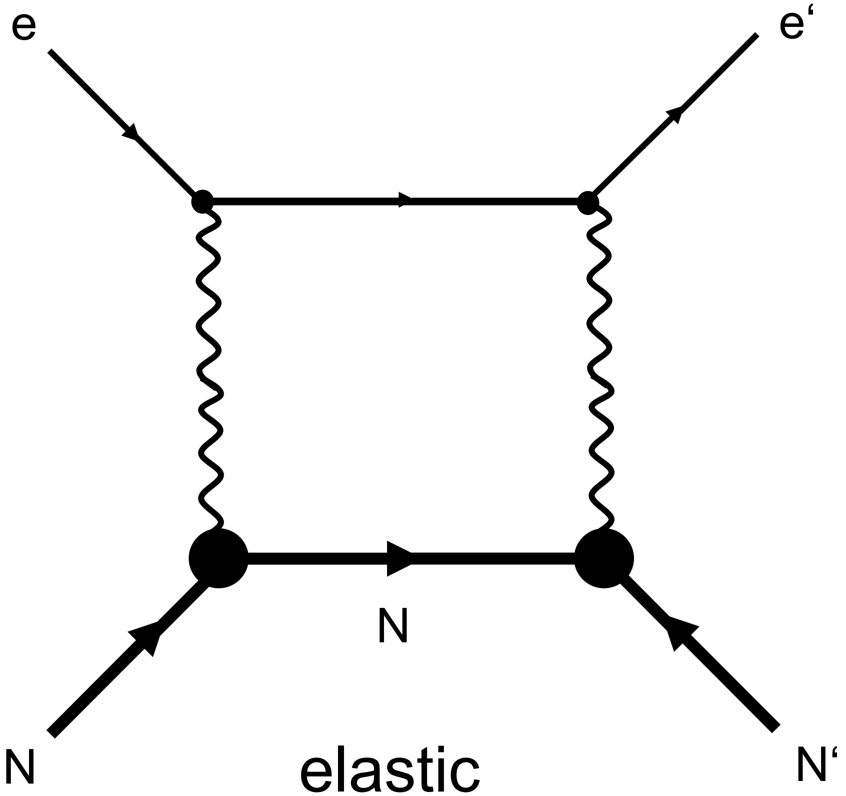
This reduction of the theoretical result is just of the right size to achieve agreement between theory and experiment.

Nucleon structure corrections

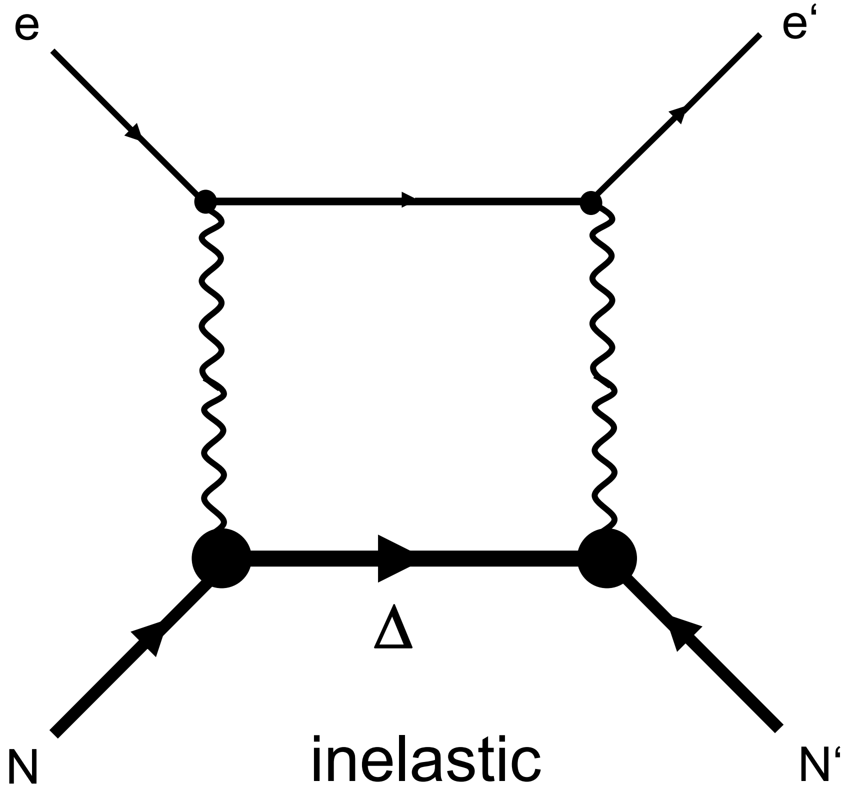
$$\Delta E^{\mathrm{HFS}}_{\mathrm{theory}} = E_F(1 + \delta_{\mathrm{QED}} + \delta_{\mathrm{recoil}} + \delta_{\mathrm{structure}})$$

$$\delta_{\mathrm{structure}} = \delta_{\mathrm{Zemach}} + \delta_{\mathrm{pol}}$$

Two photon exchange diagrams



elastic
Zemach term
δ_{Zemach}



inelastic
polarization term
δ_{pol}

Zemach radius

$$r_Z = -\frac{4}{\pi}\int_0^\infty \frac{dQ}{Q^2}\left[G_C^p(Q^2)\ \frac{G_M^p(Q^2)}{\mu_p} - 1\right]$$

subtract point nucleon limit

Zemach correction to hyperfine splitting

$$\delta_Z = -2\, r_Z / a_B \left(1 + \underbrace{0.0151}_{\text{radiative corr.}}\right)$$

