



Measurement of the
 $1s2s\ ^1S_0 - 1s2p\ ^3P_1$ Interval in Helium-like Silicon
by Fast-Beam Laser Spectroscopy

Edmund Myers, Thomas DeVore, Matthew Redshaw
(*Florida State*), David Crosby (*Oxford*)

\$\$ from US-NSF and NIST

Why study mid-Z Helium-like ions?

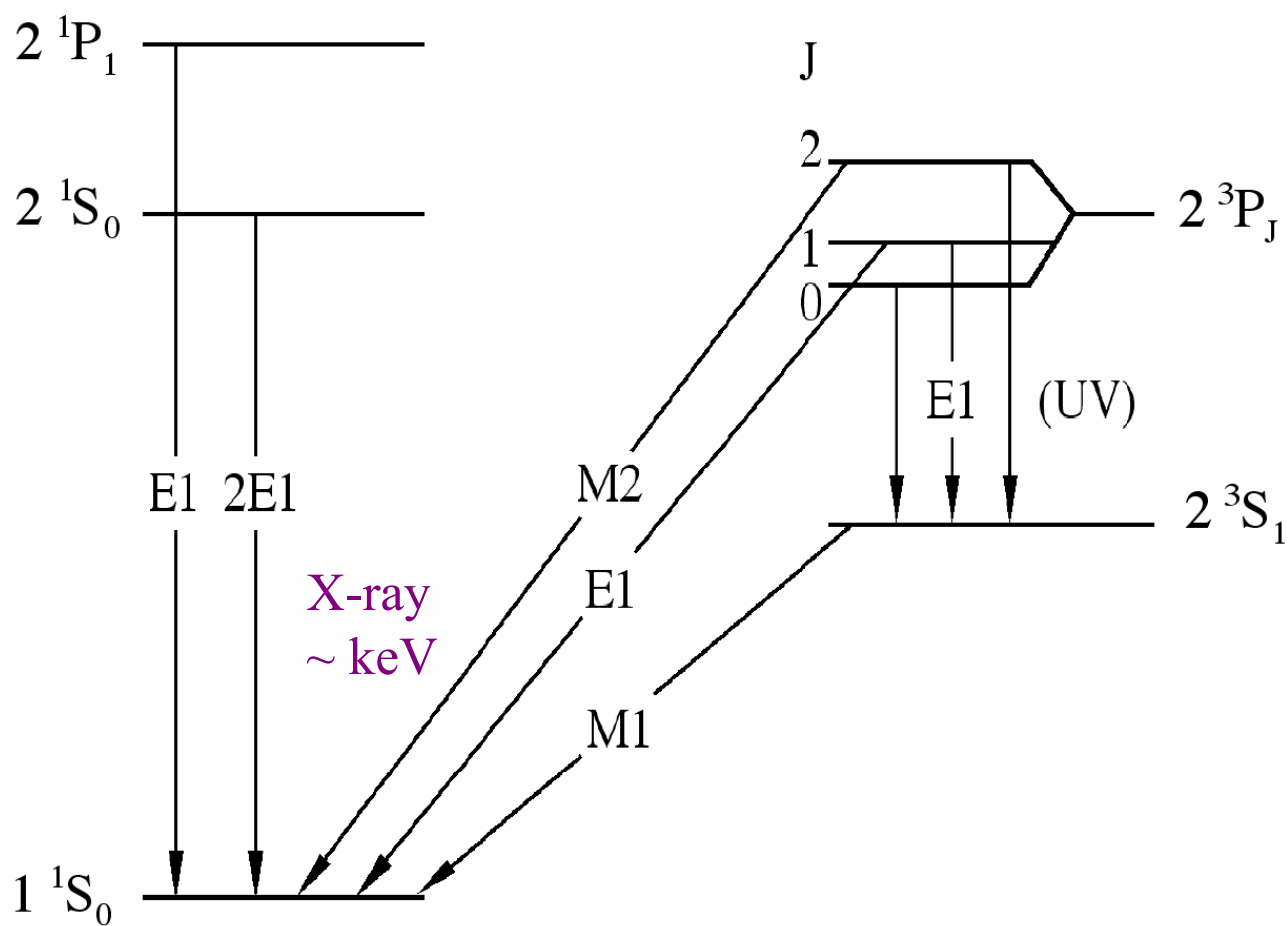
Fundamental multi-electron atom!

Only two electrons + Z/r Coulomb potential...

Test ground for *relativistic many-body theory*
Correlated electrons plus bound-state QED

Why study mid-Z Helium-like ions?

Fine structure constant from He $1s2p\ ^3P$ Fine structure



PRL 97, 013002 (2006)

Improved Theory of Helium Fine Structure

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(Received 22 February 2006; published 5 July 2006)

Improved theoretical predictions for the fine-structure splitting of 2^3P_J levels in helium are obtained by the calculation of contributions of order α^5 Ry. New results for transition frequencies $\nu_{01} = 29\,616\,943.01(17)$ kHz and $\nu_{12} = 2\,291\,161.13(30)$ kHz disagree significantly with the experimental values, indicating an outstanding problem in bound state QED.

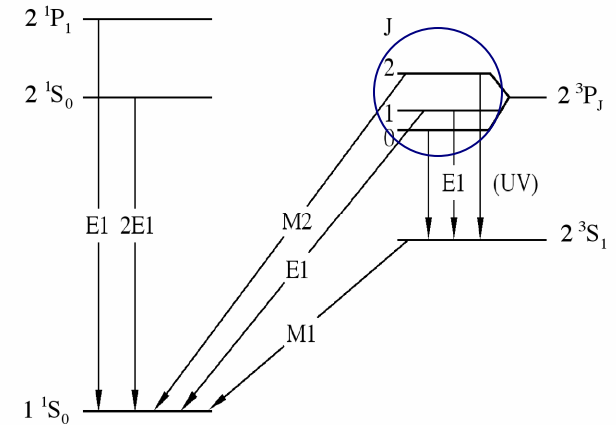
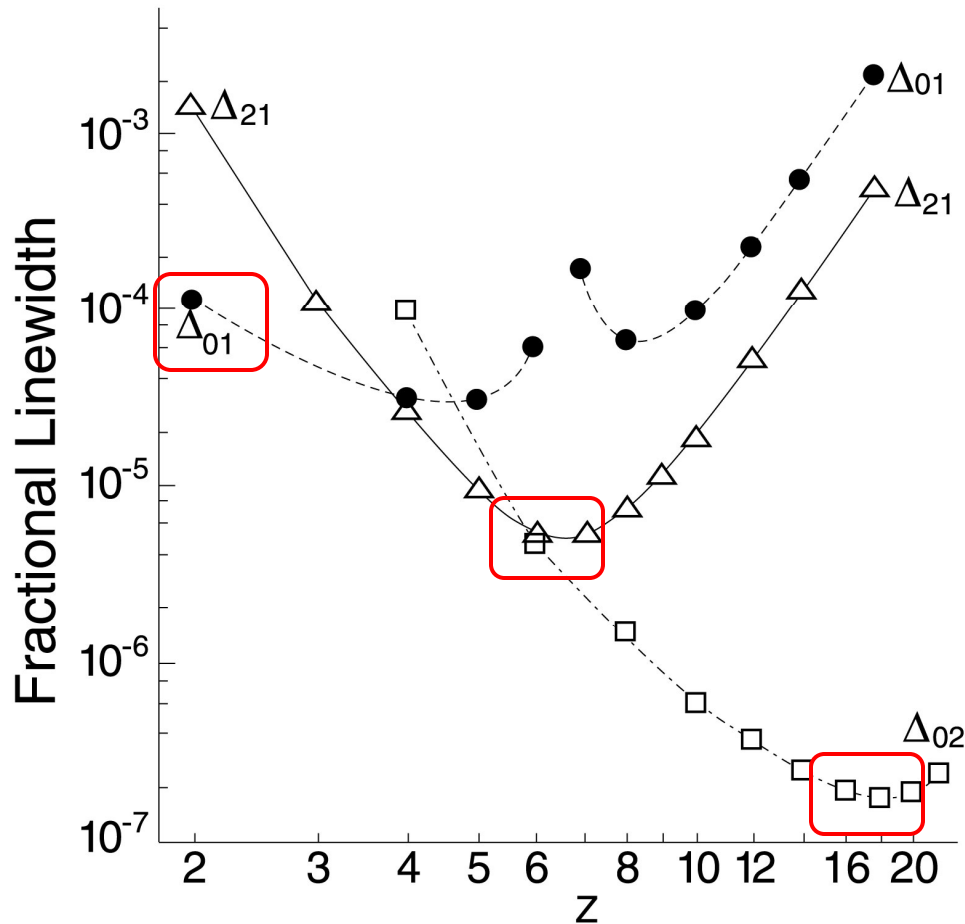
$2^3P_0 - 2^3P_1$ Fine-structure in Helium

