Measurement of the $1s2s\ ^1S_0 - 1s2p\ ^3P_1$ Interval in Helium-like Silicon by Fast-Beam Laser Spectroscopy

Edmund Myers, Thomas DeVore, Matthew Redshaw
(Florida State), David Crosby *(Oxford)*

$$ from US-NSF and NIST

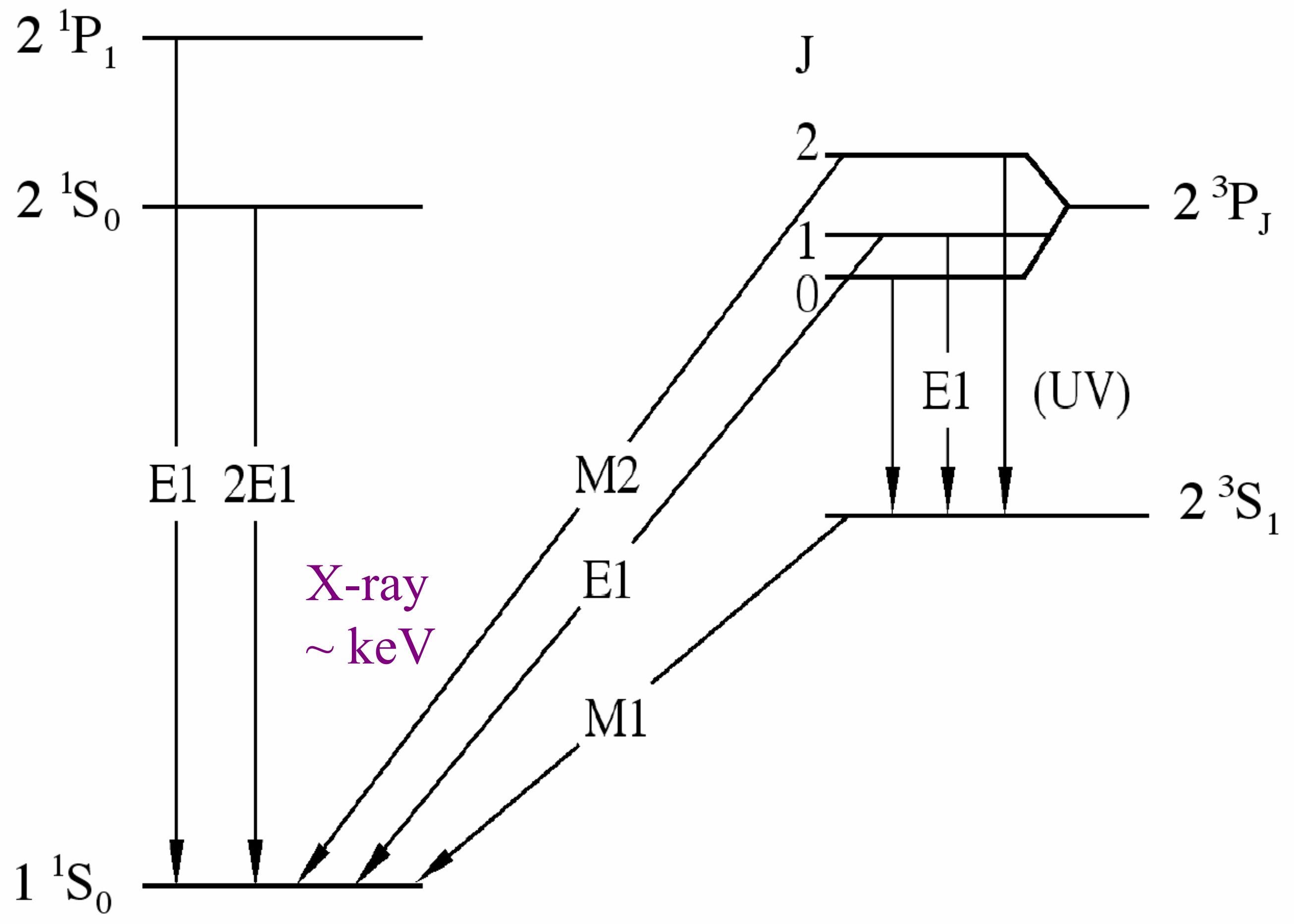
Why study mid-Z Helium-like ions?

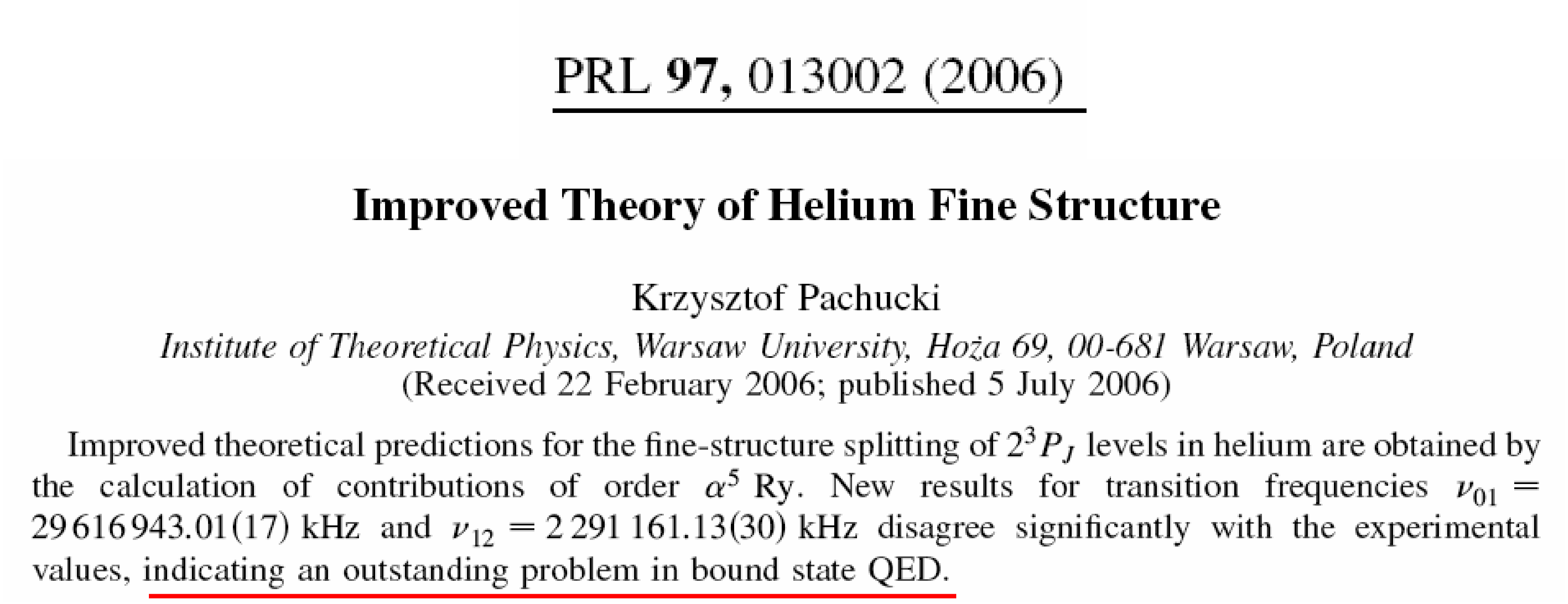
Fundamental multi-electron atom!
Only two electrons + Z/r Coulomb potential…

Test ground for ***relativistic many-body theory***
Correlated electrons plus bound-state QED

Why study mid-Z Helium-like ions?

Fine structure constant from He 1s2p ^{3}P Fine structure





PRL **97,** 013002 (2006)

Improved Theory of Helium Fine Structure

Krzysztof Pachucki
Institute of Theoretical Physics, Warsaw University, Hoża 69, 00-681 Warsaw, Poland

(Received 22 February 2006; published 5 July 2006)

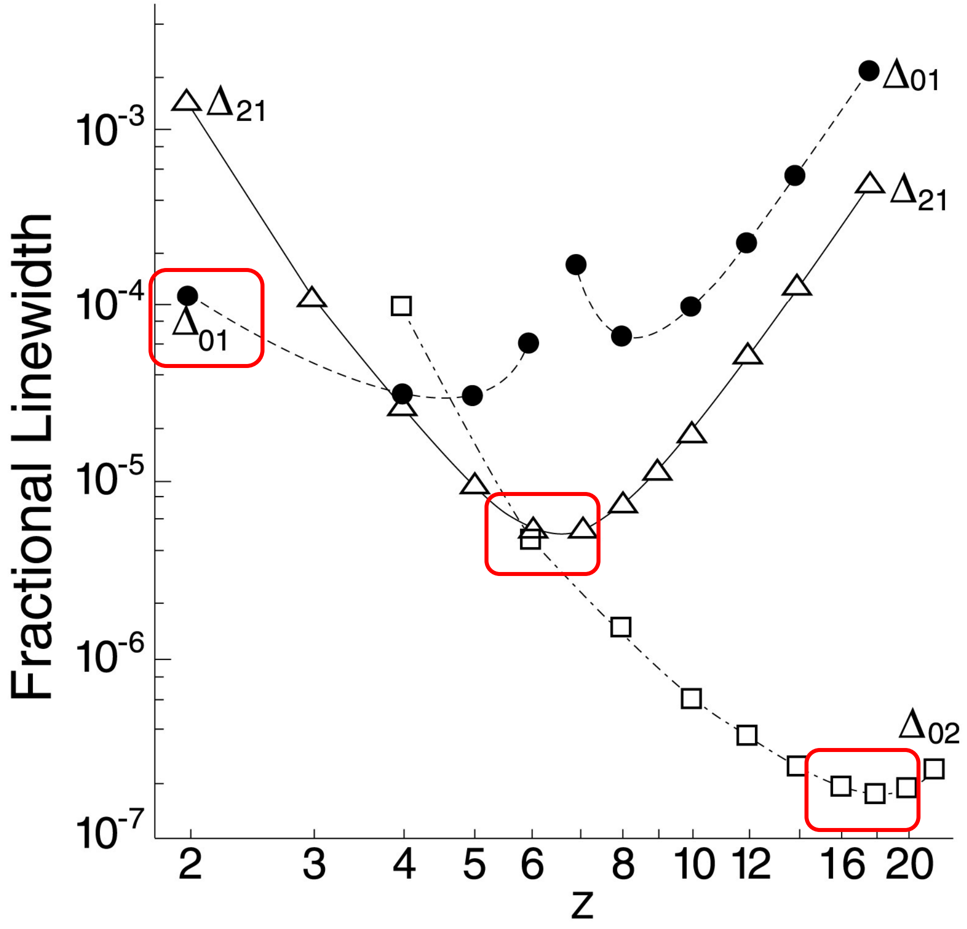
Improved theoretical predictions for the fine-structure splitting of 2^3P_J levels in helium are obtained by the calculation of contributions of order α^5 Ry. New results for transition frequencies $\nu_{01} = 29\,616\,943.01(17)$ kHz and $\nu_{12} = 2\,291\,161.13(30)$ kHz disagree significantly with the experimental values, indicating an outstanding problem in bound state QED.

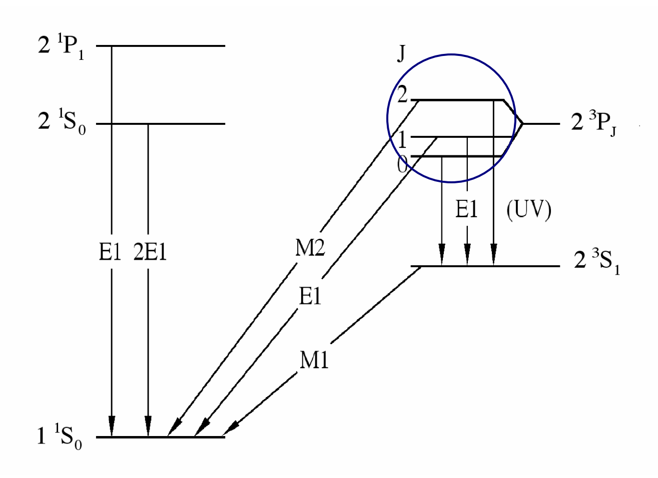
2 $^3P_0$ – 2 $^3P_1$ Fine–structure in Helium

Theory:	29,616,943.01(17) Hz
Experiment:	29,616,951.66(70) Hz

⇒ 0.3 ppm discrepancy [α (*g*-2) 0.37 ppb]

Fractional line width of 2 ^{3}P fine structure





E.G. Myers, PSAS 2000
"Hydrogen Atom"

For α from FS to compete with *g-2* need 1ppb precision

Theory starting points

$$H_{\text{Non-Rel}} = \frac{-\hbar^2}{2m}\left(\vec{\nabla}_1^2 + \vec{\nabla}_2^2\right) - \frac{Ze^2}{r_1} - \frac{Ze^2}{r_2} + \frac{e^2}{r_{12}}$$