S. G. Karshenboim, Phys. Lett. A225 (1997) 97

Deformation contribution to Zemach radius

$$r_Z = -\frac{4}{\pi}\int_0^\infty \frac{dQ}{Q^2}\left[G_C^p(Q^2)\;\frac{G_M^p(Q^2)}{\mu_p} \quad - \;1\right]$$

$$G_C^p(Q^2) = \underbrace{G_C^{IS}(Q^2)}_{\text{spherical}} - \underbrace{G_C^n(Q^2)}_{\text{deformed}}$$

$$r_Z = -\frac{4}{\pi}\int_0^\infty \frac{dQ}{Q^2}\left[\underbrace{G_C^{IS}(Q^2)\frac{G_M^p(Q^2)}{\mu_p}}_{\text{spherical}} - \underbrace{G_C^n(Q^2)\;\frac{G_M^p(Q^2)}{\mu_p}}_{\text{deformed}} \quad - \;1\right]$$

Use dipole and Galster parametrizations

$$G_D(Q^2) = \left(\frac{1}{1 + Q^2 / \Lambda^2} \right)^2$$ dipole

$$G_C^n(Q^2) = - \frac{a\,\tau}{1 + d\,\tau} G_M^n(Q^2)$$ Galster

determine Λ_{IS} and Λ_M from experimental charge and magnetic radii

charge $$\Lambda_{IS}^2 = \frac{12}{r_{IS}^2} \qquad r_{IS}^2 = r_C^2(p) + r_C^2(n)$$

magnetic $$\Lambda_M^2 = \frac{12}{r_M^2} \qquad r_M^2 = r_M^2(p) = r_M^2(n)$$

Numerical results

spherical term r_Z (spherical) = 1.0627 fm

deformation term r_Z (deformed) = 0.0456 fm

r_Z (total) = 1.1083 fm

Zemach contribution to hyperfine splitting

$$\delta_Z = -2\, r_Z / a_B = -41.86 \text{ ppm} \quad (-42.50 \text{ ppm } \text{with rad. corr.})$$

implies larger polarization contribution

$$\delta_{pol} = -(38.60 - 42.50) \text{ ppm} = 3.9 \text{ ppm}$$

Polarization contribution

$$\delta_{pol} = \frac{\alpha\, m_e}{2\pi\, m_p\, \mu_p / \mu_N}\left(\delta_1 + \delta_2\right)$$

$$\delta_1 = \frac{9}{4}\int_0^\infty \frac{dQ^2}{Q^2}\left[F_2^2(Q^2) \;\; + \frac{8\,M^2}{Q^2}\int_0^{x_{th}} dx\;\; \boldsymbol{\beta}_1\; g_1(x, Q^2) \right]$$

$$\delta_2 = -\,24\;\; M_p^2 \int_0^\infty \frac{dQ^2}{Q^4}\left[\int_0^{x_{th}} dx\;\; \boldsymbol{\beta}_2\;\; g_2(x, Q^2) \right]$$

g_1 and g_2.... spin-dependent nucleon structure functions

$$g_1(\nu,Q^2) = \frac{M_p K}{8\pi^2 \alpha \left(1 + Q^2 \big/ \nu^2\right)} \left[\sigma_{1/2}(\nu,Q^2) - \sigma_{3/2}(\nu,Q^2) + \frac{2\sqrt{Q^2}}{\nu} \sigma_{TL}(\nu,Q^2) \right]$$

$$g_2(\nu,Q^2) = \frac{M_p K}{8\pi^2 \alpha \left(1 + Q^2 \big/ \nu^2\right)} \left[-\sigma_{1/2}(\nu,Q^2) + \sigma_{3/2}(\nu,Q^2) + \frac{2\nu}{\sqrt{Q^2}} \sigma_{TL}(\nu,Q^2) \right]$$

$$g_1(\nu,Q^2) \propto \left[A_{1/2}\right]^2 \quad - \left[A_{3/2}\right]^2 + \quad \frac{2\sqrt{Q^2}}{\nu} \left[S_{1/2}{}^* \cdot A_{1/2}\right]$$

$$g_2(\nu,Q^2) \propto -\left[A_{1/2}\right]^2 \quad + \left[A_{3/2}\right]^2 + \quad \frac{2\nu}{\sqrt{Q^2}} \left[S_{1/2}{}^* \cdot A_{1/2}\right]$$

$$S_{1/2}(Q^2) \propto G_{C2}^{N\to\Delta}(Q^2)$$

nonvanishing G_{C2} increases g_1 and hence δ_{pol}
Explicit evaluation remains to be done

4. Summary

Relation between N and Δ form factors

$$G_{C2}^{p\rightarrow\Delta^+}(Q^2) = -\frac{3\sqrt{2}}{Q^2}\,G_C^n(Q^2)$$

N→ Δ charge quadrupole form factor

neutron charge form factor

Our prediction of C2/M1 based on the neutron G_C^n/G_M^n ratio

(Phys. Rev.Lett. 94, 212301 (2004))

agrees in sign and magnitude with the empirical C2/M1 ratio

(see MAID 2007 analysis EPJA 34, 69 (2007)).

Intrinsic quadrupole form factor of nucleon

Decomposition of the nucleon charge form factor in a spherically symmetric and intrinsic quadrupole part.

$$G_C^p(Q^2) = \underbrace{G_C^{IS}(Q^2)}_{\text{spherical}} - \underbrace{G_C^n(Q^2)}_{\text{deformed}}$$

Neutron charge form factor $G_C^n(Q^2)$ is a manifestation of the nucleon's intrinsic quadrupole form factor

Interpretation of observed decrease of G_C^p/G_M^p ratio

Implications for hyperfine splitting

- Hydrogen HFS is sensitive to the nonsphericity of the proton charge distribution, i.e. Its intrinsic quadrupole moment
- Zemach radius increases in absolute value due to intrinsic
- Polarization contribution increases due to $N \rightarrow \Delta$ charge quadrupole (C2) transition
- What about higher moments in the current distribution, i.e. an intrinisic magnetic octupole moment?