Theory: $29,616,943.01(17)$ Hz

Experiment: $29,616,951.66(70)$ Hz

$\Rightarrow 0.3$ ppm discrepancy [$\alpha(g-2)$ 0.37 ppb]

Fractional line width of $2\ ^3P$ fine structure



*E.G. Myers, PSAS 2000
"Hydrogen Atom"*

For α from FS to compete with $g-2$ need 1ppb precision

Theory starting points

$$H_{\text{Non-Rel}} = \frac{-\hbar^2}{2m} (\bar{\nabla}_1^2 + \bar{\nabla}_2^2) - \frac{Ze^2}{r_1} - \frac{Ze^2}{r_2} + \frac{e^2}{r_{12}}$$

Low Z: Non-Relativistic
Schrödinger Hamiltonian

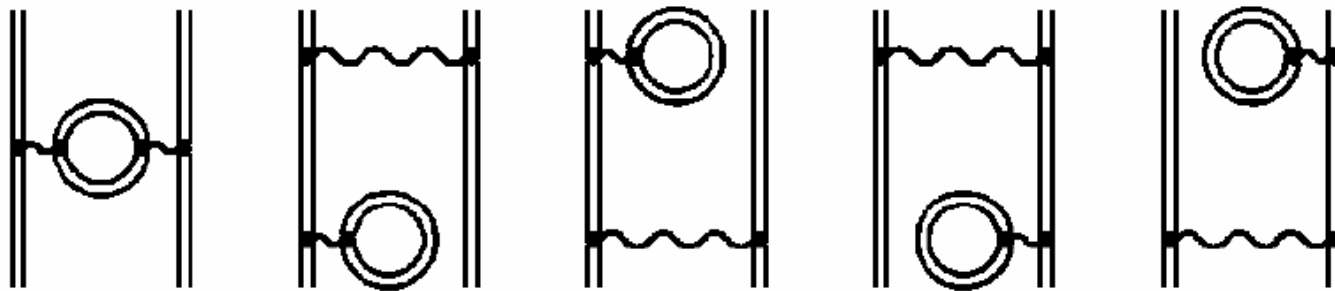
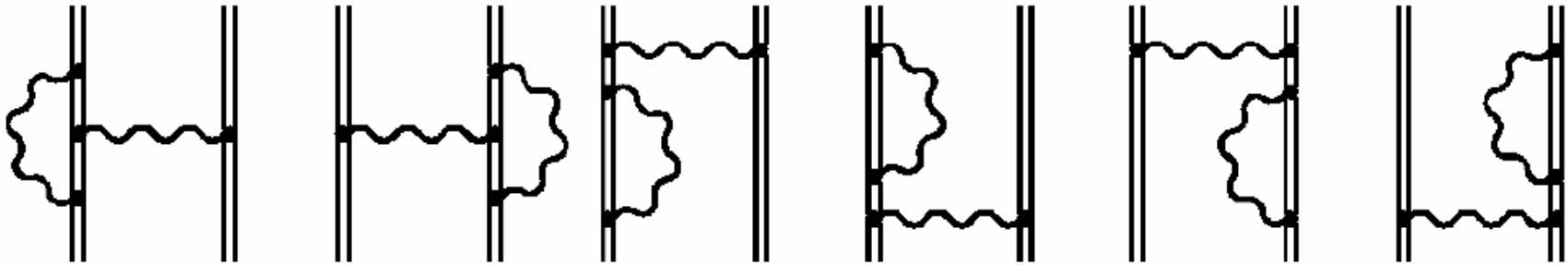
$$H_{\text{Dirac}} = c\bar{\alpha} \cdot \bar{p} + \beta mc^2 - \frac{Ze^2}{r}$$

High Z: One-electron
Dirac Hamiltonian

$$H = H_{1,\text{Dirac}} + H_{2,\text{Dirac}} + \frac{e^2}{r_{12}} + H_{\text{Breit}}$$

Mid Z: Breit Hamiltonian

$$H_{\text{Breit}} = -\frac{e^2}{2r_{12}} \left[\bar{\alpha}_1 \cdot \bar{\alpha}_2 + \frac{(\bar{\alpha}_1 \cdot \bar{r}_{12})(\bar{\alpha}_2 \cdot \bar{r}_{12})}{r_{12}^2} \right]$$



Two-electron QED corrections:
 Self-energy screening
 vacuum polarization screening

“Recent” Theory (numerical results $n=2$)

Range of Z

“Unified Method + QED”	Drake	Can JP 1988	2 - 100
“Schrodinger + QED”	Zhang, Yan, Drake	PRL 1996	2 - 12
	Busuttil, Drake	Web 2008	2 - 18

“No-pair Breit-Dirac + QED”			
RMBPT	Johnson, Sapirstein	PRA 1992	10 - 36
AOMPT	Plante, Johnson, Sapirstein	PRA 1994	3 - 100
RCI	Chen, Cheng, Johnson	PRA 1993	5 - 100
	Cheng, Chen, Johnson, Sapirstein	PRA 1994	4 - 92
	Cheng, Chen	PRA 2000	22 - 36

“Dirac + QED”			
	Artemyev, Shabaev, Yerokhin, Plunien, Soff	PRA 2005	12 -100

A/so Mohr, Sapirstein	PRA 2000, etc	
Asen, Salomonson, Lindgren	PRA 2002	“Merging MBPT with QED”
Lindgren, Salomonson, Hedendahl	PRA 2006, etc.	

UV and X-ray spectroscopy in emission?