Low Z: Non-Relativistic
Schrödinger Hamiltonian

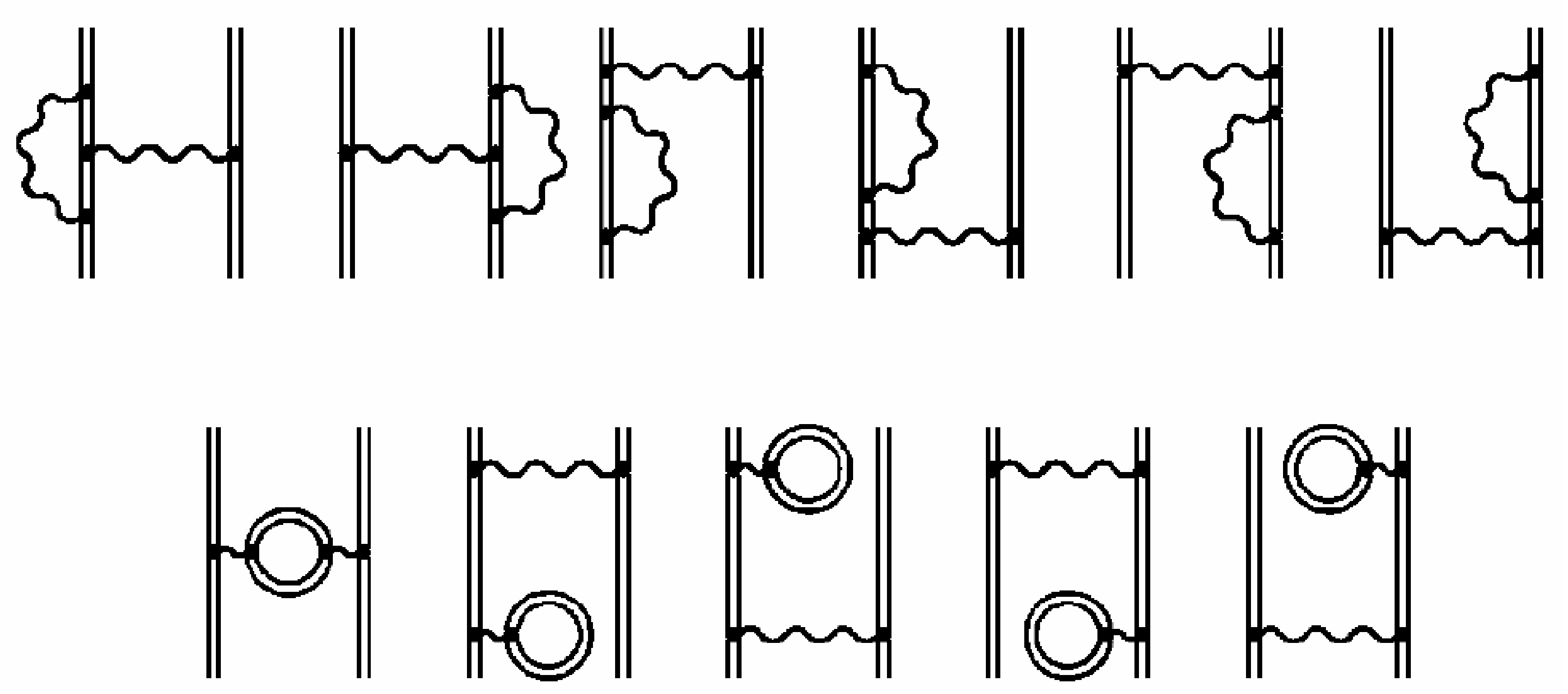
$$H_{\text{Dirac}} = c\vec{\alpha}\cdot\vec{p} + \beta mc^2 - \frac{Ze^2}{r}$$

High Z: One-electron
Dirac Hamiltonian

$$H = H_{1,\text{Dirac}} + H_{2,\text{Dirac}} + \frac{e^2}{r_{12}} + H_{\text{Breit}}$$

Mid Z: Breit Hamiltonian

$$H_{\text{Breit}} = -\frac{e^2}{2r_{12}}\left[\vec{\alpha}_1\cdot\vec{\alpha}_2 + \frac{(\vec{\alpha}_1\cdot\vec{r}_{12})(\vec{\alpha}_2\cdot\vec{r}_{12})}{{r_{12}}^2}\right]$$



Two-electron QED corrections:

Self-energy screening

vacuum polarization screening

Artemyev, A.N., *et al*, *Phys. Rev A*, **71** (2005) 062104

“Recent” Theory (numerical results n=2)

			Range of *Z*
“Unified Method + QED”	Drake	Can JP 1988	2 - 100
“Schrodinger + QED”	Zhang, Yan, Drake	PRL 1996	2 - 12
	Busuttil, Drake	Web 2008	2 - 18
“No-pair Breit-Dirac + QED”			
RMBPT	Johnson, Sapirstein	PRA 1992	10 - 36
AOMPT	Plante, Johnson, Sapirstein	PRA 1994	3 - 100
RCI	Chen, Cheng, Johnson	PRA 1993	5 - 100
	Cheng, Chen, Johnson, Sapirstein	PRA 1994	4 - 92
	Cheng, Chen	PRA 2000	22 - 36
“Dirac + QED”			
	Artemyev, Shabaev, Yerokhin, Plunien, Soff	PRA 2005	12 -100

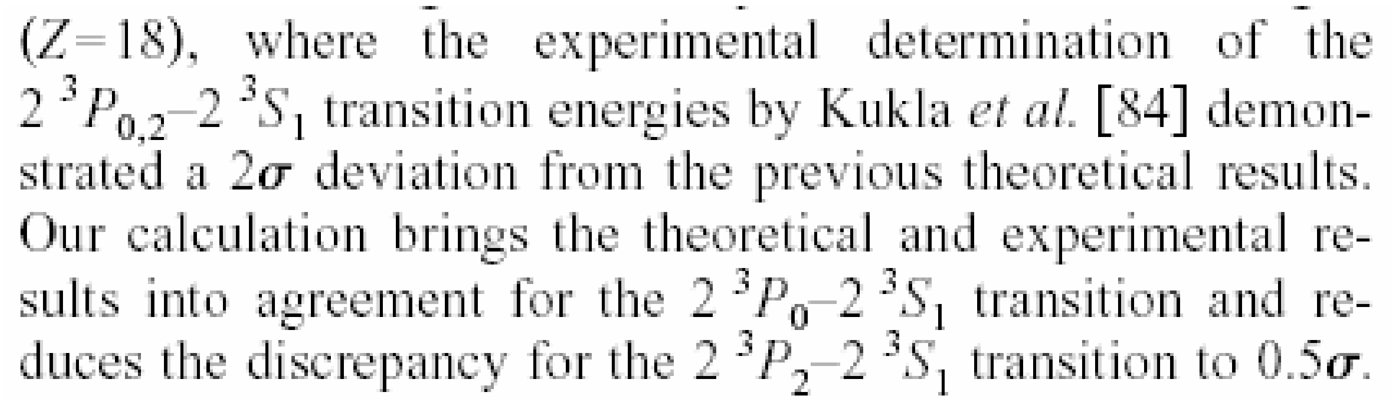
Also Mohr, Sapirstein PRA 2000, etc
Asen, Salomonson, Lindgren PRA 2002 **“Merging MBPT with QED”**
Lindgren, Salomonson, Hedendahl PRA 2006, etc.

UV and X-ray spectroscopy in emission?

Artemyev, Shabaev, Yerokhin, Plunien and Soff, PRA 71, 2005

(Theory Paper) Compares experiment and theory for $12 < Z < 100$

Only one non-laser experimental result has precision to differentiate Artemyev et al from Plante et al and Chen et al.

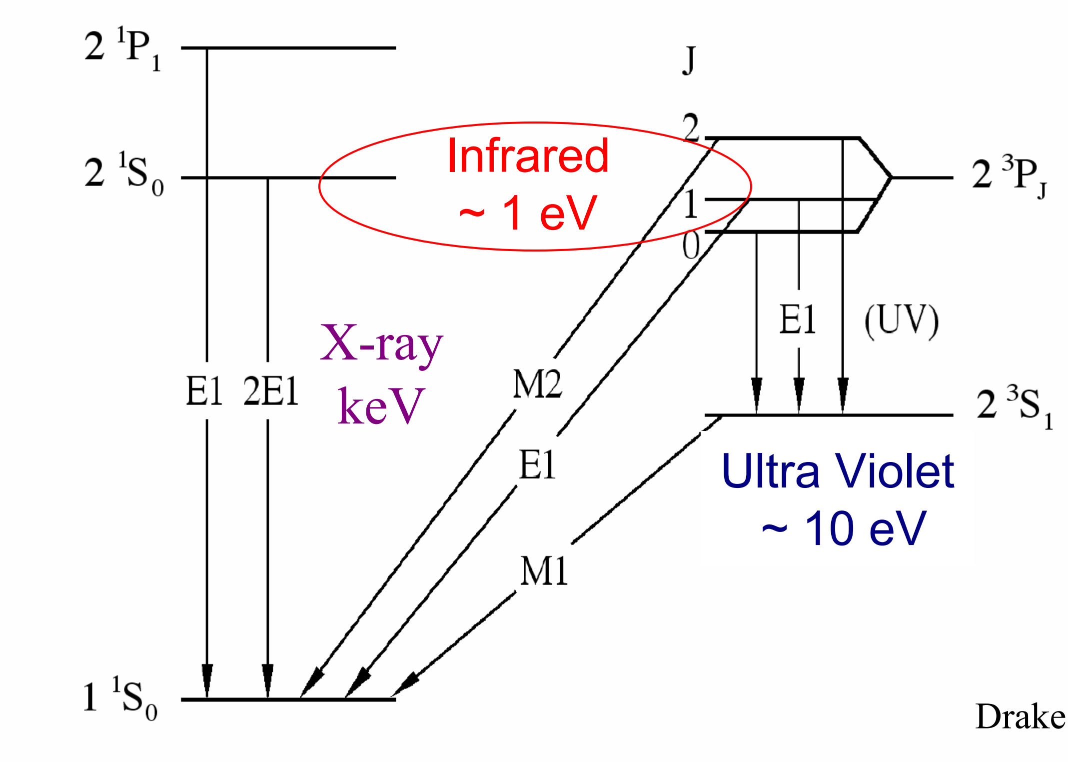


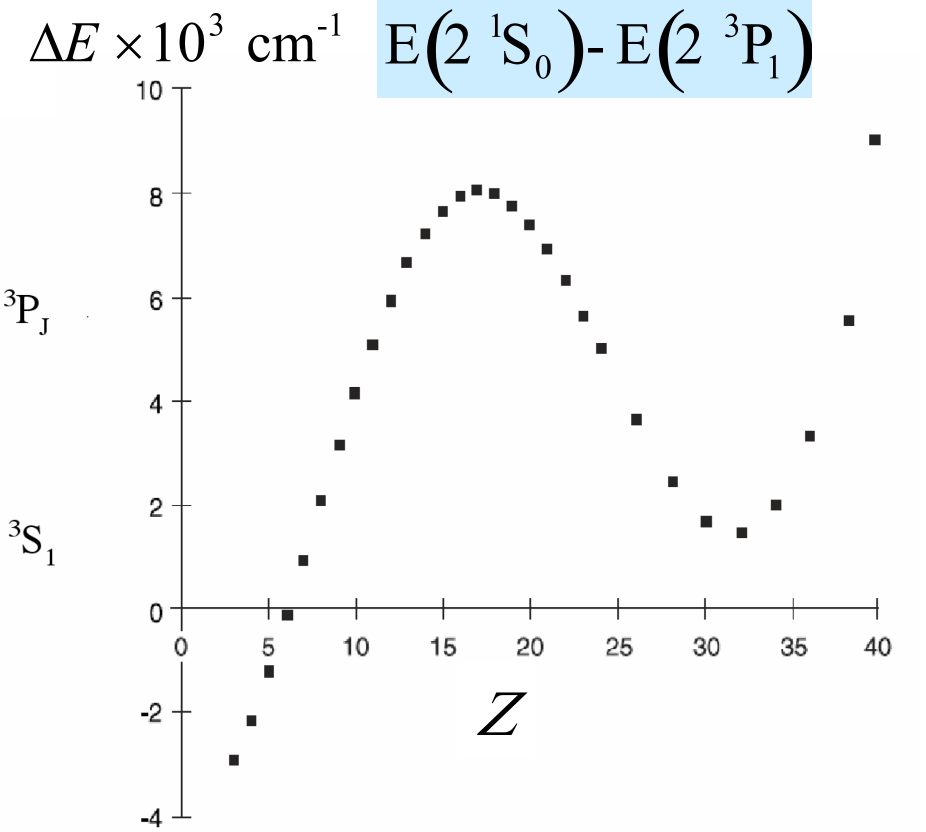
$(Z=18)$, where the experimental determination of the $2\,^3P_{0,2}$–$2\,^3S_1$ transition energies by Kukla *et al.* [84] demonstrated a 2σ deviation from the previous theoretical results. Our calculation brings the theoretical and experimental results into agreement for the $2\,^3P_0$–$2\,^3S_1$ transition and reduces the discrepancy for the $2\,^3P_2$–$2\,^3S_1$ transition to 0.5σ.

Kukla, Livingston, Suleiman, Berry, Dunford, Gemmell, Kanter, Cheng, Curtis PRA 51 1995

Why measure the $2\ ^1S_0$ - $2\ ^3P_1$ interval?