END

Thank you for your attention.

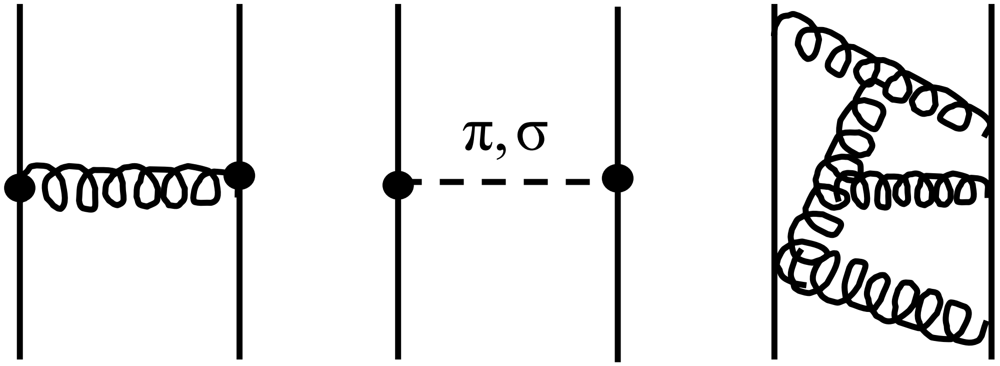
Back up material

Constituent quark model

Hamiltonian: $$H = T_{[1]} + V_{[2]}$$

$$V_{[2]}(\vec{r}_i, \vec{r}_j, \vec{\sigma}_i, \vec{\sigma}_j, \vec{\tau}_i, \vec{\tau}_j, \vec{\lambda}_i, \vec{\lambda}_j) = V_{[2]}^{\text{gluon}}(i,j) \; + \; V_{[2]}^{\pi,\sigma}(i,j) \; + \; V_{[2]}^{\text{conf}}(i,j)$$

$\vec{r}_i \cdots$ space
$\vec{\sigma}_i \cdots$ spin
$\vec{\tau}_i \cdots$ isospin
$\vec{\lambda}_i \cdots$ color

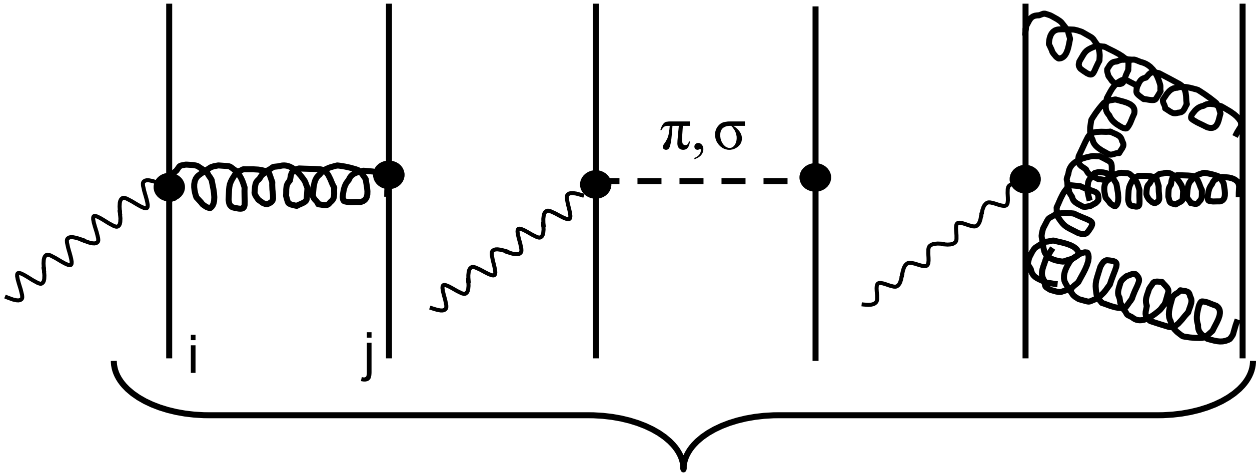


	asymptotic freedom	chiral symmetry	confinement
range:	short	intermediate	long
	$\sim \vec{\sigma}_1 \cdot \vec{\sigma}_2 \; \delta(\vec{r})$	$e^{-m_\pi r}/r$	$\sim r$

Electromagnetic currents

$$\vec{j} = \vec{j}_{[1]} + \vec{j}_{[2]}$$

$$\vec{j}_{[2]}(\vec{r}_i, \vec{r}_j) = \vec{j}^{\,\mathrm{gluon}}_{[2]}(i,j) \quad + \quad \vec{j}^{\,\pi,\sigma}_{[2]}(i,j) \quad + \quad \vec{j}^{\,\mathrm{conf}}_{[2]}(i,j)$$



gluon, meson, confinement
exchange currents $j_{[2]}$

Buchmann, Hernandez, Yazaki: Phys. Lett. B 269 (1991); Nucl. Phys. A 569 (1994) 661

Continuity equation for electromagnetic current

$$\vec{\nabla} \cdot \vec{j}(\vec{x}) + i\left[H, \rho(\vec{x})\right] = 0$$

continuity equation for total current

$$\vec{\nabla} \cdot \vec{j}_{[1]}(\vec{x}) + i\left[T_{[1]}, \rho_{[1]}(\vec{x})\right] = 0$$