Artemyev, Shabaev, Yerokhin, Plunien and Soff, PRA 71, 2005

(Theory Paper) Compares experiment and theory for $12 < Z < 100$

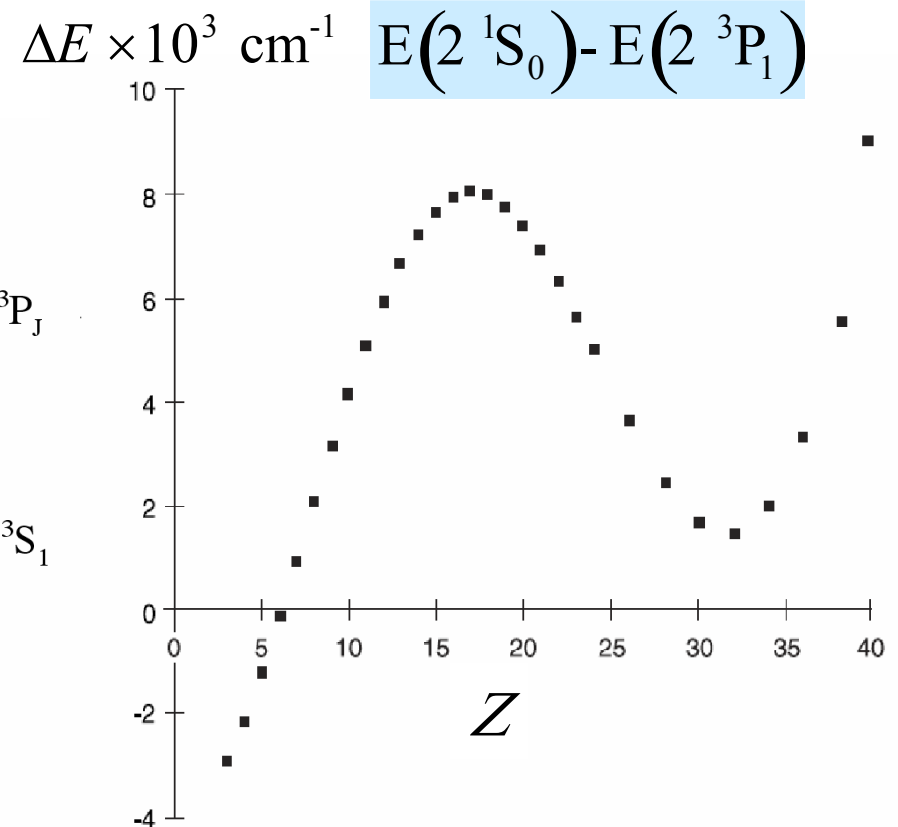
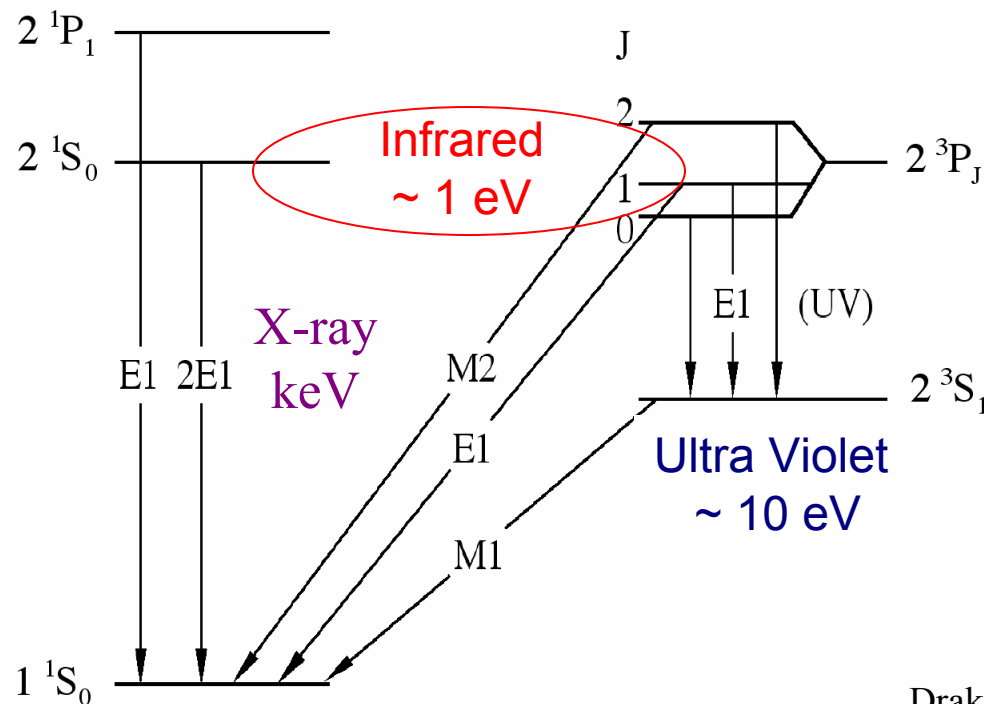
Only one non-laser experimental result has precision to differentiate Artemyev et al from Plante et al and Chen et al.

($Z=18$), where the experimental determination of the $2^3P_{0,2}-2^3S_1$ transition energies by Kukla *et al.* [84] demonstrated a 2σ deviation from the previous theoretical results. Our calculation brings the theoretical and experimental results into agreement for the $2^3P_0-2^3S_1$ transition and reduces the discrepancy for the $2^3P_2-2^3S_1$ transition to 0.5σ .

Kukla, Livingston, Suleiman, Berry, Dunford, Gemmell, Kanter, Cheng, Curtis PRA 51 1995

Why measure the $2\ ^1S_0 - 2\ ^3P_1$ interval?

- In IR for $Z < 40$, partly allowed E1 \Rightarrow laser spectroscopy
 \Rightarrow small interval \Rightarrow high *absolute* precision
- S-state \Rightarrow sensitive to QED



Drake, G.W., *Canadian Journal of Physics*, **88** (1988) 586

Why Helium-like Silicon?

Mean Lifetime

Want highest Z possible

$\tau (2^3P_1)$ falls rapidly $\sim Z^{-10}$

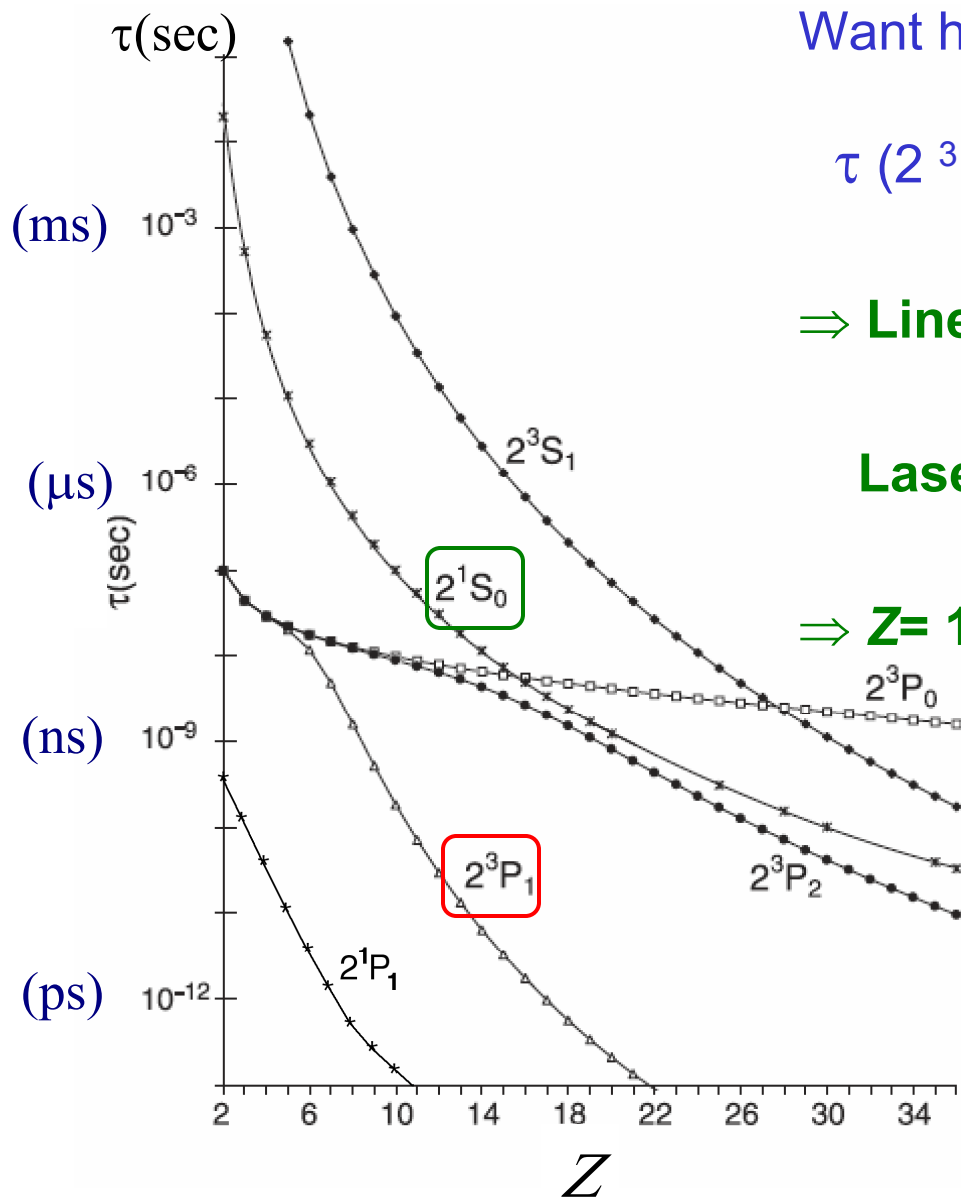
\Rightarrow Line width increases as Z^{10}