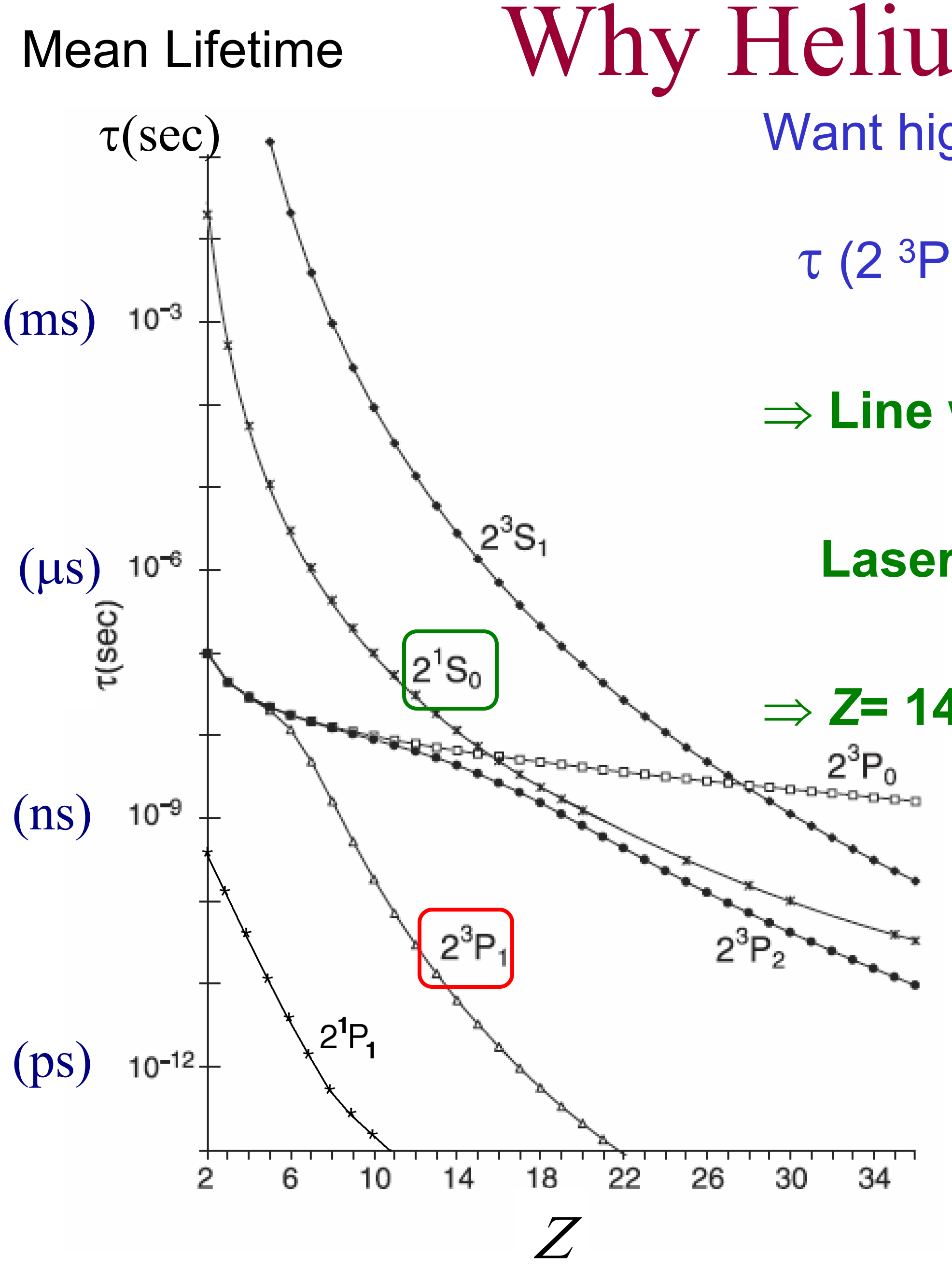
- In IR for Z < 40, partly allowed E1 ⇒ laser spectroscopy
  ⇒ small interval ⇒ high *absolute* precision
- S-state ⇒ sensitive to QED





Drake, G.W., *Canadian Journal of Physics*, **88** (1988) 586

Why Helium-like Silicon?



Want highest Z possible

$\tau\ (2\ ^3P_1)$ falls rapidly ~ Z^{-10}

⇒ **Line width increases as Z^{10}**

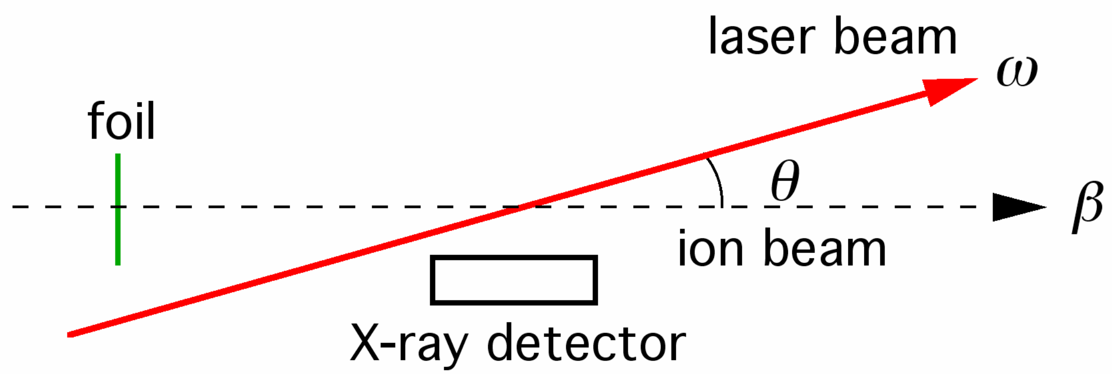
Laser induced transition rate falls ~ Z^{-6}

⇒ **Z= 14 is experimental compromise**

QED contributions to

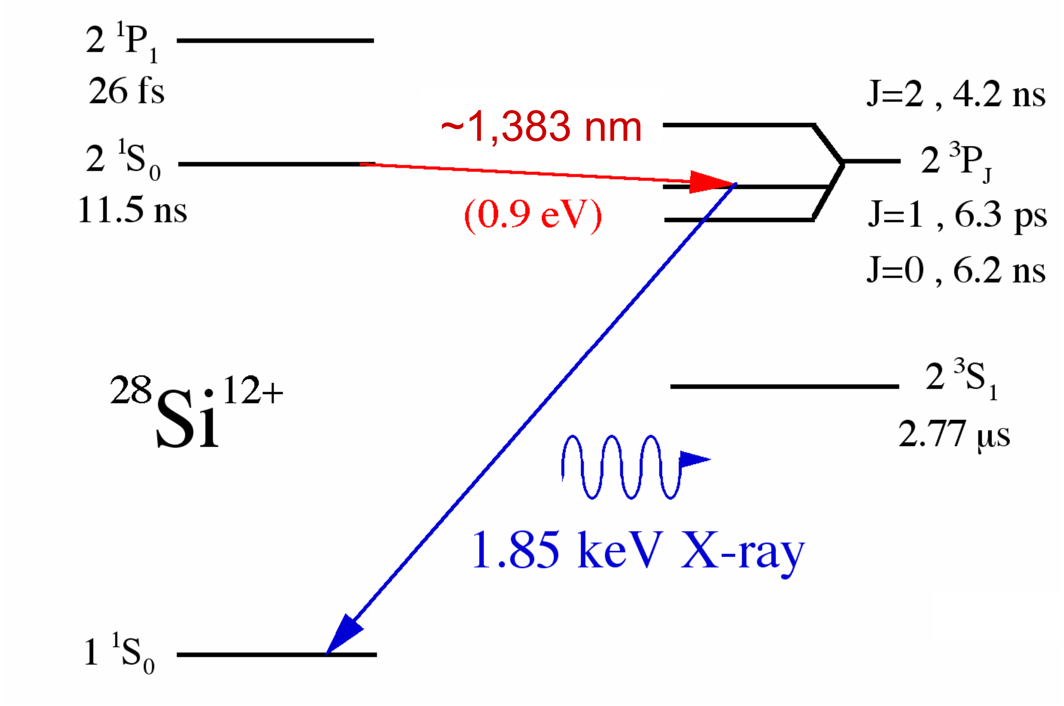
$2\ ^1S_0 - 2\ ^3P_1$ in Si^{12+} ~ 6%

Fast-Beam Laser Resonance Technique

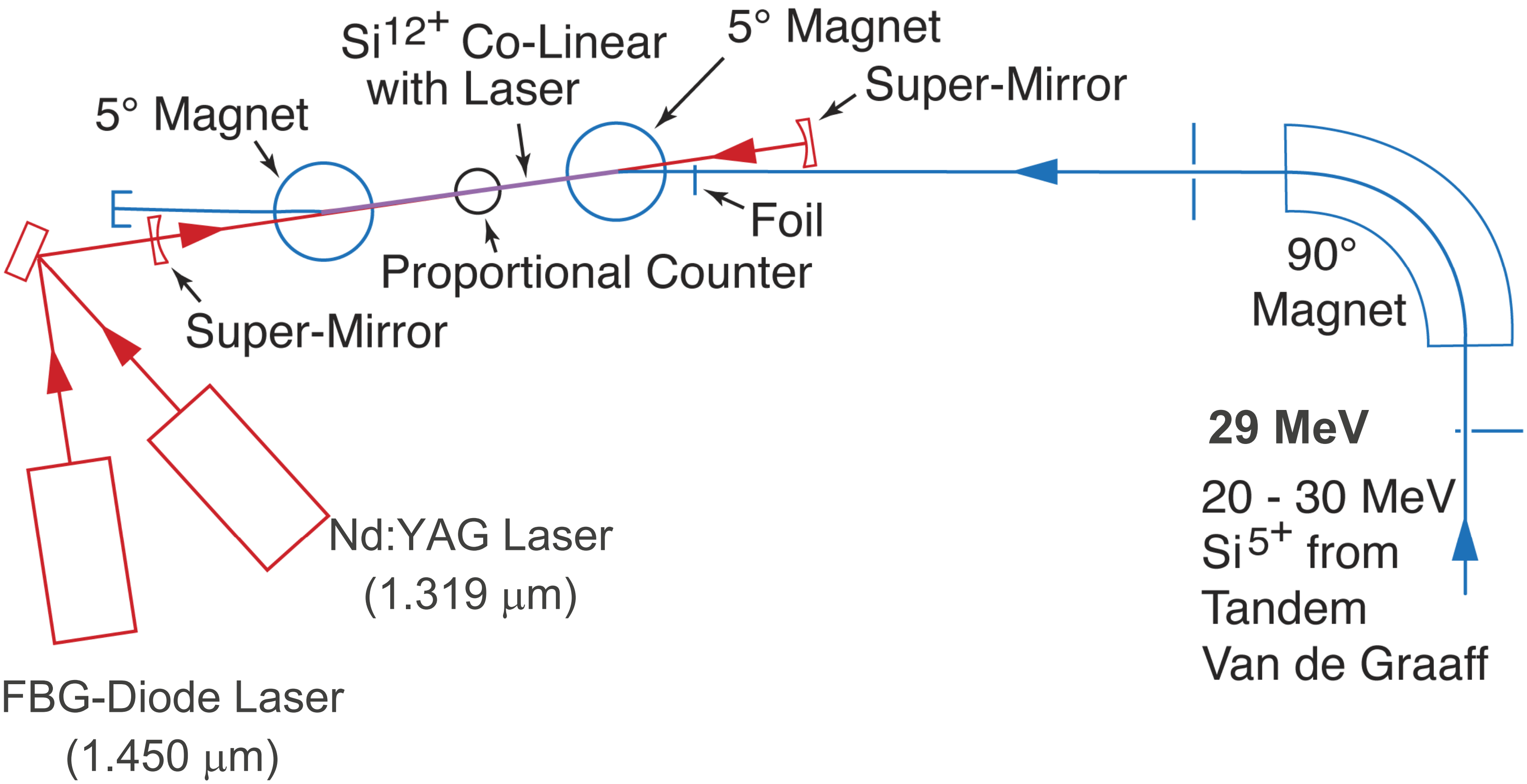


v/c = 5%

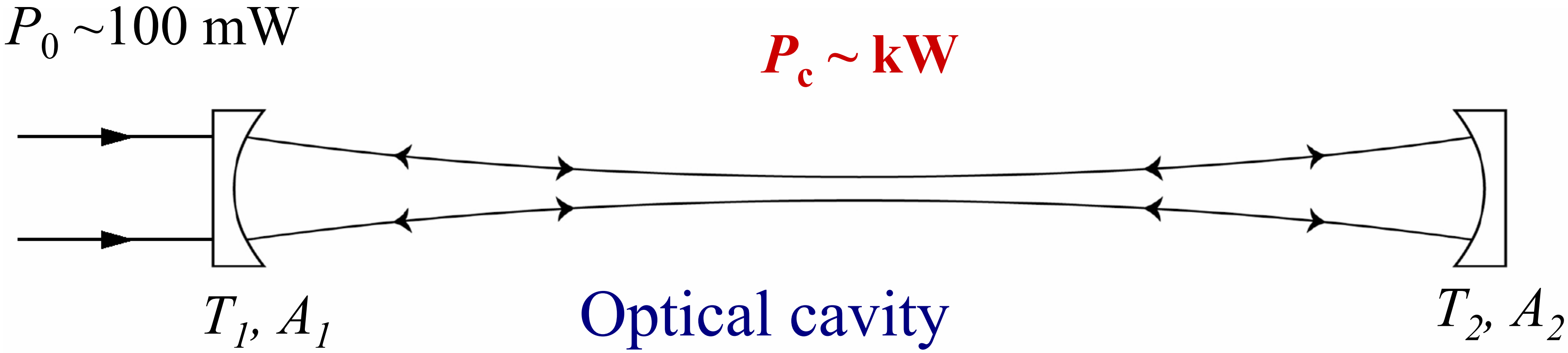
Relativistic Doppler formula $\omega' = \omega_l \gamma (1 \pm \beta \cos\theta)$