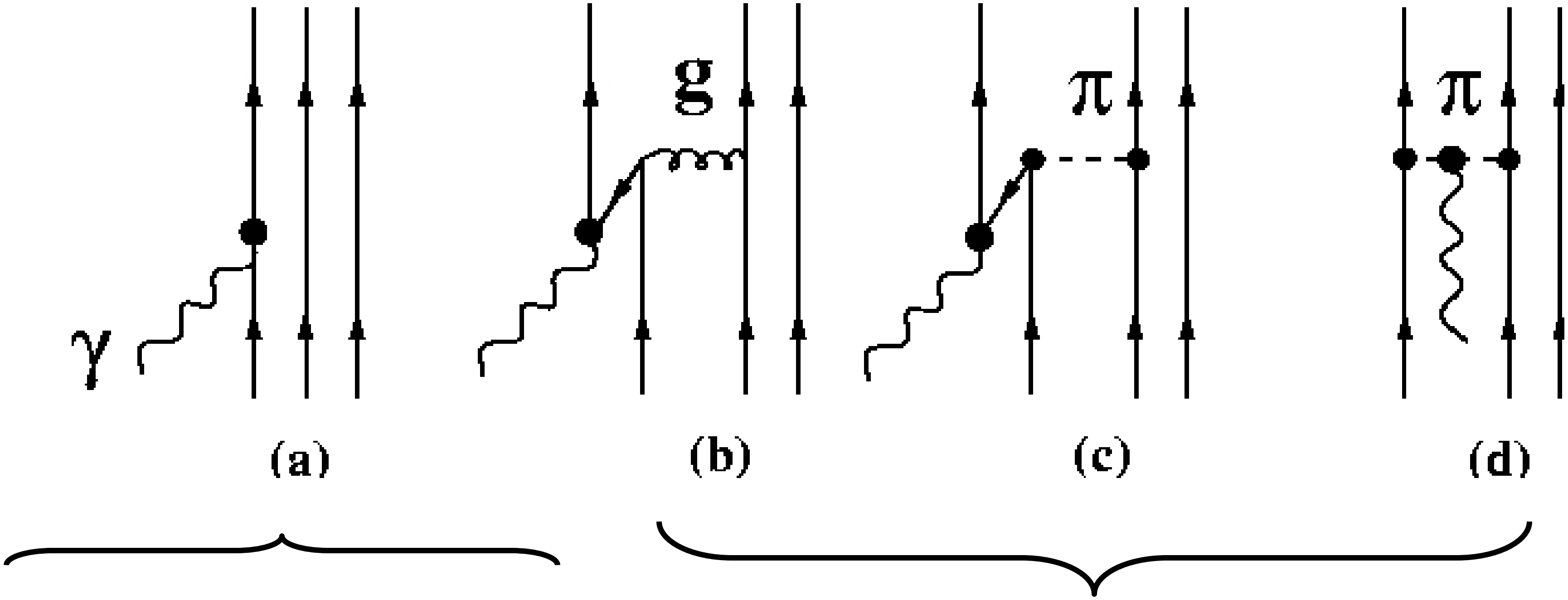
continuity equation for one-body current

$$\vec{\nabla} \cdot \vec{j}_{[2]}(\vec{x}) + i\left[V_{[2]}, \rho_{[1]}(\vec{x})\right] = 0$$

connection between potential and exchange currents

A.B., Leidemann, Arenhoevel, NPA 443 (1985) 726

Origin of two-body operators



one-quark operator

two-quark operators
(exchange currents)

elimination of quark-antiquark and gluon degrees of freedom
→ two-quark operators

Spin-flavor selection rules for charge density operator

$$M = \left\langle 56 \right| \rho_R \left| 56 \right\rangle$$

$M \neq 0$ only if ρ_R transforms according to one of the representations R on the right hand side

$$\overline{56} \times 56 = \underset{\substack{\uparrow \\ \text{0-body}}}{1} + \underset{\substack{\uparrow \\ \text{1-body}}}{35} + \underset{\substack{\uparrow \\ \text{2-body}}}{\boxed{405}} + \underset{\substack{\uparrow \\ \text{3-body}}}{2695}$$

Spin-flavor symmetry breaking

For example, spin-flavor symmetry breaking two-body operators can be constructed from direct products of one-body operators.

$$35 \times 35 = 1 + 35 + 35 + 189 + 280 + \overline{280} + \boxed{405}$$

However, only the **405** dimensional representation appears in the the direct product **56 x 56**. Therefore, an allowed two-body operator must transform according to the **405.**

Decomposition of SU(6) tensor into SU(3) and SU(2) tensors

$$\begin{aligned} 405 = (1,1) + \boxed{(8,1)} + (27,1) \\ +\,2\,(8,3) + (10,3) + (10,3) + (27,3) \\ (1,5) + \boxed{(8,5)} + (27,5) \end{aligned}$$

scalar J=0

vector J=1

tensor J=2

First entry: dimension of SU(3) flavor operator
Second entry: dimension of SU(2) spin operator 2J+1

Charge operator transforms as flavor octet.
Coulomb multipoles have even rank (odd dimension) in spin space.

Spin scalar (8,1) and spin tensor (8,5) are the only components of the SU(6) tensor **405** that can then contribute to $\rho_{[2]}$.

same value for the entire multiplet 56

$$M = \left\langle 56 \right| \rho_{405} \left| 56 \right\rangle = \left\langle 56 \right\| \rho_{405} \left\| 56 \right\rangle \cdot (\text{CG coefficient})$$

provides relations
between matrix elements
of different components
of 405 tensor

Explains why there is a constant ratio between the spin scalar and spin tensor charge density operators and why their matrix elements on the 56 dimensional baryon ground state representation are related.