Laser induced transition rate falls $\sim Z^{-6}$

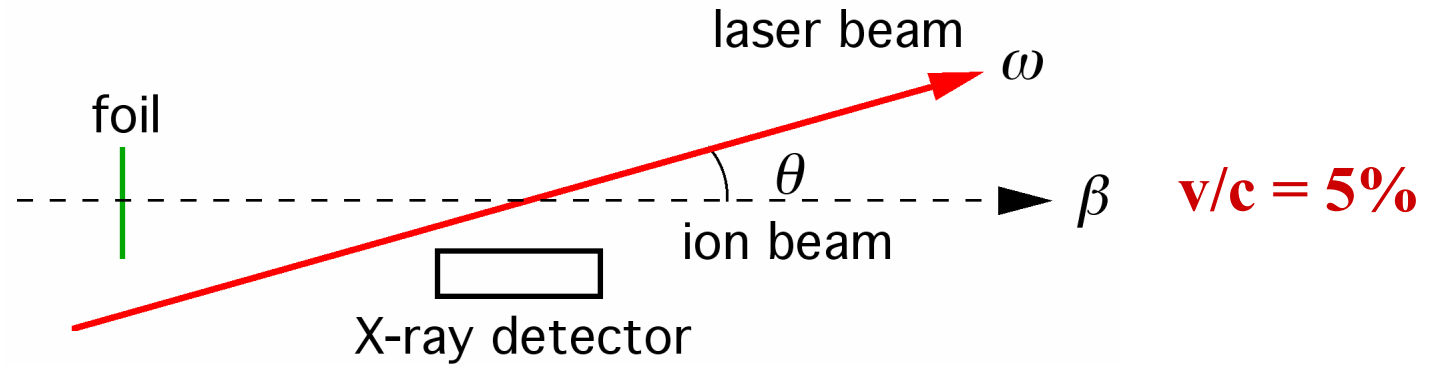
$\Rightarrow Z=14$ is experimental compromise

QED contributions to

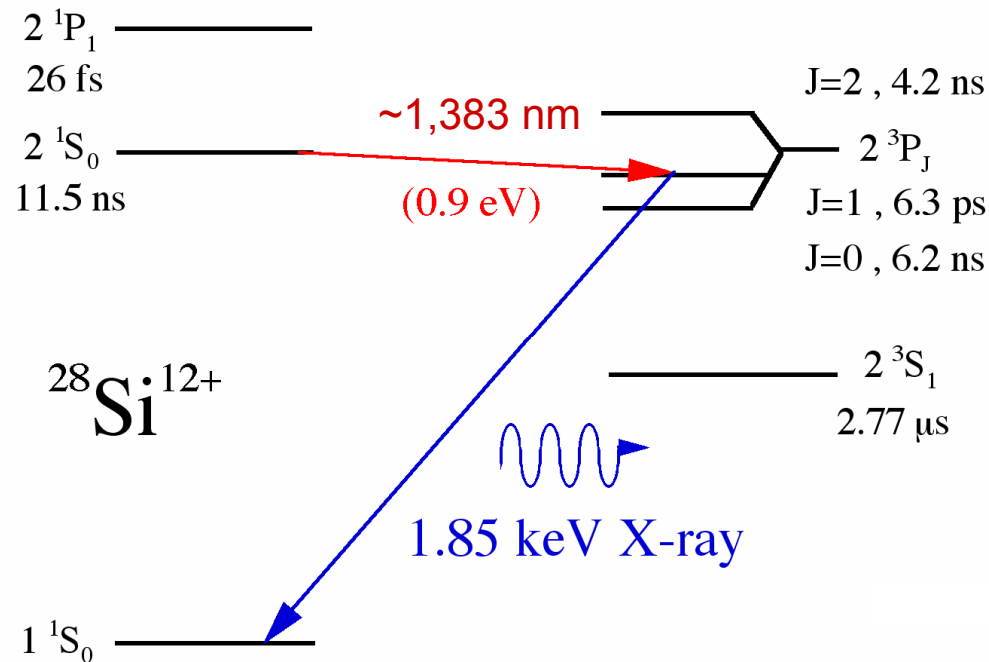
$2^1S_0 - 2^3P_1$ in $\text{Si}^{12+} \sim 6\%$



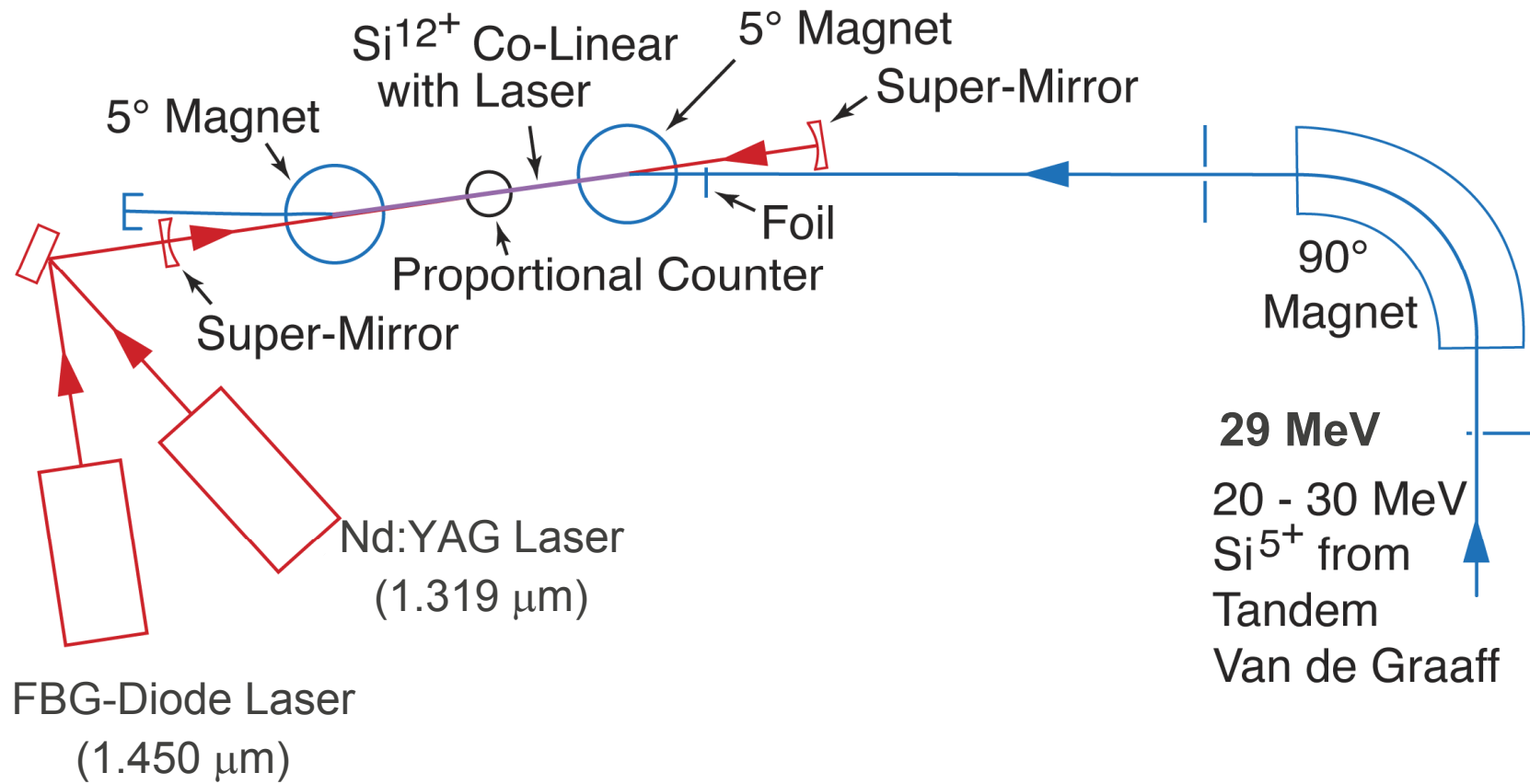
Fast-Beam Laser Resonance Technique



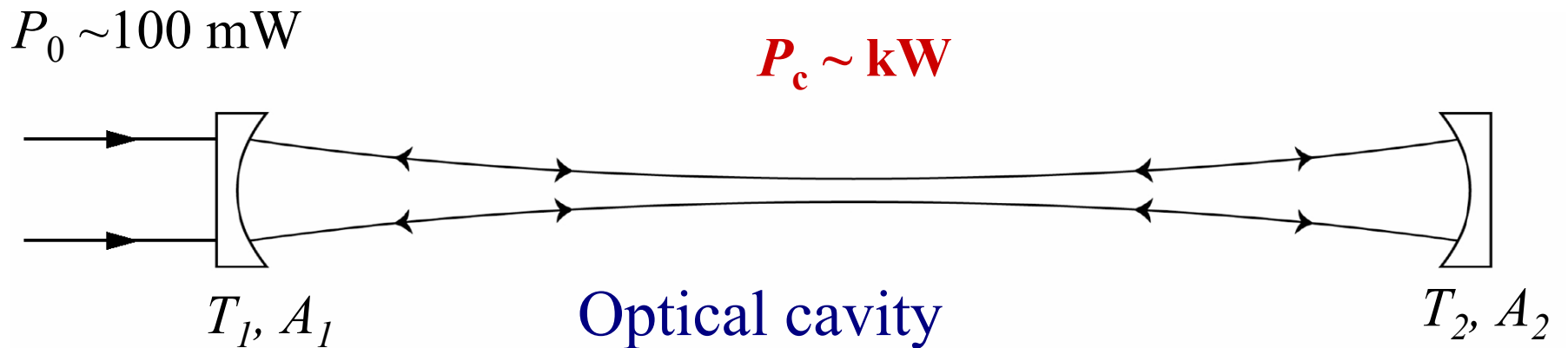
Relativistic Doppler formula $\omega' = \omega_l \gamma (1 \pm \beta \cos \theta)$