Experimental Setup



High-Finesse Power-Build-up Cavity



Optical cavity

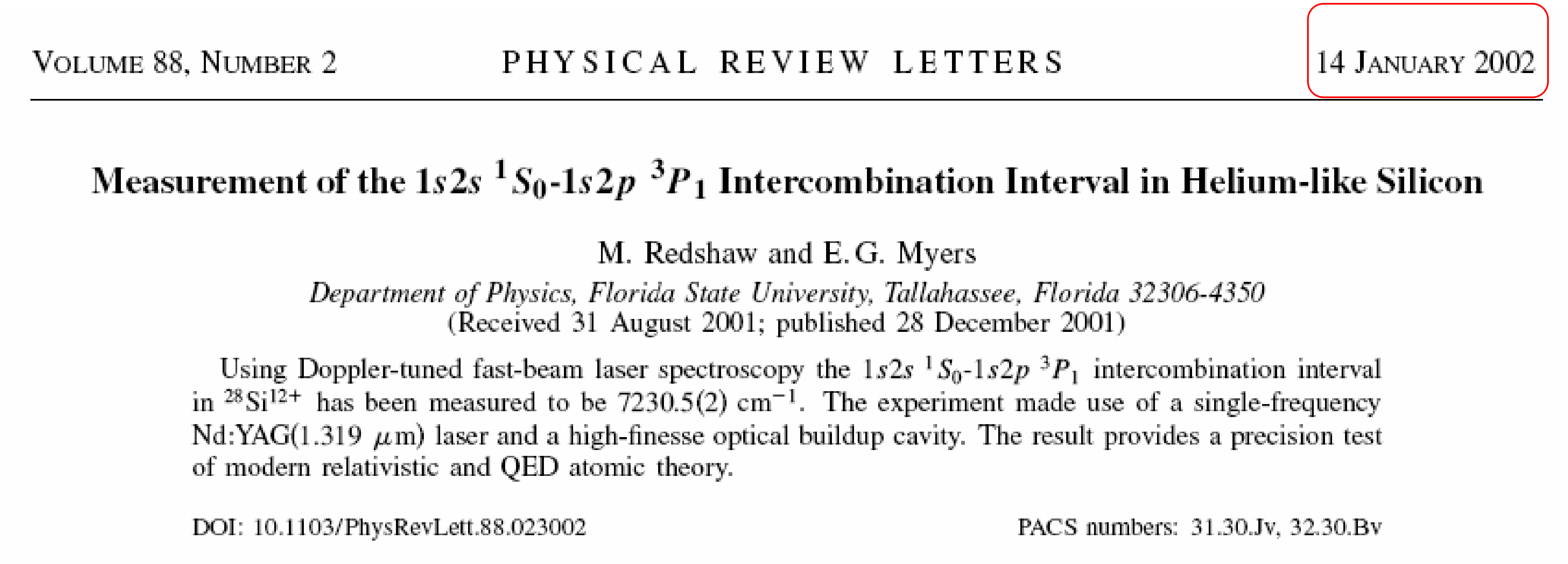
Super mirrors: R > 99.995%

Resonance width ~ 3 kHz !

$$\frac{P_c}{P_0} = \frac{4T_1}{\left(A_1 + A_2 + T_1 + T_2\right)^2}$$

$T \leq$ 50 ppm

Previous Measurement:



VOLUME 88, NUMBER 2 PHYSICAL REVIEW LETTERS 14 JANUARY 2002

Measurement of the 1s2s $^1S_0$-1s2p $^3P_1$ Intercombination Interval in Helium-like Silicon

M. Redshaw and E. G. Myers
Department of Physics, Florida State University, Tallahassee, Florida 32306-4350

(Received 31 August 2001; published 28 December 2001)

Using Doppler-tuned fast-beam laser spectroscopy the 1s2s $^1S_0$-1s2p $^3P_1$ intercombination interval in ^{28}Si^{12+} has been measured to be 7230.5(2) cm^{-1}. The experiment made use of a single-frequency Nd:YAG(1.319 μm) laser and a high-finesse optical buildup cavity. The result provides a precision test of modern relativistic and QED atomic theory.

DOI: 10.1103/PhysRevLett.88.023002 PACS numbers: 31.30.Jv, 32.30.Bv

- Single co-propagating laser at 1,319 nm
- 28 ppm precision
- Limited by uncertainty in beam velocity

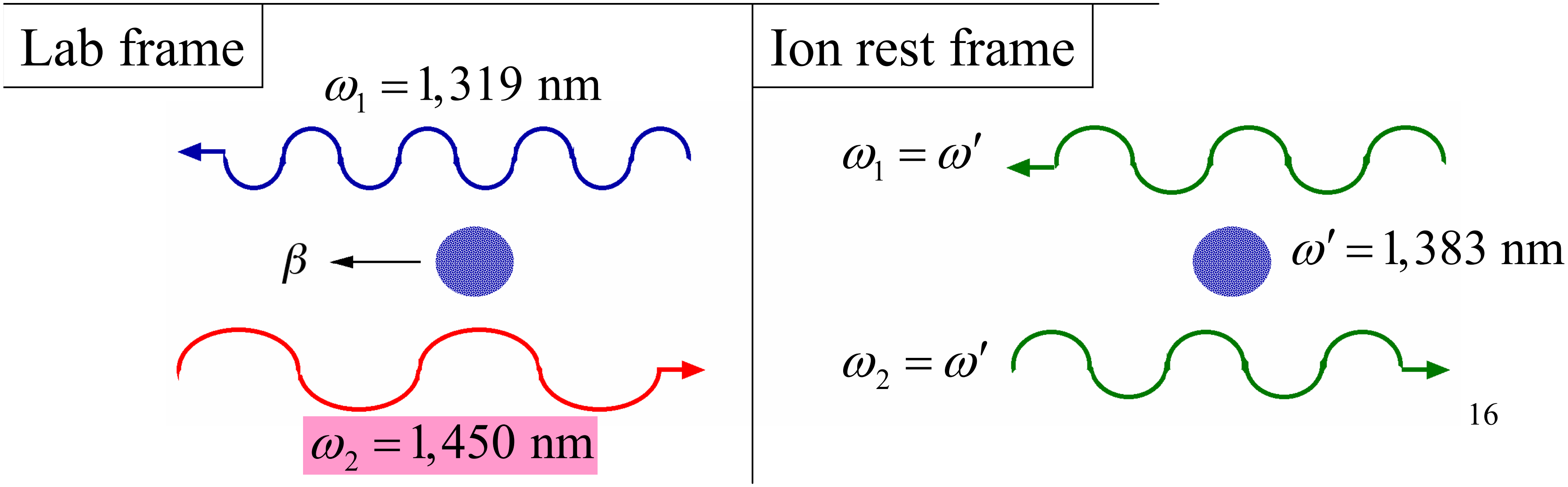
Two laser Doppler-shift cancellation

(c.f. Li^+ $2\,^3S_1 - 2\,^3P_J$ at TSR, ESR storage rings to test special relativity)

Co-propagating: $\omega_1' = \omega_1\gamma_1\left(1-\beta_1\cos\theta_1\right)$

Counter-propagating: $\omega_2' = \omega_2\gamma_2\left(1+\beta_2\cos\theta_2\right)$

Doppler free result: $\omega' = \sqrt{\omega_1\omega_2}$ $\qquad \beta_1 = \beta_2\ ,\ \theta_1 = \theta_2 = 0$



Lab frame

$\omega_1 = 1{,}319\ \text{nm}$

β

$\omega_2 = 1{,}450\ \text{nm}$

Ion rest frame

$\omega_1 = \omega'$

$\omega' = 1{,}383\ \text{nm}$

$\omega_2 = \omega'$

Doppler-shift cancellation cont…

In practice, we can easily have

$$\left|\beta_1 - \beta_2\right| < 0.0005 \quad , \quad \left|\theta_1\right|, \left|\theta_2\right| < 0.01$$

Hence, $$\omega' = \left(\omega_1\omega_2\right)^{1/2}\left[1 + \underline{f\left\{\Delta p,\ \bar{p},\ \Delta\left(\theta^2\right),\ \overline{\theta^2}\right\}}\right]^{1/2}$$

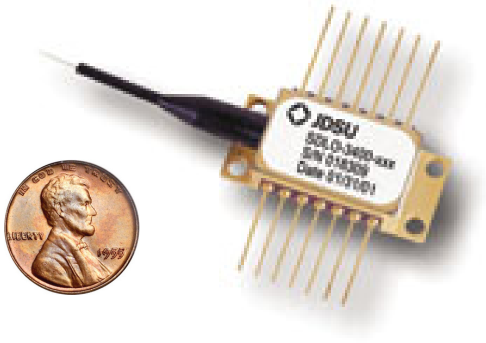
$$f \approx \Delta p\left(1 + \frac{\Delta p}{2} - \frac{\bar{p}^2}{2}\right) - \frac{\Delta\left(\theta^2\right)}{2}\bar{p}\left(1 + \frac{\bar{p}^2}{4}\right) + \overline{\theta^2}\left(\bar{p}^2 - \frac{\Delta p}{2}\right) + \cdots$$

where, $\boldsymbol{\Delta p \equiv (\beta_2\gamma_2 - \beta_1\gamma_1)}$ = difference in ion beam rigidity.

Mainly sensitive to Δp

Thompson, J.K., Howie, D.J.H., and Myers, E.G., *Phys. Rev. A.*, **57** (1998) 180

300 mW, fiber coupled, diode lasers at 1,450 nm (pumps for Raman amplifiers)