A. Buchmann, AIP conference proceedings 904 (2007) 110

Comparison with data

use two-parameter Galster formula for G_C^n

$$G_C^n(Q^2) = -\frac{a\,\tau}{1+d\,\tau}\,G_M^n(Q^2) \qquad G_C^n(Q^2) = \mu_n\,G_D(Q^2)$$

$$\frac{C2}{M1}(Q^2) = \frac{|\vec{q}|}{Q}\;\frac{M_N}{2Q}\;\frac{a\,\tau}{1+d\,\tau}$$

$$\tau \equiv \frac{Q^2}{4M_N^2}$$

$a \sim r_n^2$ neutron charge radius

$d \sim r_n^4$ 4th moment of $\rho_n(r)$

Grabmayr and Buchmann, Phys. Rev. Lett. 86 (2001) 2237

Limiting values

$$\frac{C2}{M1}(Q^2 = 0) = -\frac{M_\Delta^2 - M_N^2}{2M_\Delta}\ \frac{M_N}{12}\ \frac{r_n^2}{\mu_n} = -0.031$$

$$\frac{C2}{M1}(Q^2 \to \infty) = -\frac{1}{4}\ \frac{M_N}{M_\Delta}\ \frac{a}{d} = -\left[\underset{\substack{\uparrow \\ \text{d=2.8}}}{0.06} - \underset{\substack{\uparrow \\ \text{d=0.8}}}{0.21}\right]$$

best fit of data (MAID 2007) with d=1.75 $\frac{C2}{M1}(Q^2 \to \infty) = -0.10$

Angular momentum selection rules

$J_i + J_{op} \rightarrow J_f$

Nucleon J=1/2

$$\left\langle \frac{1}{2} \middle| Q^{[2]} \middle| \frac{1}{2} \right\rangle = Q_N \equiv 0$$

$1/2 + 2 \not\rightarrow 1/2$

no spectroscopic quadrupole moment

Delta J=3/2

$$\left\langle \frac{3}{2} \middle| Q^{[2]} \middle| \frac{3}{2} \right\rangle = Q_\Delta$$

$3/2 + 2 \rightarrow 3/2$

spectroscopic quadrupole moment exists

Nucleon → Delta J=3/2

$$\left\langle \frac{3}{2} \middle| Q^{[2]} \middle| \frac{1}{2} \right\rangle = Q_{N\rightarrow\Delta}$$

$1/2 + 2 \rightarrow 3/2$

transition quadrupole moment exists

Nucleon model calculations of Q_0

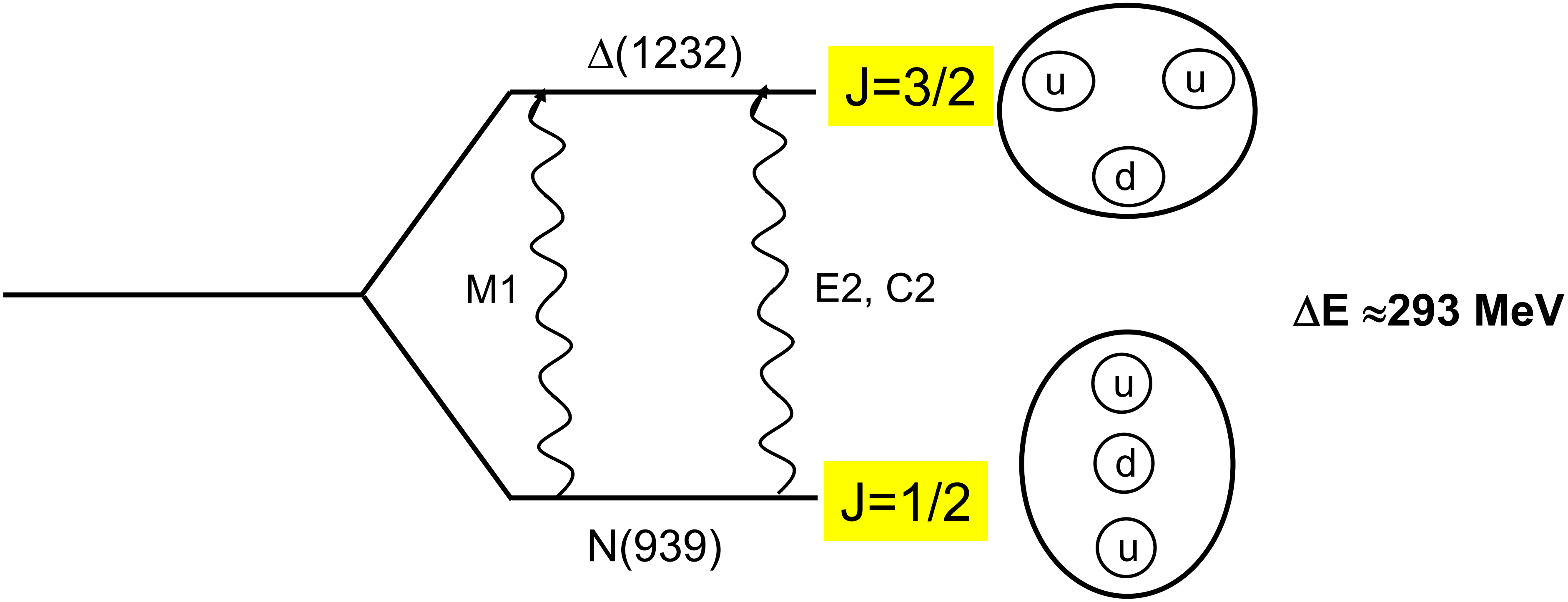
Calculation of Q_0 in three different nucleon models

- quark model
- pion-nucleon model
- collective model

All three models lead to qualitatively the same result for Q_0:

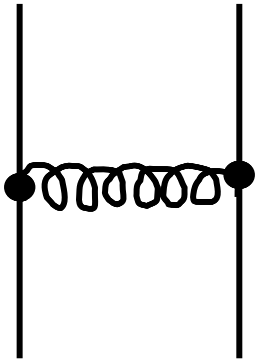
Neutron charge radius determines the sign und size of the intrinsic N und Δ quadrupole moments.