Experimental Setup



High-Finesse Power-Build-up Cavity



Super mirrors: $R > 99.995\%$

Resonance width $\sim 3 \text{ kHz} !$

$$\frac{P_c}{P_0} = \frac{4T_1}{(A_1 + A_2 + T_1 + T_2)^2}$$

$$T \leq 50 \text{ ppm}$$

Previous Measurement:

VOLUME 88, NUMBER 2

PHYSICAL REVIEW LETTERS

14 JANUARY 2002

Measurement of the $1s2s\ ^1S_0$ - $1s2p\ ^3P_1$ Intercombination Interval in Helium-like Silicon

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(Received 31 August 2001; published 28 December 2001)

Using Doppler-tuned fast-beam laser spectroscopy the $1s2s\ ^1S_0$ - $1s2p\ ^3P_1$ intercombination interval in $^{28}\text{Si}^{12+}$ has been measured to be $7230.5(2)\text{ cm}^{-1}$. The experiment made use of a single-frequency Nd:YAG($1.319\ \mu\text{m}$) laser and a high-finesse optical buildup cavity. The result provides a precision test of modern relativistic and QED atomic theory.

DOI: 10.1103/PhysRevLett.88.023002

PACS numbers: 31.30.Jv, 32.30.Bv

- Single co-propagating laser at 1,319 nm
- 28 ppm precision
- Limited by uncertainty in beam velocity

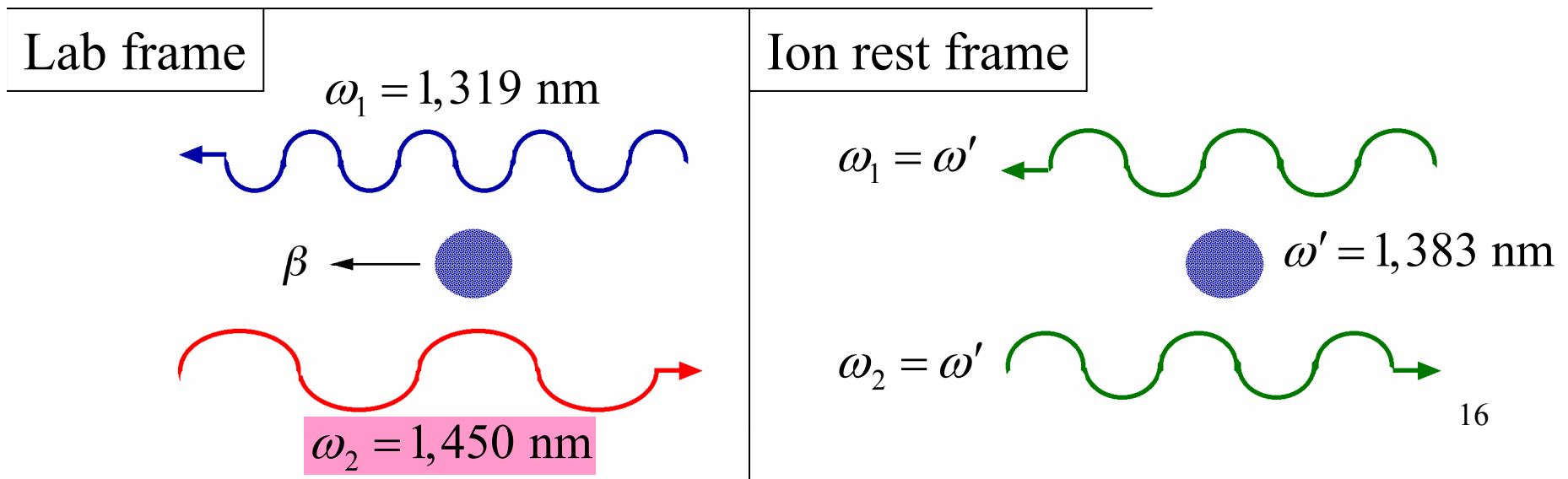
Two laser Doppler-shift cancellation

(c.f. $\text{Li}^+ 2^3S_1 - 2^3P_J$ at TSR, ESR storage rings to test special relativity)

Co-propagating: $\omega'_1 = \omega_1 \gamma_1 (1 - \beta_1 \cos \theta_1)$

Counter-propagating: $\omega'_2 = \omega_2 \gamma_2 (1 + \beta_2 \cos \theta_2)$

Doppler free result: $\omega' = \sqrt{\omega_1 \omega_2}$ $\beta_1 = \beta_2$, $\theta_1 = \theta_2 = 0$



Doppler-shift cancellation cont...

In practice, we can easily have

$$|\beta_1 - \beta_2| < 0.0005 \quad , \quad |\theta_1|, |\theta_2| < 0.01$$

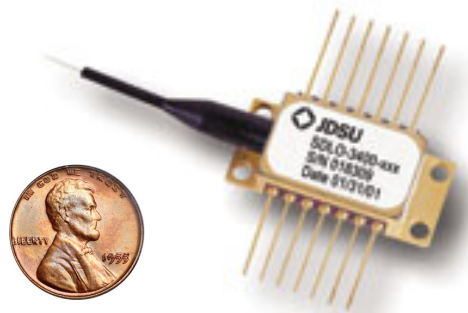
Hence,
$$\omega' = (\omega_1 \omega_2)^{1/2} \left[1 + f \left\{ \Delta p, \bar{p}, \Delta(\theta^2), \bar{\theta}^2 \right\} \right]^{1/2}$$

$$f \approx \Delta p \left(1 + \frac{\Delta p}{2} - \frac{\bar{p}^2}{2} \right) - \frac{\Delta(\theta^2)}{2} \bar{p} \left(1 + \frac{\bar{p}^2}{4} \right) + \bar{\theta}^2 \left(\frac{\bar{p}^2}{2} - \frac{\Delta p}{2} \right) + \dots$$

where, $\Delta p \equiv (\beta_2 \gamma_2 - \beta_1 \gamma_1)$ = difference in ion beam rigidity.

Mainly sensitive to Δp

300 mW, fiber coupled, diode lasers at 1,450 nm
(pumps for Raman amplifiers)