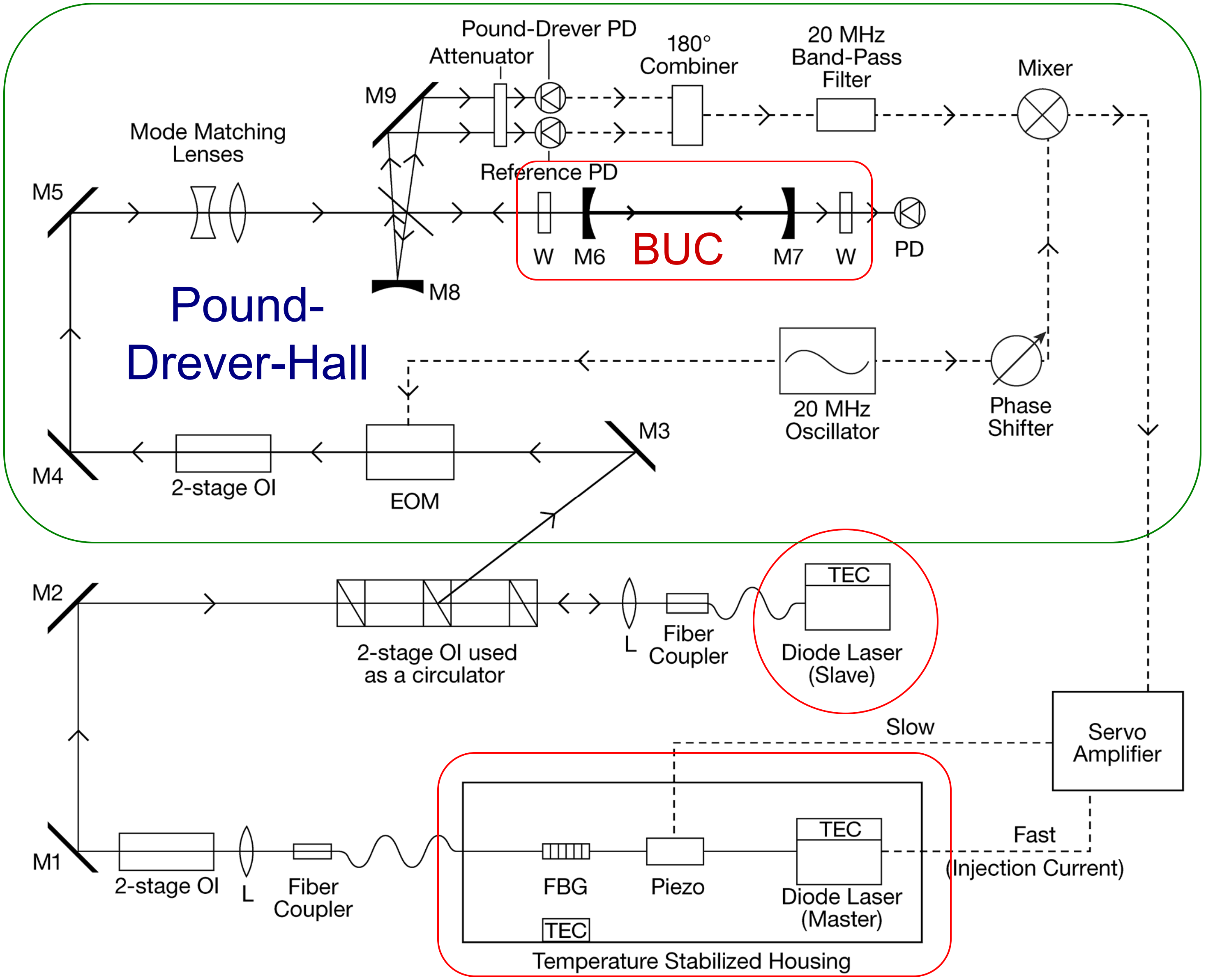


Not intended for single-frequency operation…

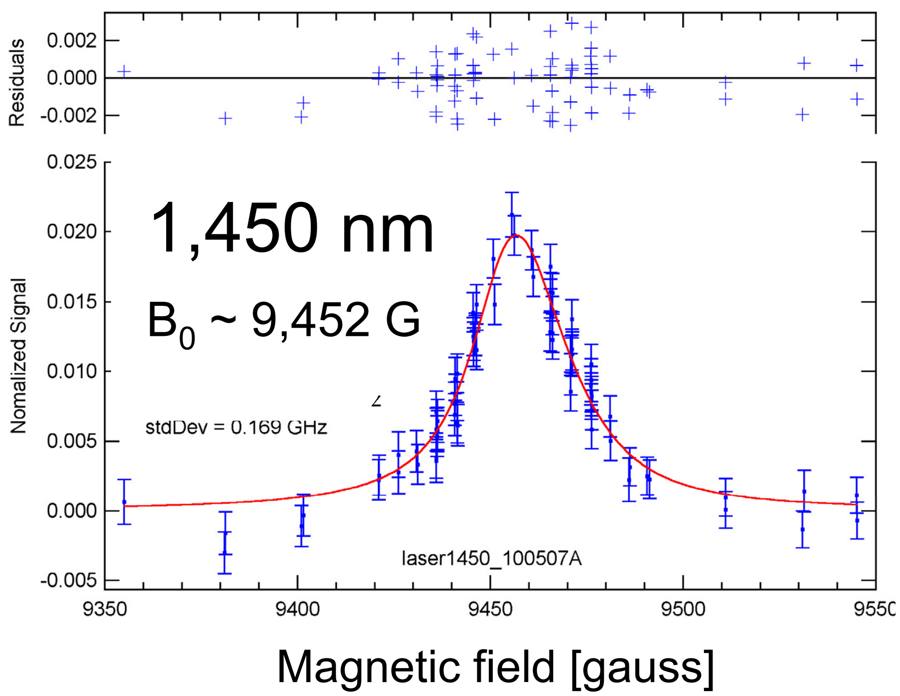
⇒Use Fiber-Bragg Grating to force single mode operation

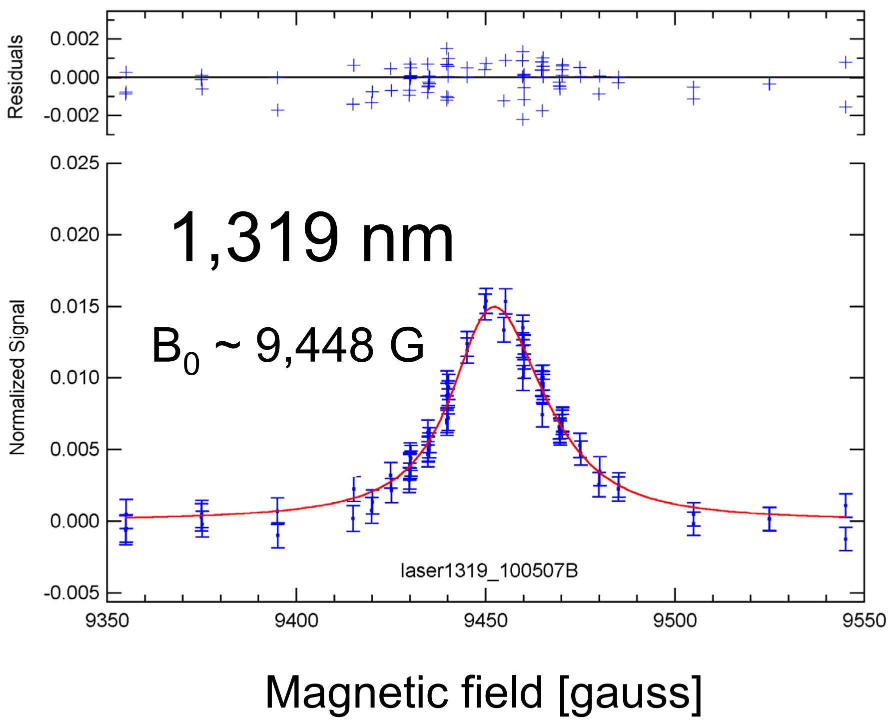
Acknowledgement: David Shiner, Ali Khademian, U. North Texas

1,450 nm, ~200 mW, locked to Power BUC

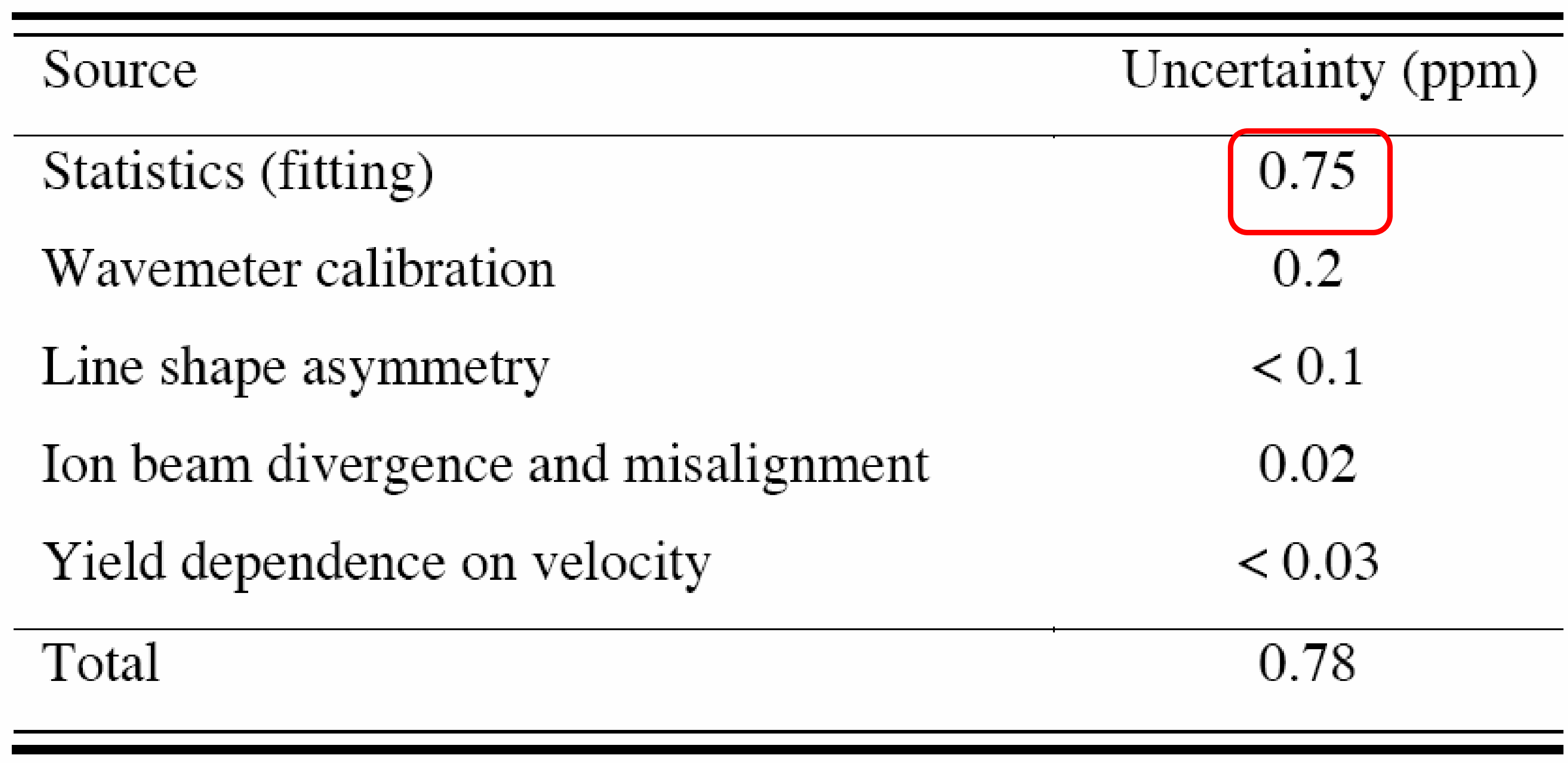


Laser-induced resonances





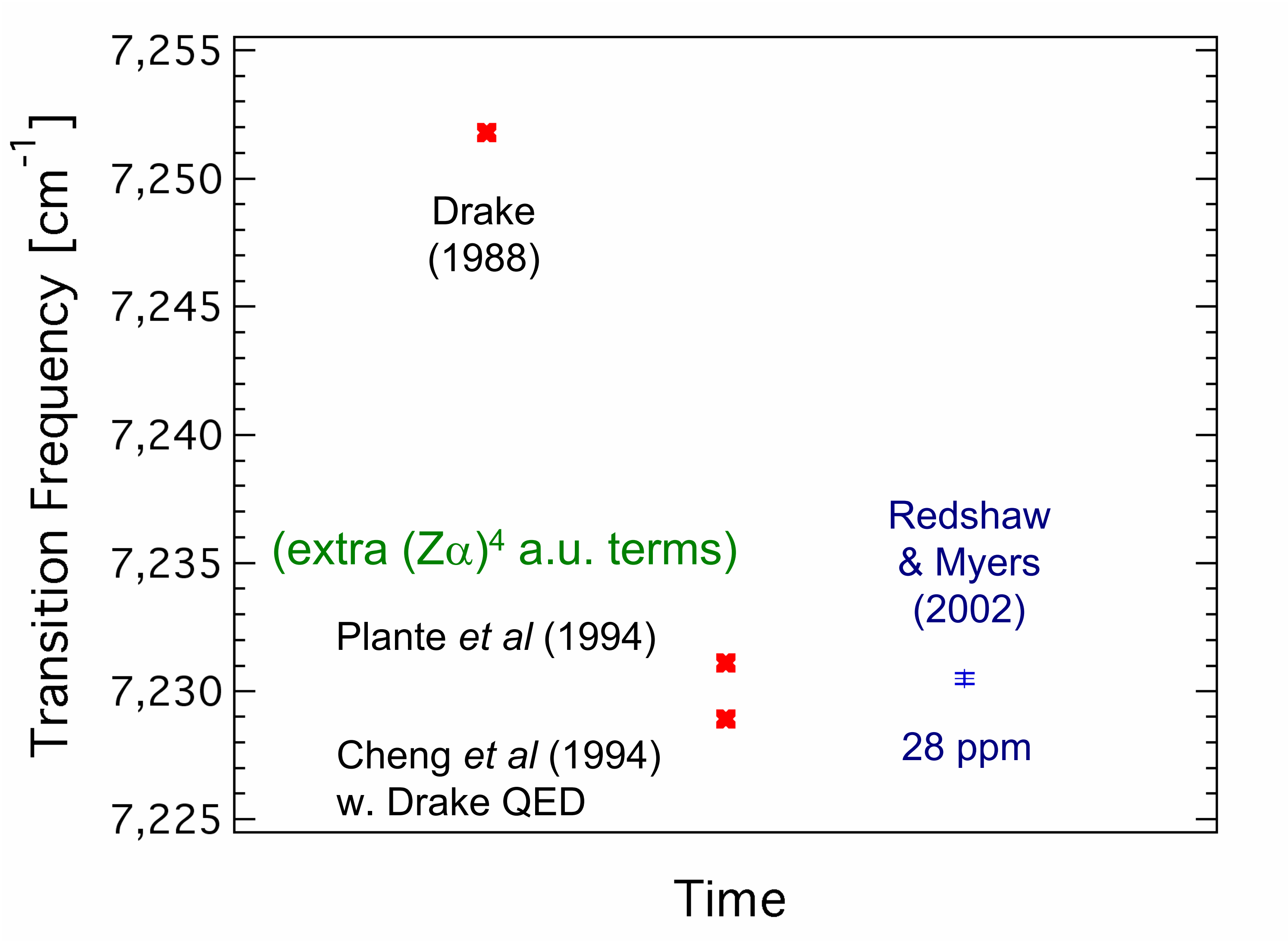
Error Budget



Source	Uncertainty (ppm)
Statistics (fitting)	0.75
Wavemeter calibration	0.2
Line shape asymmetry	< 0.1
Ion beam divergence and misalignment	0.02
Yield dependence on velocity	< 0.03
Total	0.78