Δ(1232) resonance



The Delta (1232) resonance is the lowest excited state of nucleon with the same quark content as the ground state.

Gluon exchange potential between quarks



quark hyperfine interaction
causes N-Δ mass splitting

quark tensor force
causes D state admixture in
N and Δ wave functions

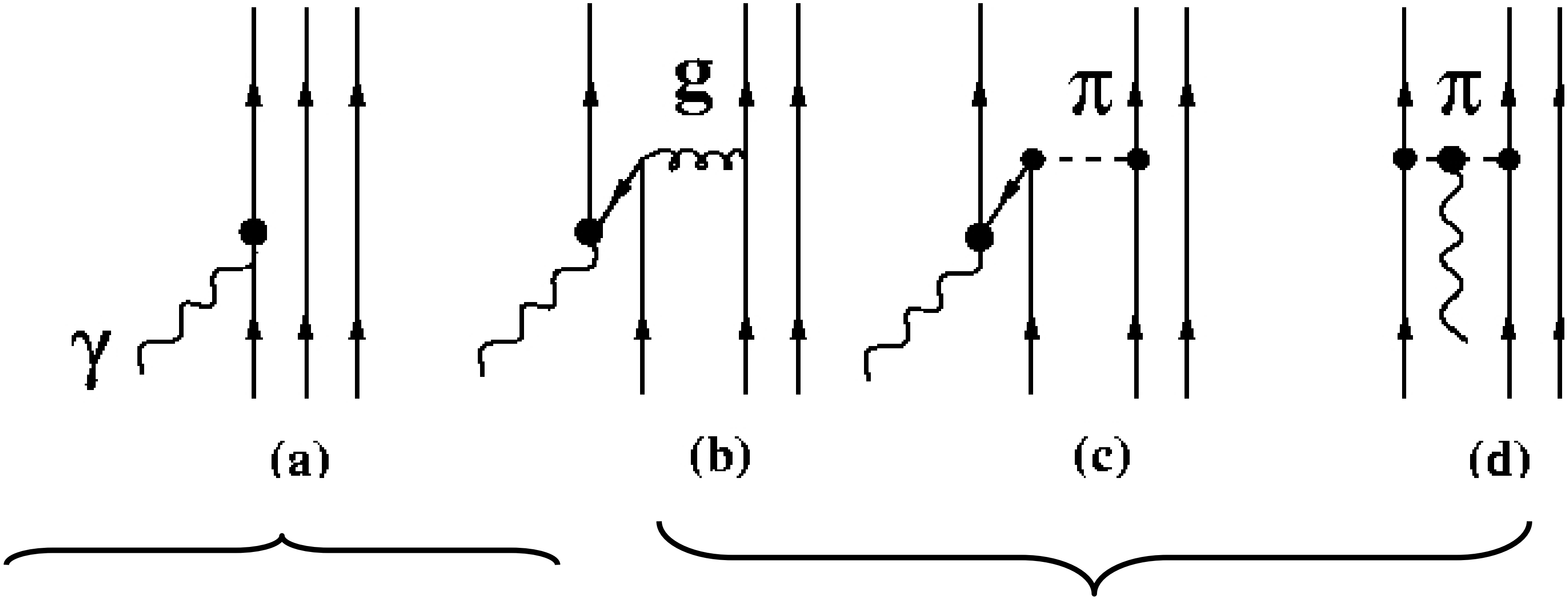
$$V^{\text{gluon}} = \alpha_S \left\{ \frac{1}{r} - \frac{\pi}{m_q^2}\left(1 + \frac{2}{3}\vec{\sigma}_i \cdot \vec{\sigma}_j\right)\delta(\vec{r}) - \frac{1}{4m_q^2}\,\frac{1}{r^3}\left(3\,\vec{\sigma}_i \cdot \hat{r}\,\vec{\sigma}_j \cdot \hat{r} - \vec{\sigma}_i \cdot \vec{\sigma}_j\right) + \cdots \right\}$$

central

tensor

- typical size of D-state probability in nucleon and Delta $P_D(N) \approx P_D(\Delta) \approx 0.2\%$,
- too small to account for experimental E2 and C2 transition strengths
- quark-antiquark degrees of freedom cause nonspherical charge distribution