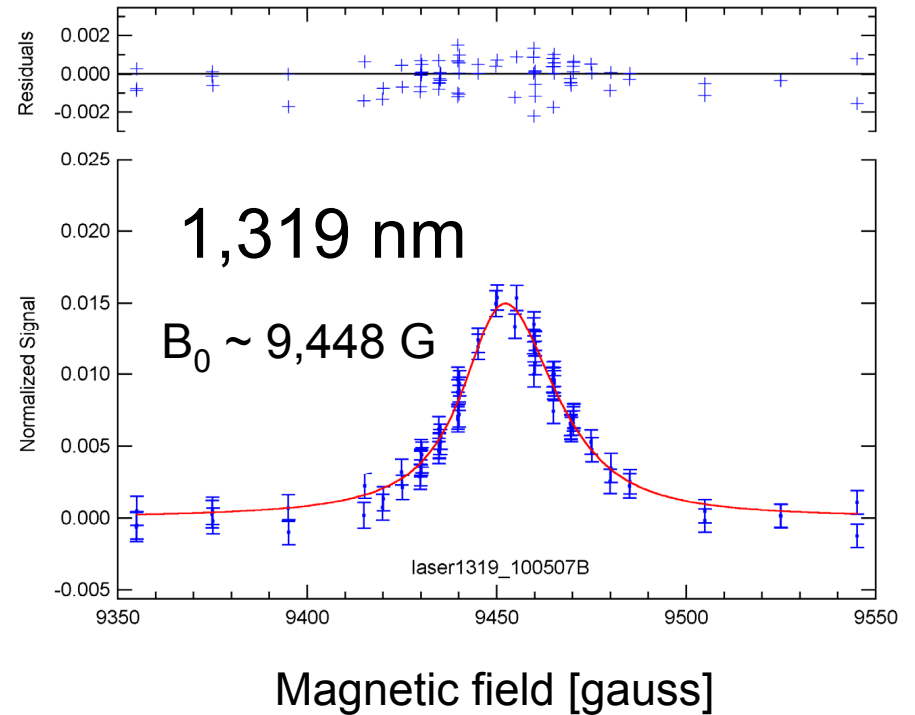
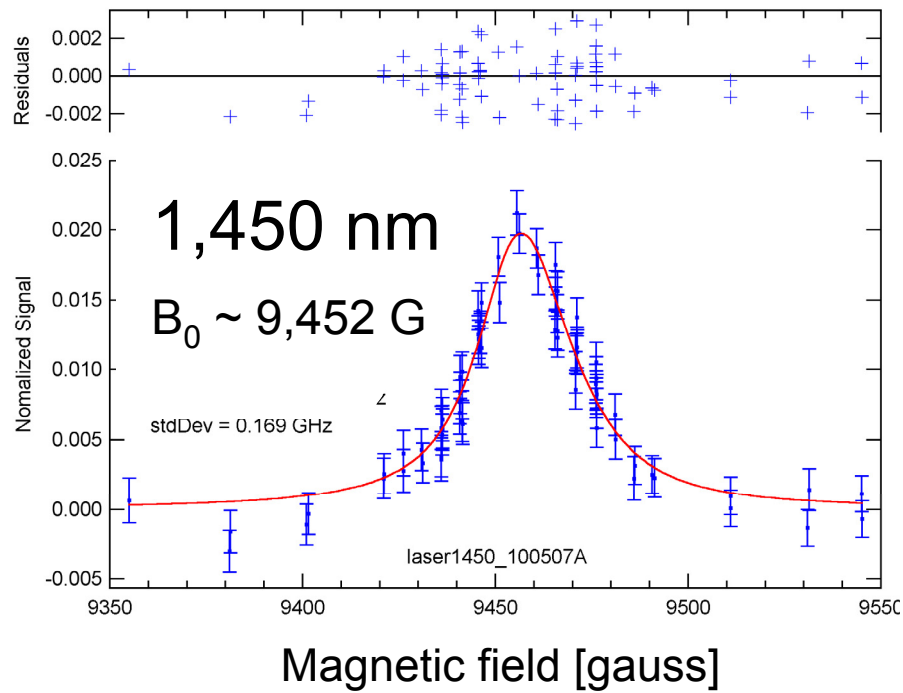


Not intended for single-frequency operation...

⇒ Use Fiber-Bragg Grating to force single mode operation

Acknowledgement: David Shiner, Ali Khademian, U. North Texas

Laser-induced resonances

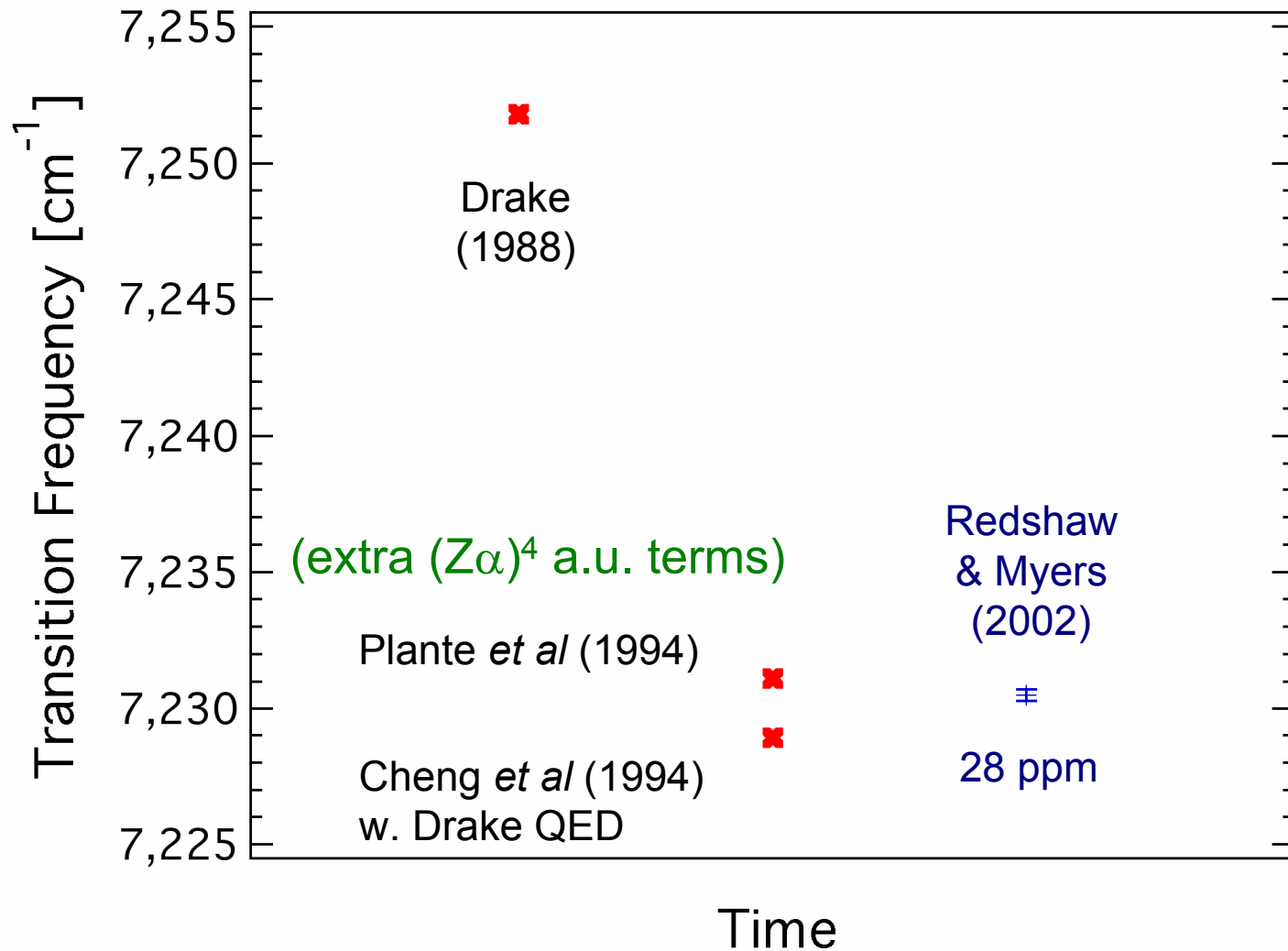


Error Budget

Source	Uncertainty (ppm)
Statistics (fitting)	0.75
Wavemeter calibration	0.2
Line shape asymmetry	< 0.1
Ion beam divergence and misalignment	0.02
Yield dependence on velocity	< 0.03
Total	0.78