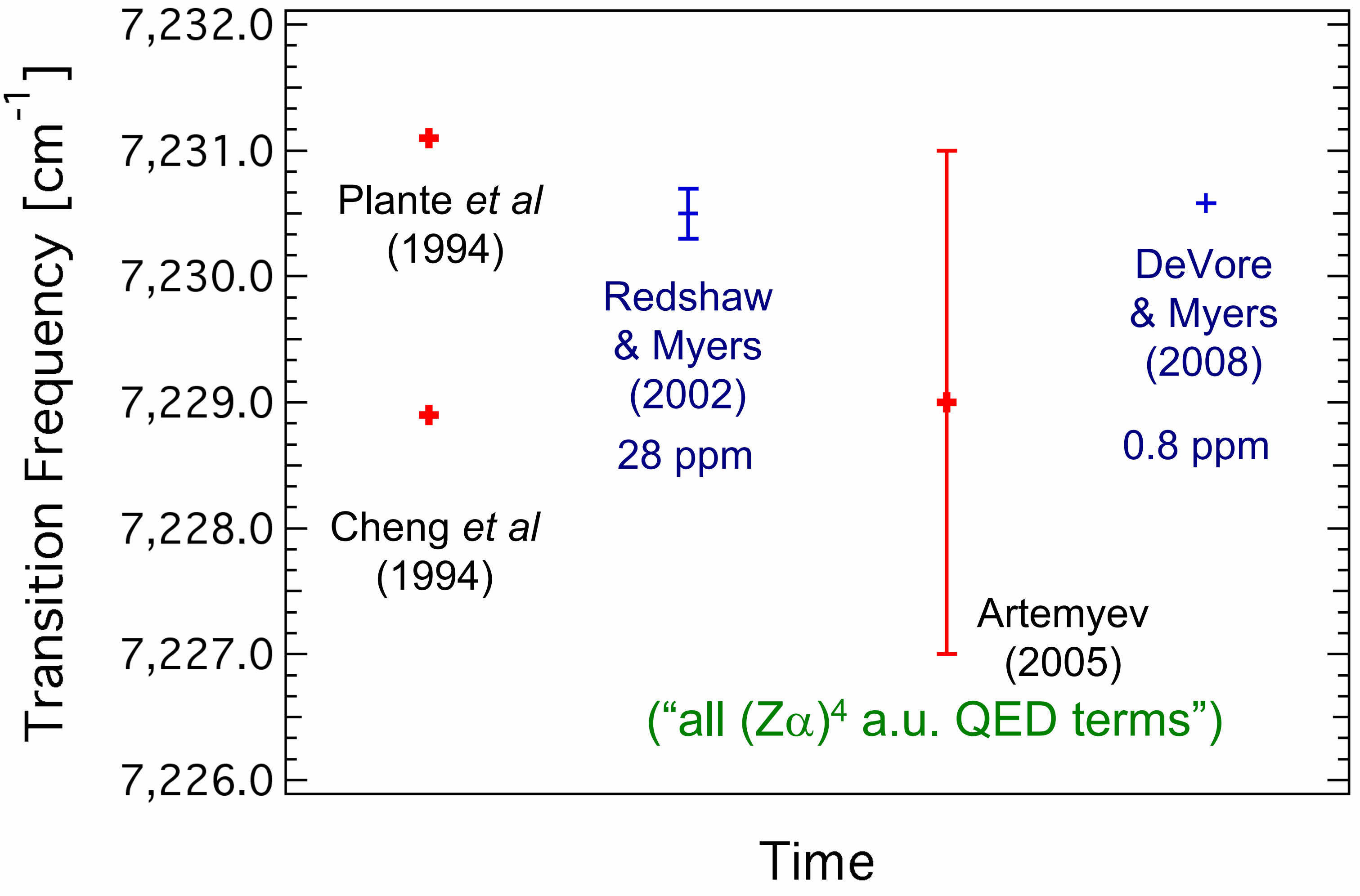
$^{28}Si^{12+}$ 1s2s $^1S_0$ – 1s2p $^3P_1$ Theory and Experiment

circa 2002



$^{28}\mathrm{Si}^{12+}$ 1s2s $^1S_0$ – 1s2p $^3P_1$ Theory and Experiment

circa 2008



$^{28}\mathrm{Si}^{12+}$ 1s2s $^1S_0$ – 1s2p $^3P_1$ Results

	(units cm^{-1})	
This Experiment ('08)	7230.585(6)	x30
Previous Experiment ('02)	7230.5(2)	improvement
Closest Theory:		
Plante, Johnson and Sapirstein('94)	7231.1	
Theory – Experiment	**0.515(6)**	**(70 ppm, 90 σ!)**

Expt'l uncertainty = 2.5 x 10^{-4} $(Z\alpha)^4$ atomic units
13 ppm of QED corrections
0.25% of nuclear size correction

Status of laser spectroscopy of moderate-*Z* He-like ions

All (low *n*) laser spectroscopy on He-like ions

	Transitions	Ions	**Z**
1)	$2\,^3S_1 - 2\,^3P_{0,1,2}$	$^{6,7}Li^+$, $^9Be^{2+}$, $^{11}B^{3+}$	≤ 5
2)	$2\,^3P_2 - 2\,^3P_1$	$^{19}F^{7+}$	9
	$2\,^3P_0 - 2\,^3P_1$	$^{24}Mg^{10+}$	12
3)	$2\,^1S_0 - 2\,^3P_{0,1}$	$^{14,15}N^{5+}$	7
	$2\,^1S_0 - 2\,^3P_1$	$^{28}Si^{12+}$	14

“Moderate Z”

2^3 P Fine Structure (units cm^{-1})

	N^{5+} 0-1	**F^{7+} 1-2**	**Mg^{10+} 0-1**
Experiment (FSU)	**8.6707(7)**	**957.8730(12)** **(1.2 ppm)**	**833.133(15)**
Zhang et al ‘96	**8.686(20)**	957.840(80)	832.335
Plante et al ‘94	8.73(2)	**957.87(2)**	**833.1(2)**
Chen et al ’93	**8.67(2)**	957.85(2)	833.3(2)
σ(theory) /σ(expt)	**~28**	**~17**	**~13**

E.G. Myers, PSAS 2000, “Hydrogen Atom”

$2\,^1S_0 - 2\,^3P_1$ Intercombination (units cm^{-1})

	N^{5+}	**Si^{12+}**	$(Z\alpha)^4$au?
Experiment	**986.3180(7)**	**7230.585(6)**	
Drake ‘88	**986.579**	7251.8	some
Cheng et al ‘94	993.6	7264.7	most
Cheng with Drake QED	985.9	7228.9	most
Plante et al ‘94	984.7	**7231.1**	most
Artemyev et al ’05		7229(2)	**all**
|Expt–Closest Theory| /σ(expt)	**~370**	**~90**	

Laser spectroscopy of He-like ions:

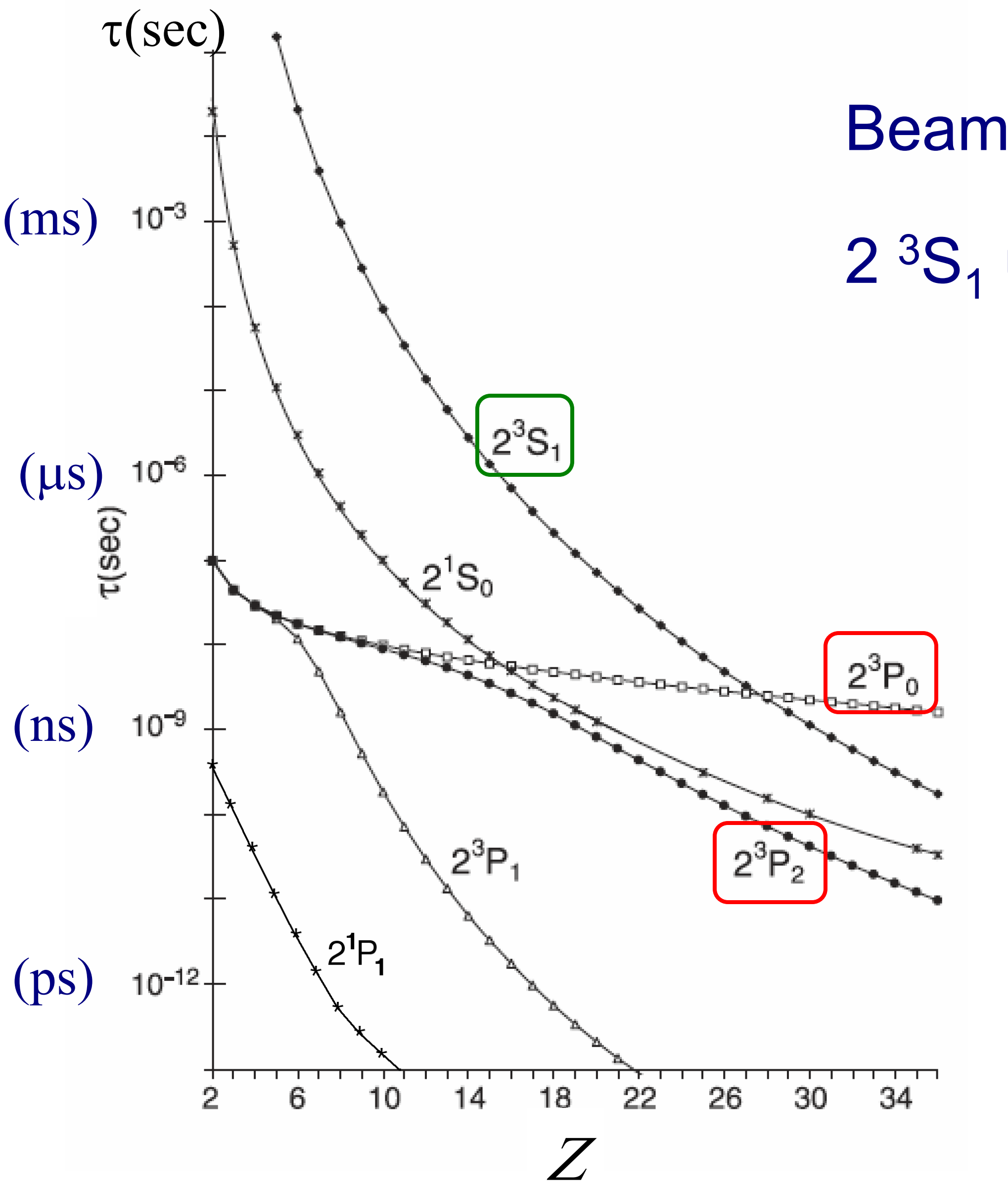
Future Directions

Low-Z: C^{4+} $2\ {}^3S - 2\ {}^3P_{0,1,2}$ (227 nm)
$2\ {}^1S - 2\ {}^3P_1$ (79 μm)

Higher-Z: Ca^{18+} $2\ {}^3S - 2\ {}^3P_{0,2}$ (59 nm, 47 nm)
VUV lasers: harmonic generation, mixing, fs comb ?
Cooled ions in a storage ring?

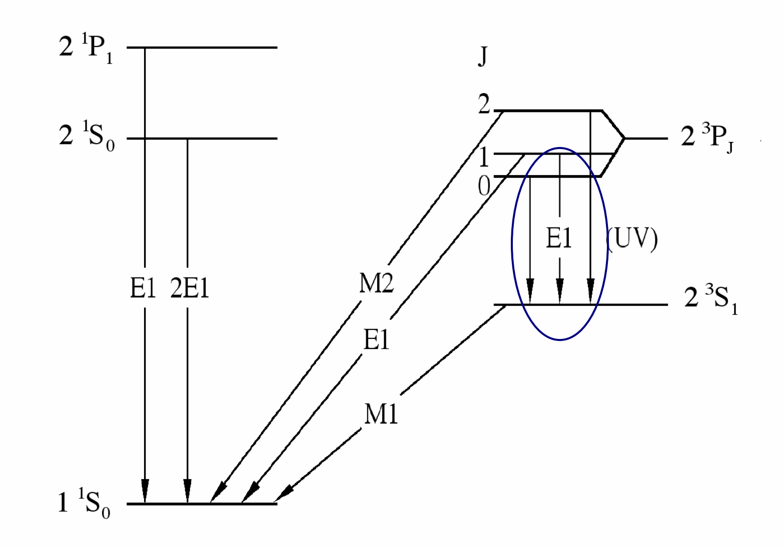
Precise measurement of He-like fine structure for FS constant ?