Two-body operators: exchange currents



one-quark operator

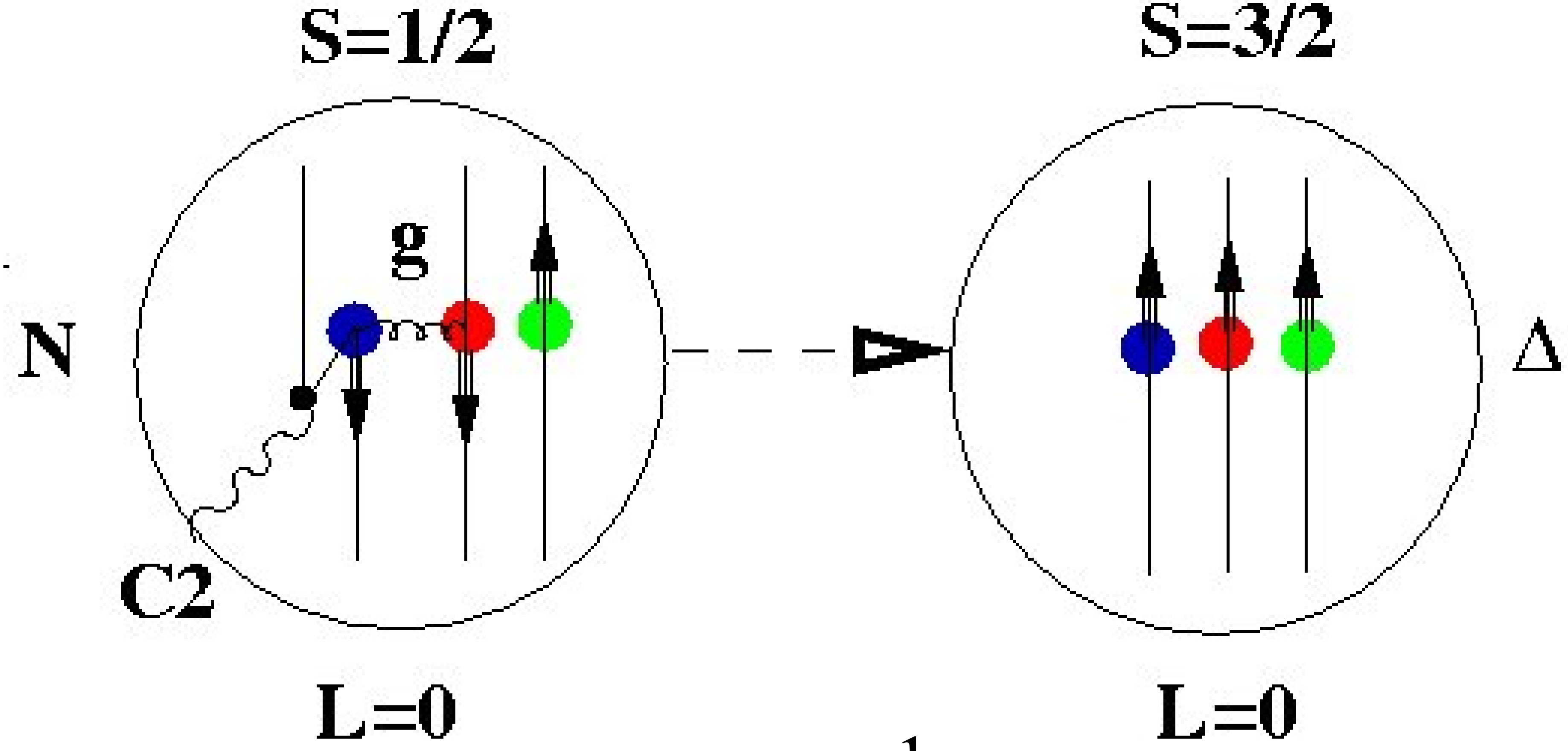
two-quark operators
(exchange currents)

elimination of quark-antiquark and gluon degrees of freedom

→ two-quark operators

these dominate E2 and C2 transitions to Delta (1232)