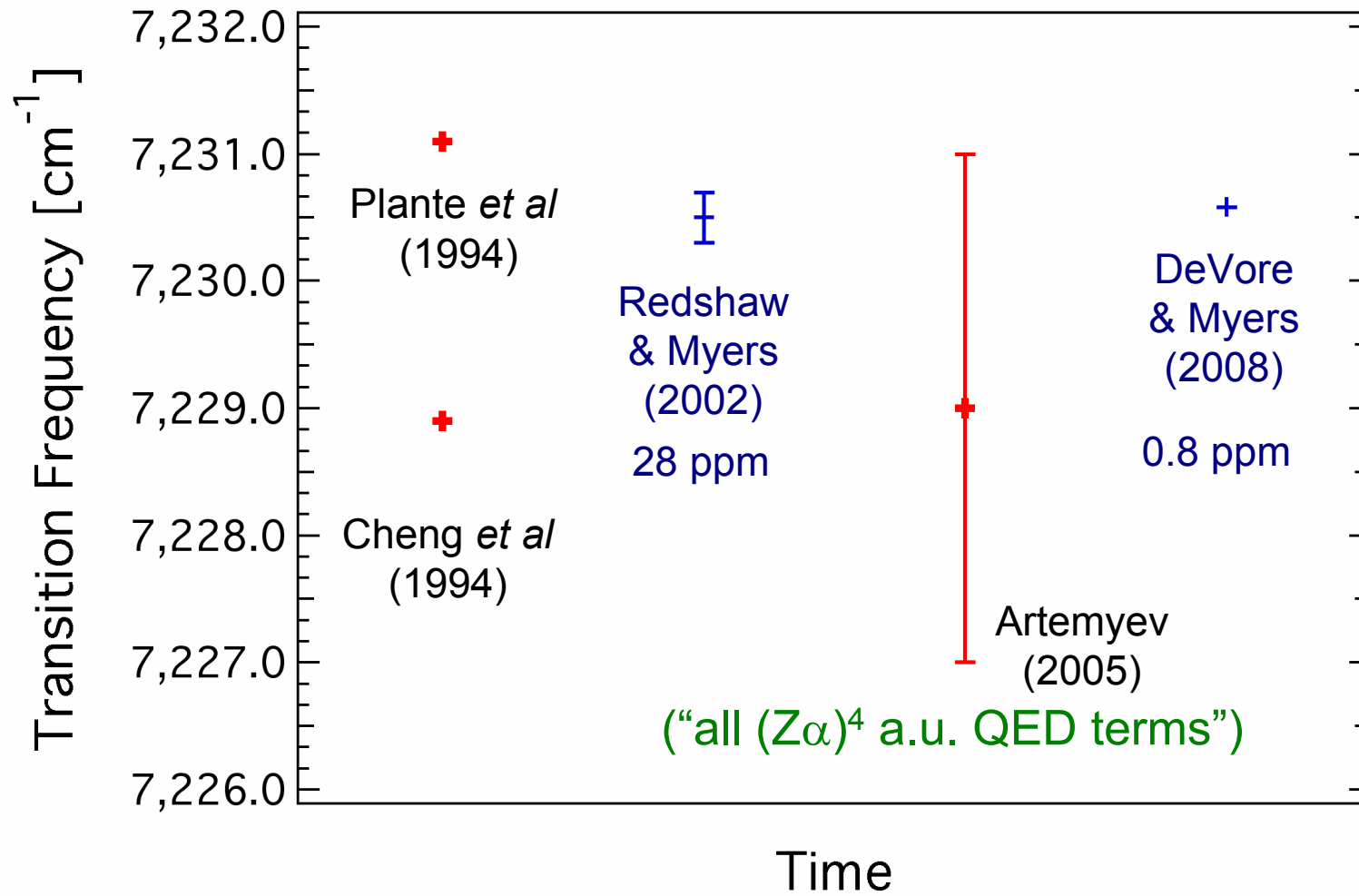
$^{28}\text{Si}^{12+}$ $1s2s\ ^1S_0 - 1s2p\ ^3P_1$ Theory and Experiment

circa 2002



$^{28}\text{Si}^{12+}$ $1s2s\ ^1S_0 - 1s2p\ ^3P_1$ Theory and Experiment

circa 2008



$^{28}\text{Si}^{12+}$ $1s2s\ ^1S_0 - 1s2p\ ^3P_1$ Results

	(units cm^{-1})	
This Experiment ('08)	7230.585(6)	x30
Previous Experiment ('02)	7230.5(2)	improvement
<i>Closest Theory:</i>		
Plante, Johnson and Sapirstein('94)	7231.1	
Theory – Experiment	0.515(6)	(70 ppm, 90 σ!)

Expt'l uncertainty = $2.5 \times 10^{-4} (Z\alpha)^4$ atomic units
13 ppm of QED corrections
0.25% of nuclear size correction

Status of laser spectroscopy of moderate-Z He-like ions

All (low n) laser spectroscopy on He-like ions

	Transitions	Ions	Z
1)	$2\ ^3S_1 - 2\ ^3P_{0,1,2}$	$6,7\text{Li}^+, \ ^9\text{Be}^{2+}, \ ^{11}\text{B}^{3+}$	≤ 5
2)	$2\ ^3P_2 - 2\ ^3P_1$	$^{19}\text{F}^{7+}$	9
	$2\ ^3P_0 - 2\ ^3P_1$	$^{24}\text{Mg}^{10+}$	12
3)	$2\ ^1S_0 - 2\ ^3P_{0,1}$	$^{14,15}\text{N}^{5+}$	7
	$2\ ^1S_0 - 2\ ^3P_1$	$^{28}\text{Si}^{12+}$	14

“Moderate Z”

2^3P Fine Structure (units cm^{-1})

	N^{5+} 0-1	F^{7+} 1-2	Mg^{10+} 0-1
Experiment (FSU)	8.6707(7)	957.8730(12) (1.2 ppm)	833.133(15)
Zhang et al '96	8.686(20)	957.840(80)	832.335
Plante et al '94	8.73(2)	957.87(2)	833.1(2)
Chen et al '93	8.67(2)	957.85(2)	833.3(2)
$\sigma(\text{theory}) / \sigma(\text{expt})$	~28	~17	~13

E.G. Myers, PSAS 2000, "Hydrogen Atom"

2 ¹S₀ – 2 ³P₁ Intercombination (units cm⁻¹)

	N⁵⁺	Si¹²⁺	
Experiment	986.3180(7)	<u>7230.585(6)</u>	
			(Zα) ⁴ au?
Drake '88	986.579	7251.8	some
Cheng et al '94	993.6	7264.7	most
Cheng with Drake QED	985.9	7228.9	most
Plante et al '94	984.7	7231.1	most
Artemyev et al '05		7229(2)	all
 Expt–Closest Theory / σ(expt)	~370	~90	

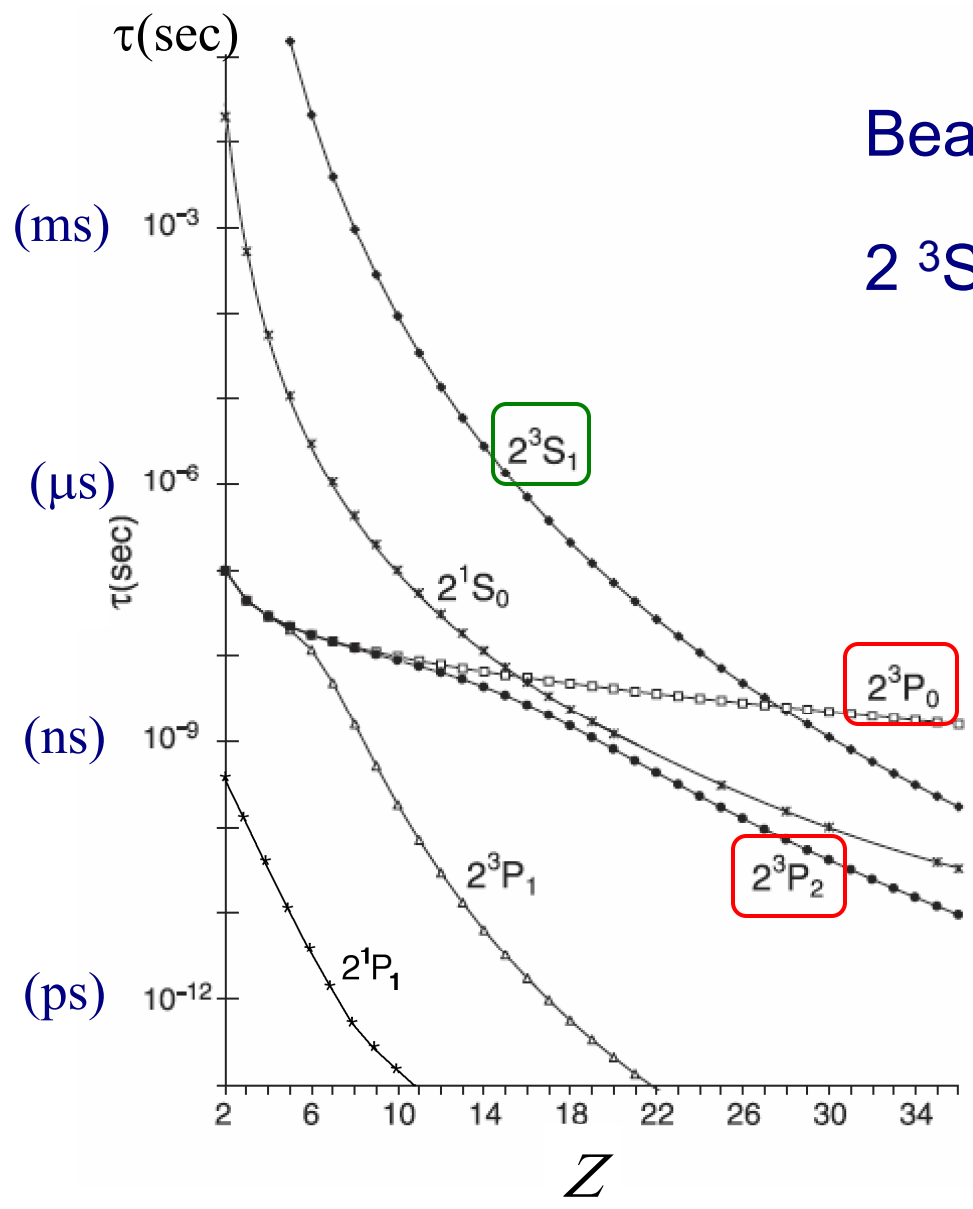
Laser spectroscopy of He-like ions: *Future Directions*