Mean lifetimes of $n = 2$ levels in helium-like ions



Beam velocities ~few cm/ns

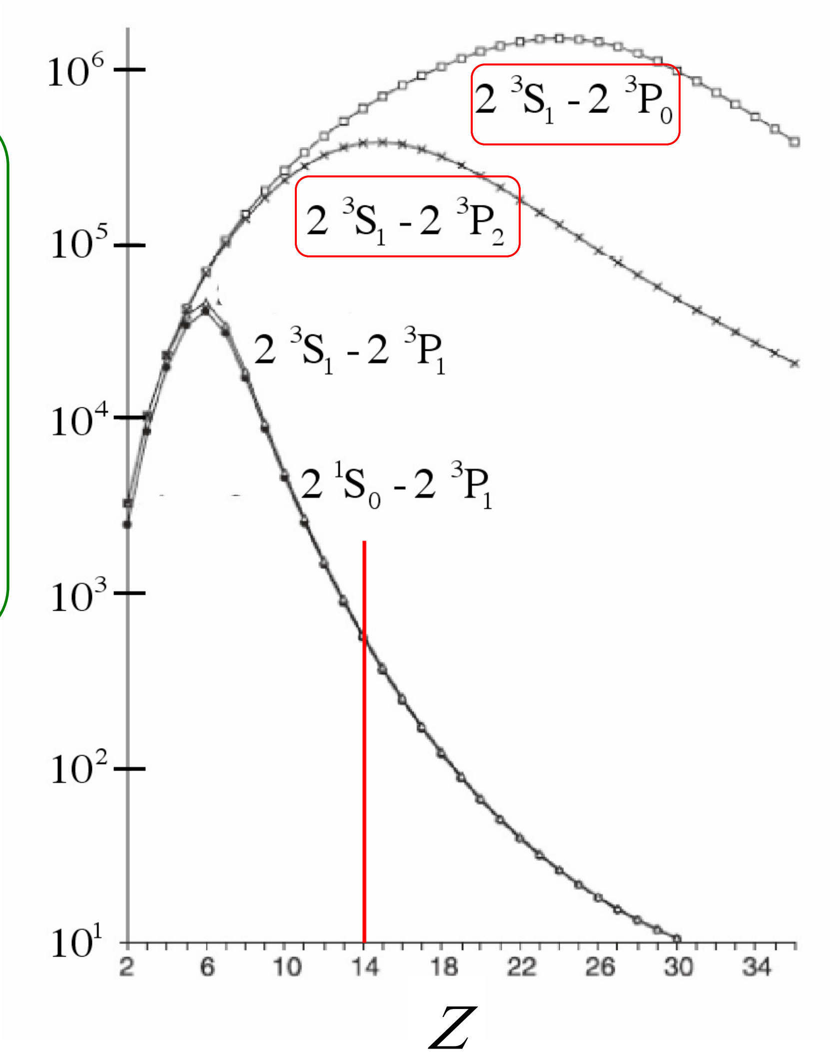
$2\ ^3S_1$ useful initial level for Z < 30



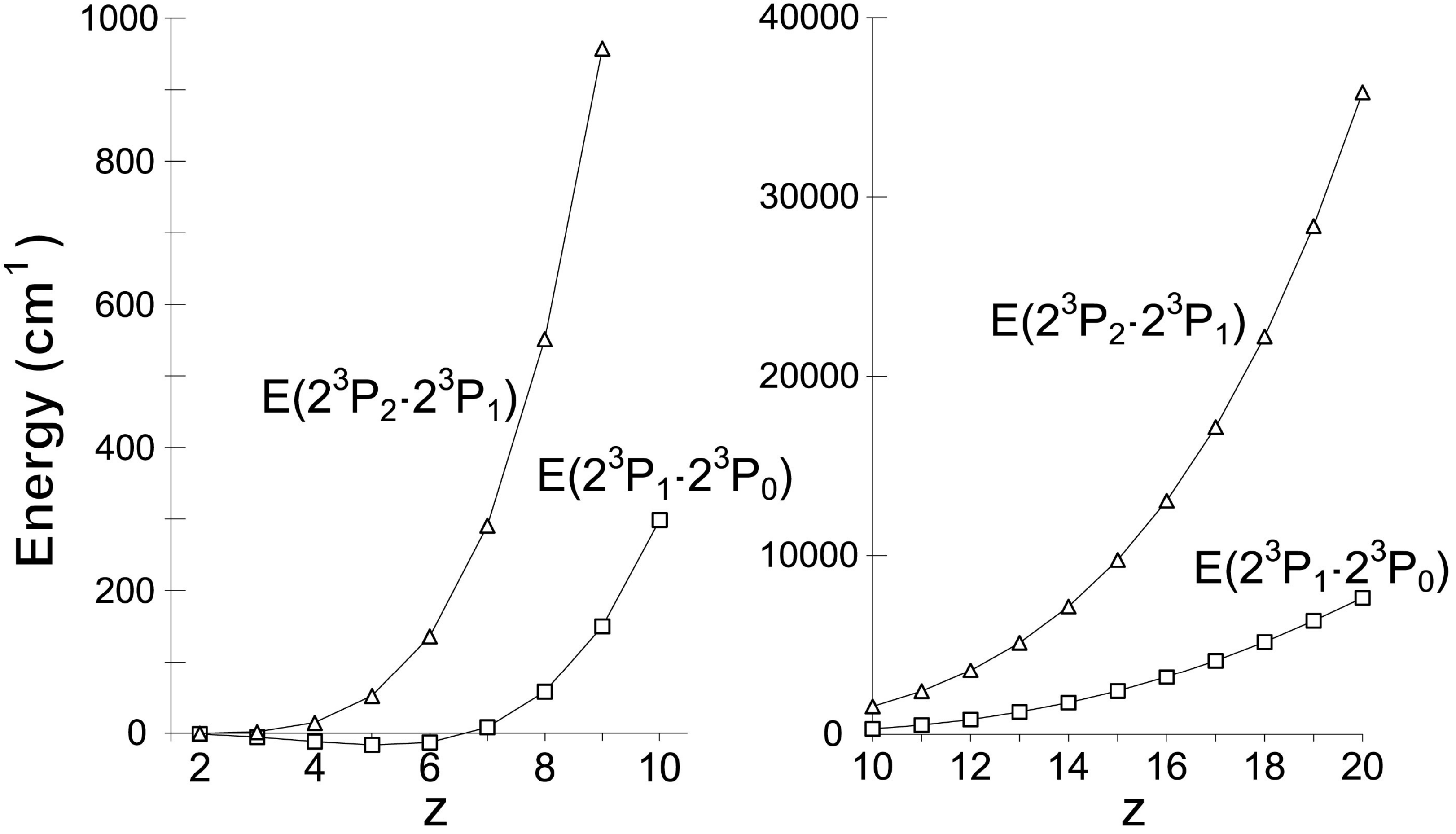
QED Sensitivity of $2\ ^3S - 2\ ^3P_{0,2}$

"QED Sensitivity"

$$\frac{\Delta E_{\rm QED}}{\hbar\Gamma}$$

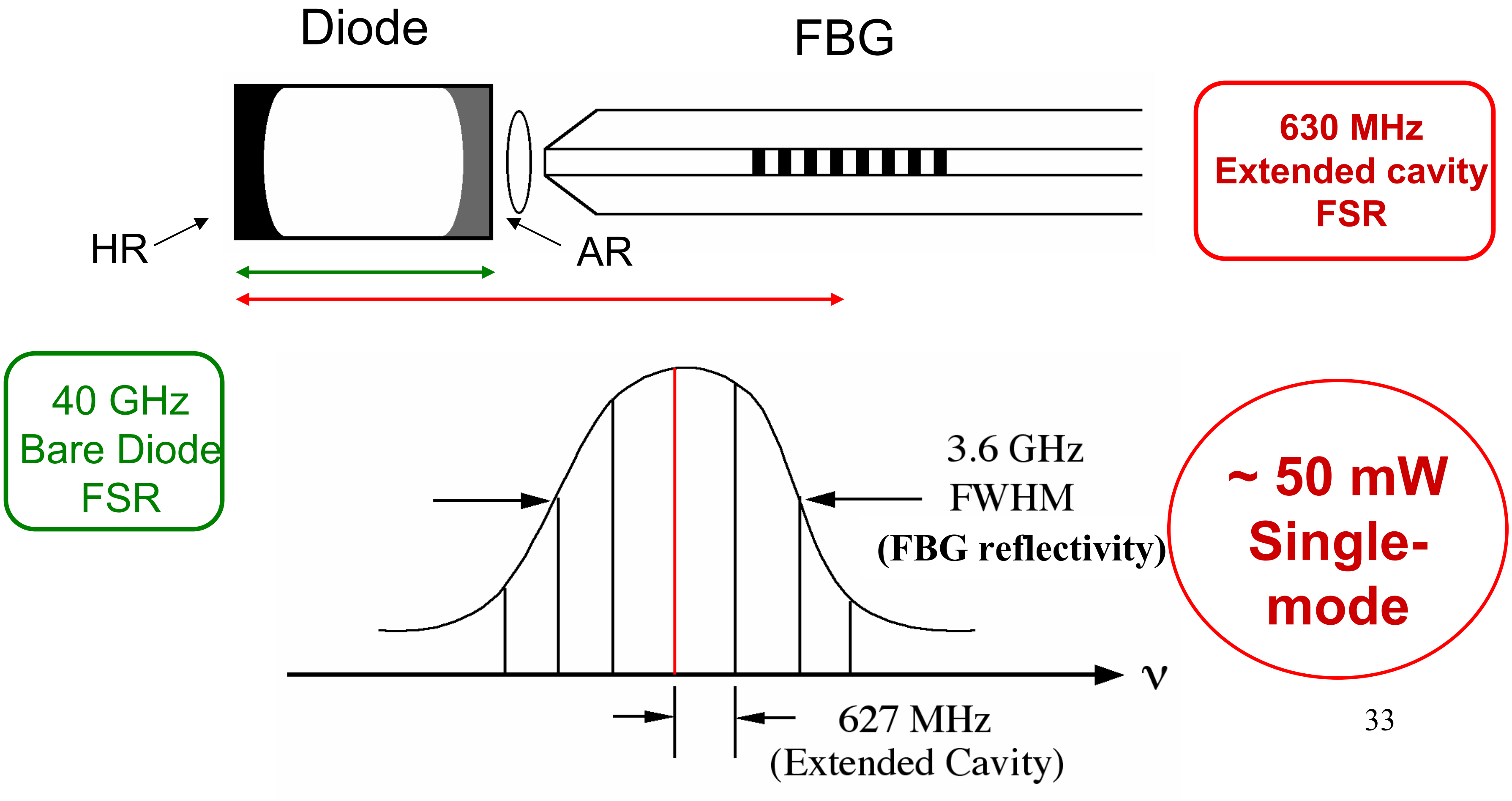


E.G. Myers, PSAS 2000
"Hydrogen Atom"


Energy (cm^{-1})
1000
800
600
400
200
0
2
4
6
8
10
Z
$E(2^3P_2 \cdot 2^3P_1)$
$E(2^3P_1 \cdot 2^3P_0)$
40000
30000
20000
10000
0
10
12
14
16
18
20
Z
$E(2^3P_2 \cdot 2^3P_1)$
$E(2^3P_1 \cdot 2^3P_0)$

Enforcing single-frequency operation & lasing at 1,450 nm (*Shiner, Khademian*)