Double spin flip

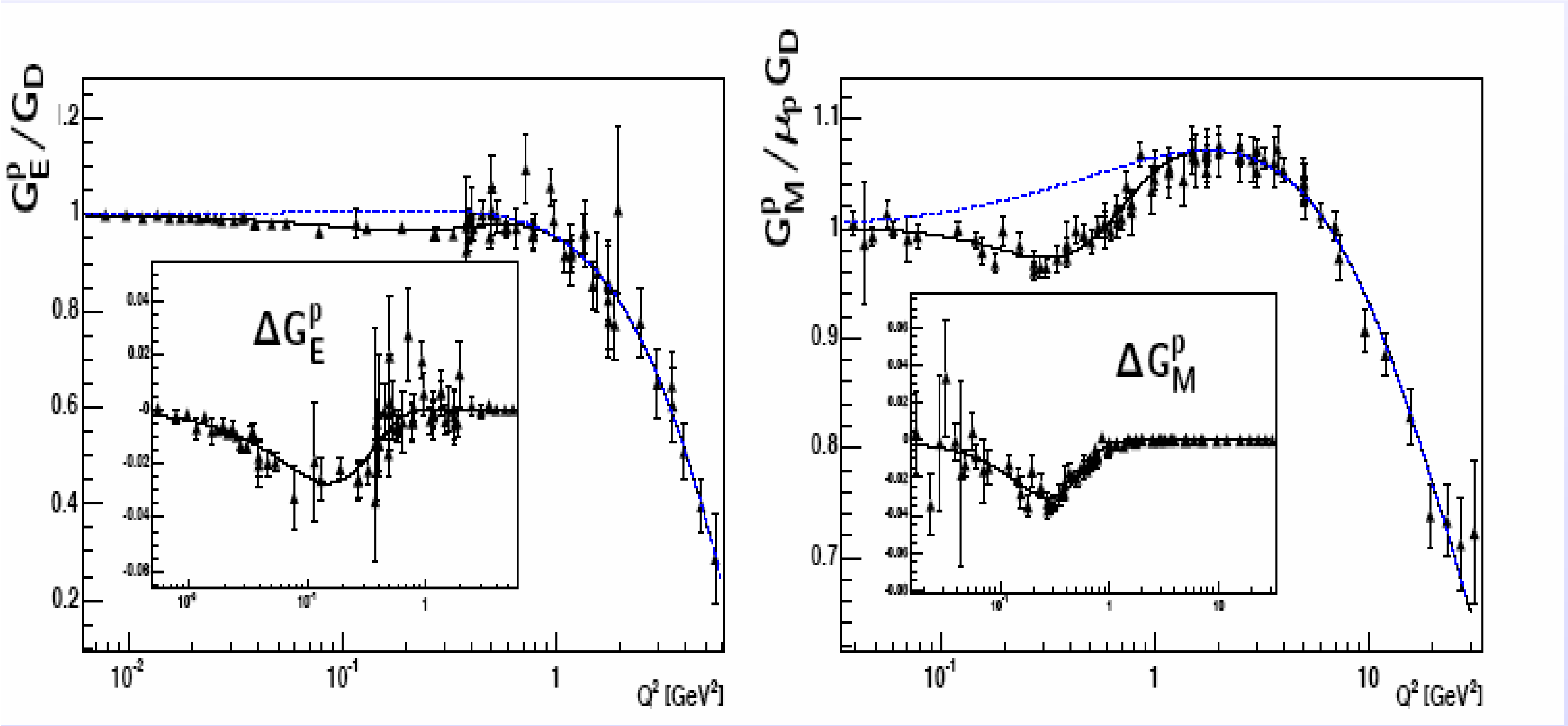


$$Q^{p\rightarrow\Delta^+} = \frac{1}{\sqrt{2}}\ r_n^2$$

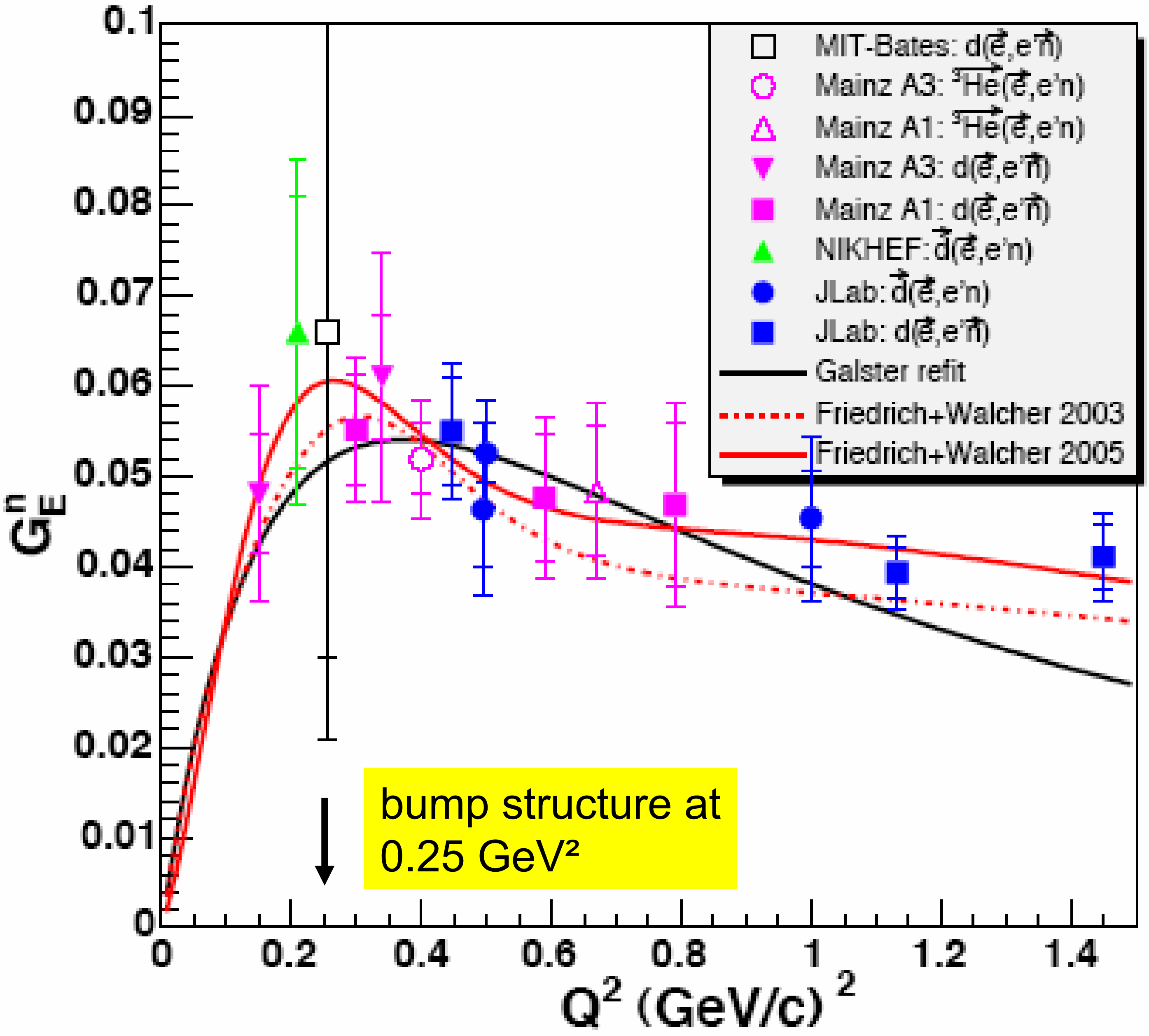
A. J. Buchmann, E. Hernandez, A. Faessler, Phys. Rev. C55 (1997) 448

Observation by Friedrich and Walcher

all nucleon elastic form factors
have a dip structure at around Q²= 0.25 GeV²



neutron charge form factor



bump structure at
0.25 GeV²

Dip structure at low Q^2

The decomposition $G_C^p(Q^2) = \underbrace{G_C^{IS}(Q^2)}_{\text{spherical}} - \underbrace{G_C^n(Q^2)}_{\text{deformed}}$

suggests an explanation of the

- sign
- size
- width

of the dip structure observed in $G_C^p(Q^2)$ at $Q^2 \sim 0.25$ GeV^2

Intrinsic quadrupole form factor

The neutron charge form factor is an observable manifestation of the intrinsic quadrupole deformation of the nucleon.

$$G_C^n(Q^2) = \frac{1}{6}\ Q^2\ \underbrace{G_2^n(Q^2)}_{\text{intrinsic quadrupole form factor of the nucleon}}$$

The intrinsic quadrupole form factor also affects $G_C{}^p(Q^2)$

→ dip structure at low Q^2
→ fall off at high Q^2