Fiber-Bragg grating: narrow-band, wavelength selective reflector



630 MHz Extended cavity FSR

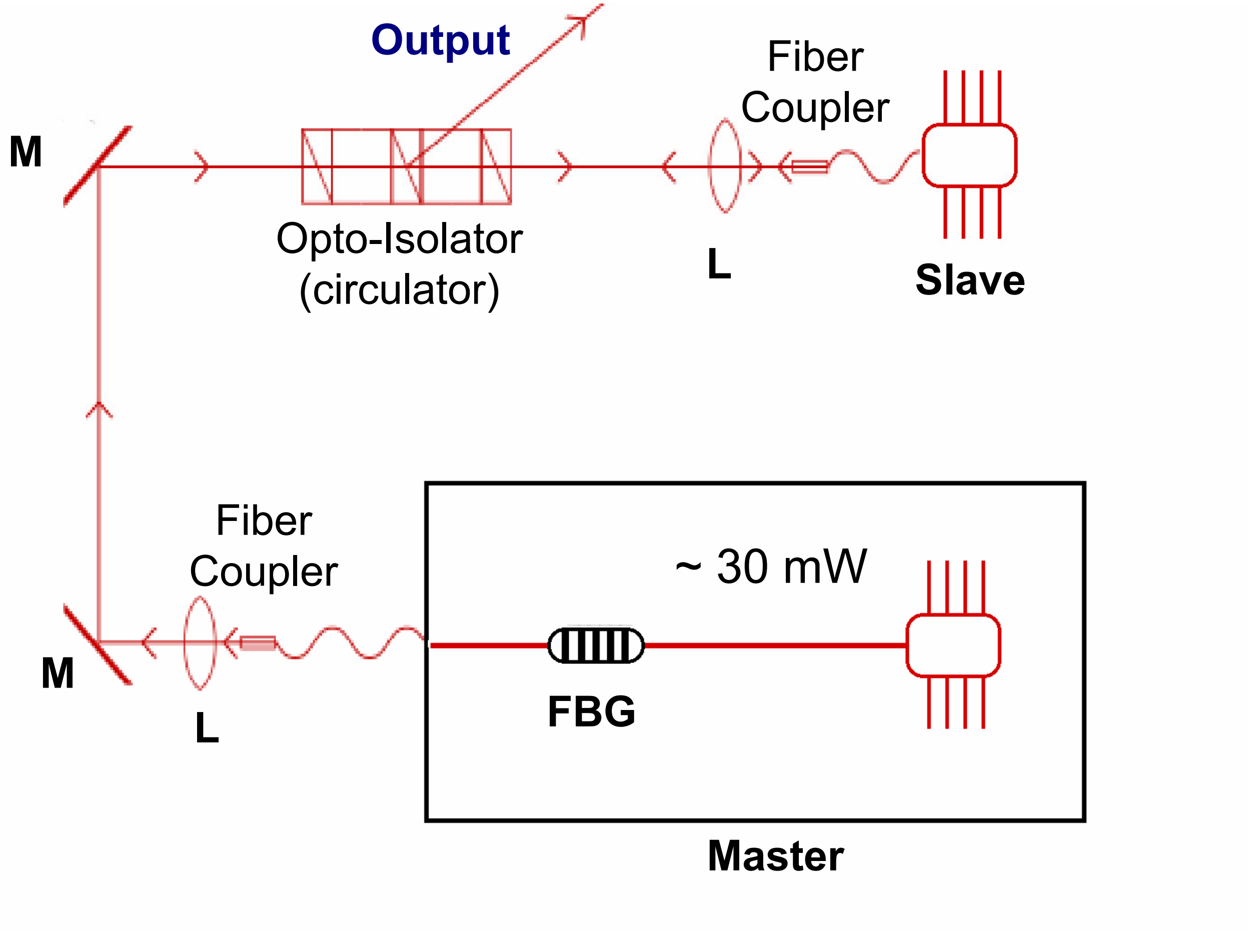
40 GHz Bare Diode FSR

~ 50 mW Single-mode

Power amplification: Injection-locking

Master laser: stable, single-frequency, low power (~ 30 mW)

Slave laser: less stable, similar λ, higher power (> 200 mW)



1,450 nm
Single-mode
~ 200 mW

Why study energy levels in Helium-like ions?

Helium-like ions vs Helium

Contributions to Energy levels vary with powers of Z

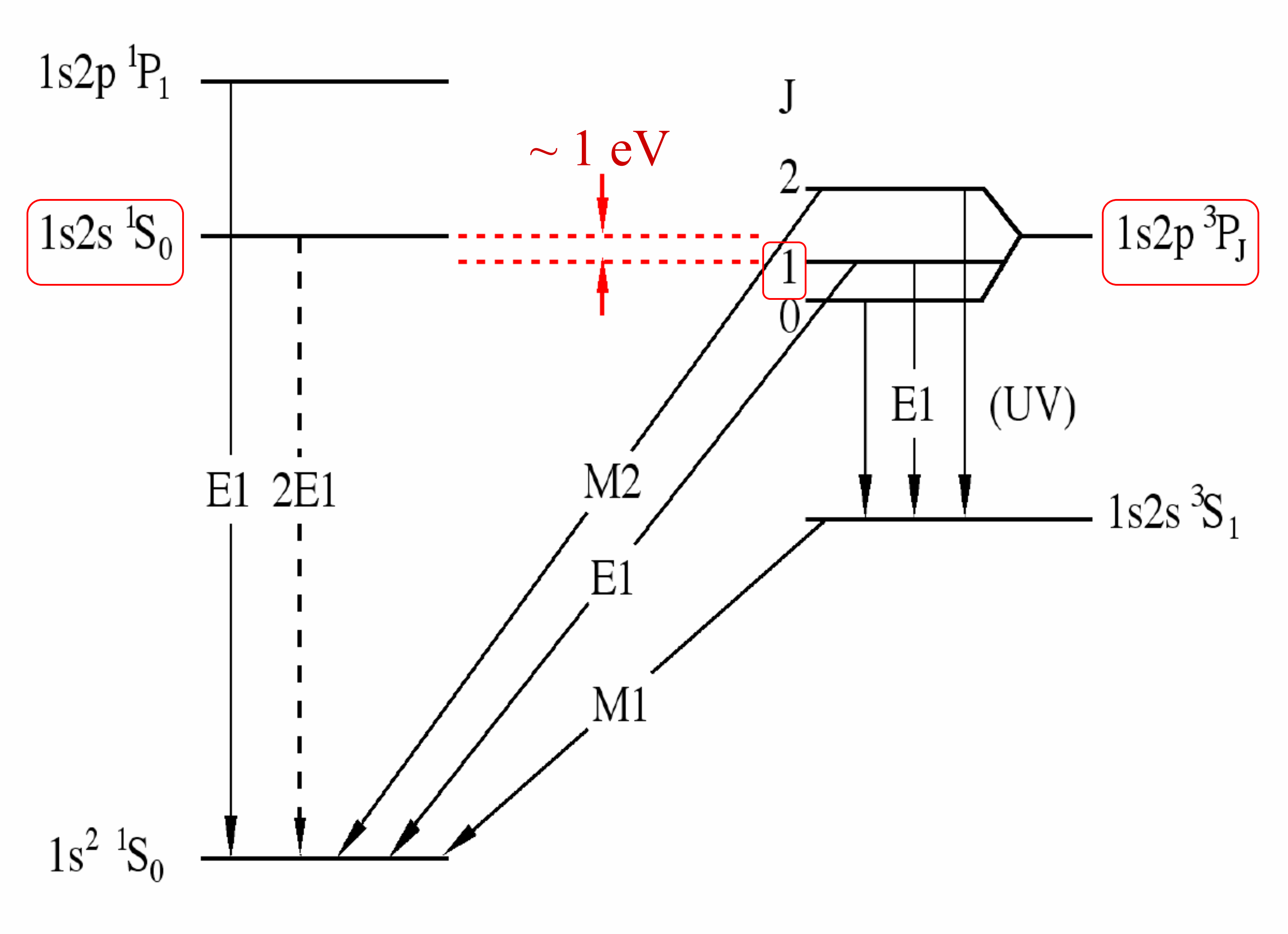
Relativistic effects: $\sim (Z\alpha)^4 mc^2\ ,\ (Z\alpha)^6 mc^2 \ldots$

QED effects: $\sim \alpha \ln\left[(Z\alpha)^{-2}\right](Z\alpha)^4 mc^2 \ldots$

Helium-like ions vs Hydrogen-like ions

1. No equivalent Dirac equation
2. QED corrections much more complicated

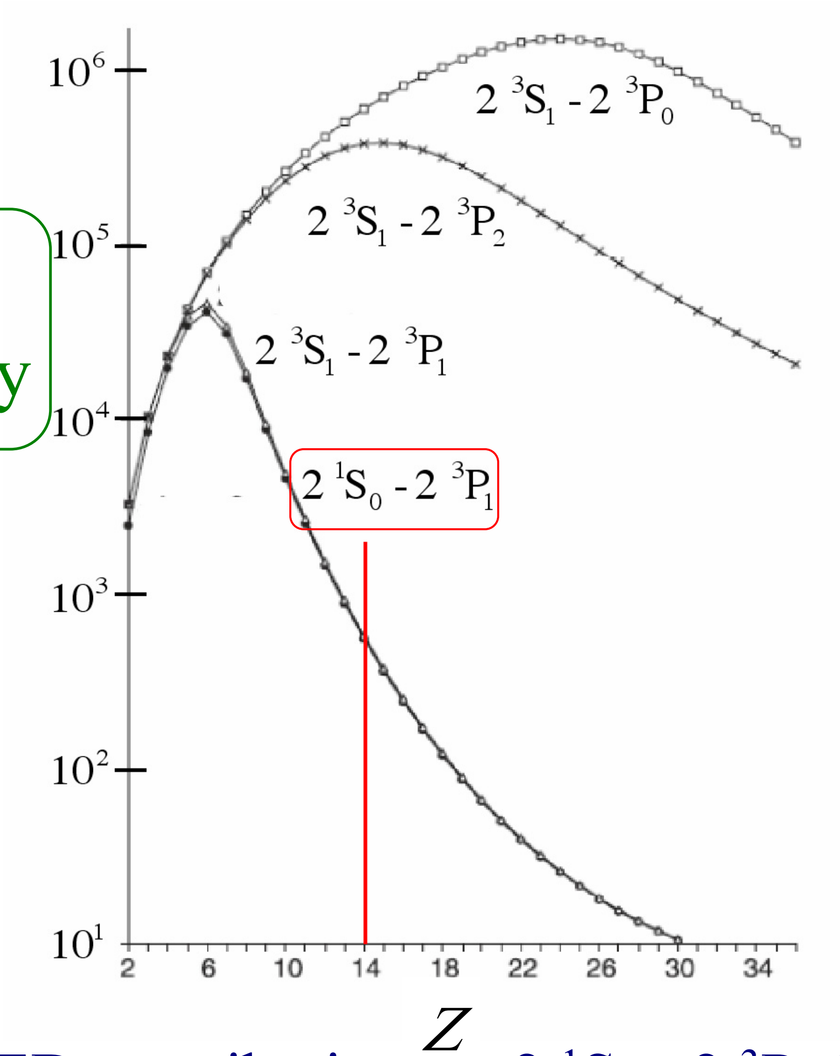
He-Like ions, *n* = 1,2



Why helium-like silicon?

QED sensitivity

$$\frac{\Delta E_{\text{QED}}}{\hbar\Gamma}$$



Want highest Z possible,

BUT

- $\tau(2\ ^3P_1)$ falls rapidly ~ Z^{-10}
- laser-induced $\frac{d\text{Prob}}{dt}$

  falls ~ Z^{-6}

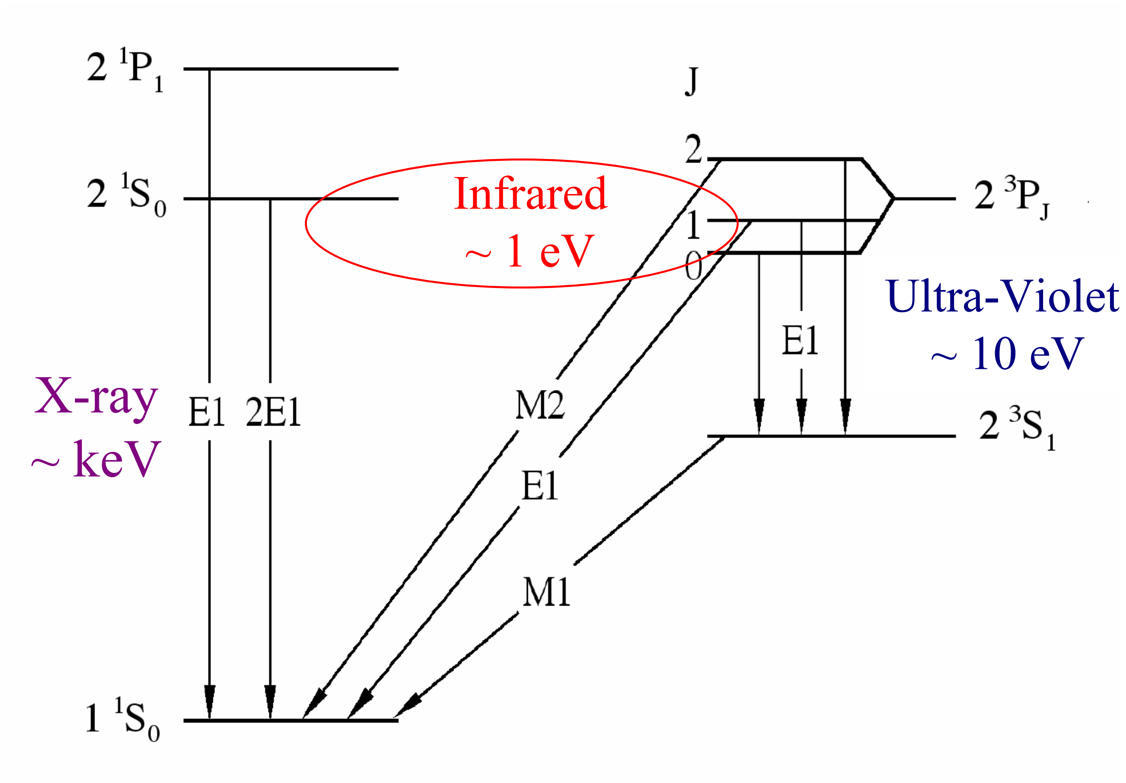
  (for a given laser intensity)

QED contributions to $2\ ^1S_0 - 2\ ^3P_1$ in silicon ~ <u>6%</u>

Why Si^{12+} 1s2s $^1S_0$ – 1s2p $^3P_1$?

- QED and relativistic effects increase with *Z*
- But, theory is more accurate than **all** UV, X-Ray spectroscopy!
- High absolute precision ⇒ **small interval + laser spectroscopy**

Si^{12+} is highest *Z* for “precise” laser spectroscopy



Our Measurement

$1s2s\ ^1S_0 - 1s2p\ ^3P_1$ interval in helium-like silicon, $^{28}Si^{12+}$