Low-Z: C^{4+} $2^3S - 2^3P_{0,1,2}$ (227 nm)
 $2^1S - 2^3P_1$ (79 μm)

Higher-Z: Ca^{18+} $2^3S - 2^3P_{0,2}$ (59 nm, 47 nm)
VUV lasers: harmonic generation, mixing, fs comb ?
Cooled ions in a storage ring?

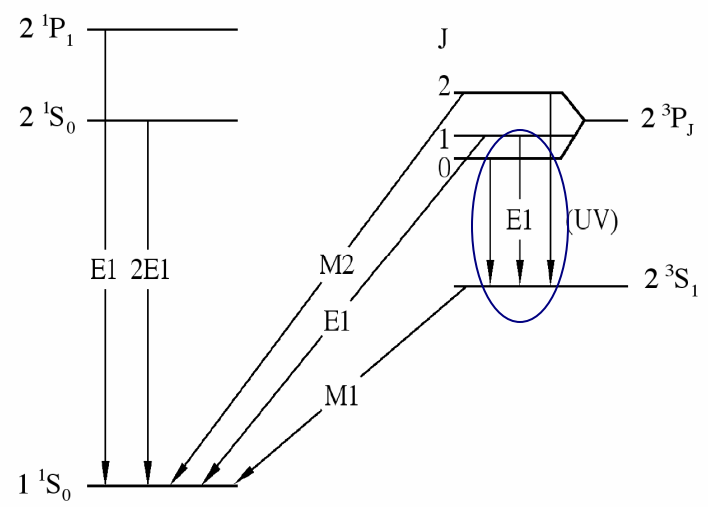
Precise measurement of He-like fine structure for FS constant ?

Mean lifetimes of $n = 2$ levels in helium-like ions



Beam velocities \sim few cm/ns

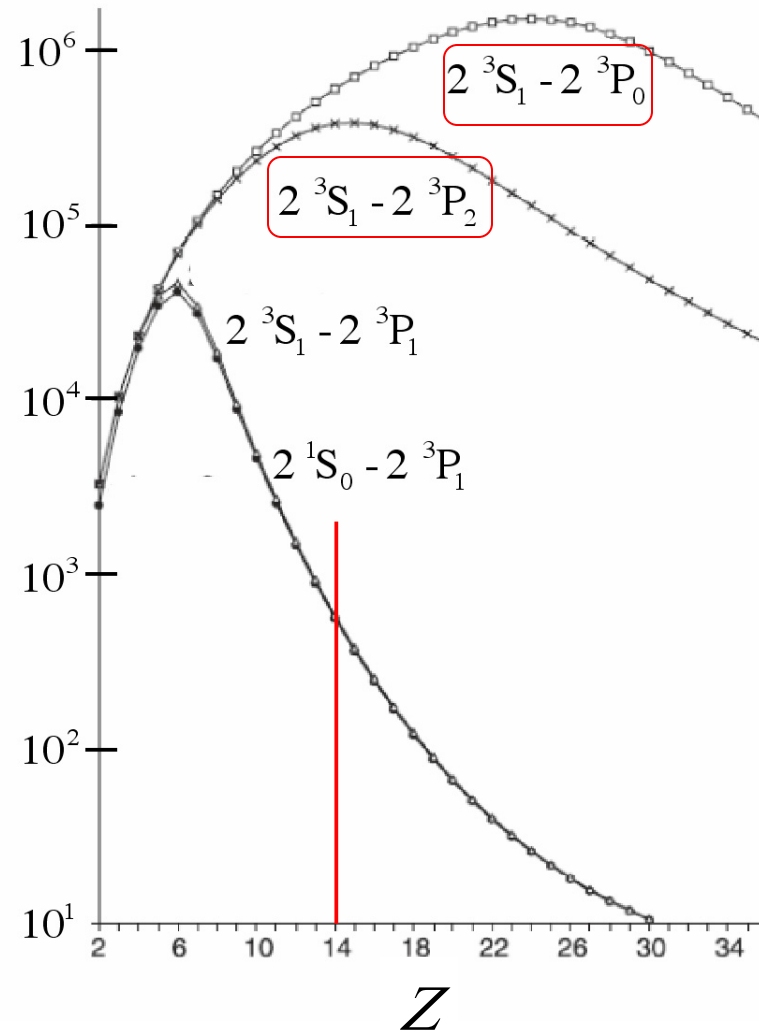
2^3S_1 useful initial level for $Z < 30$



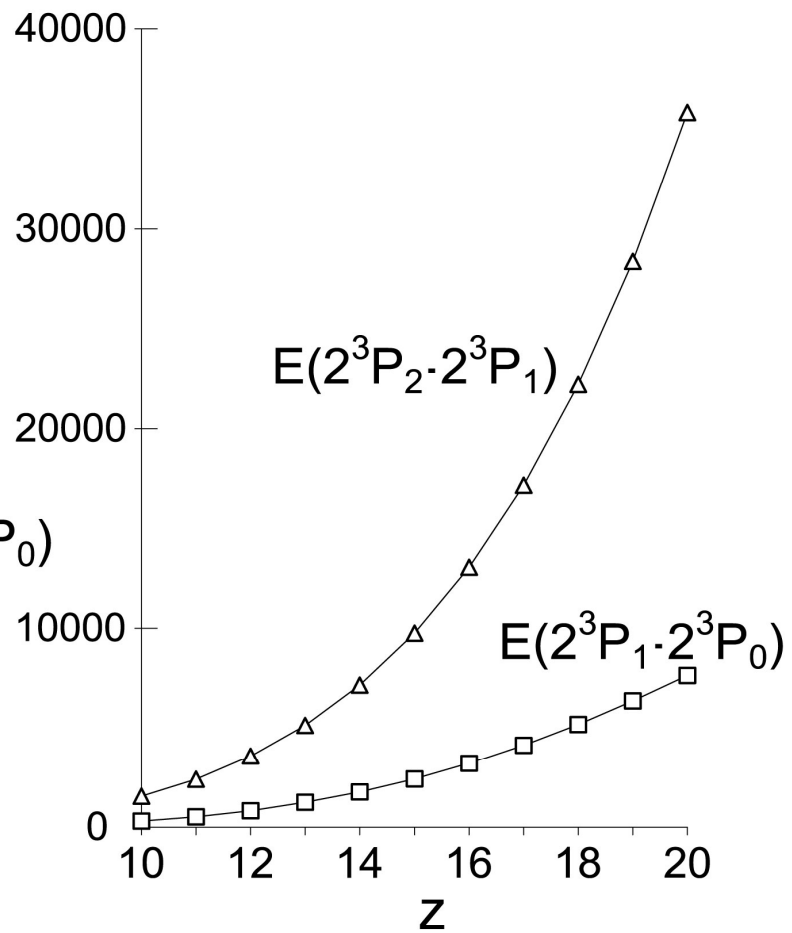
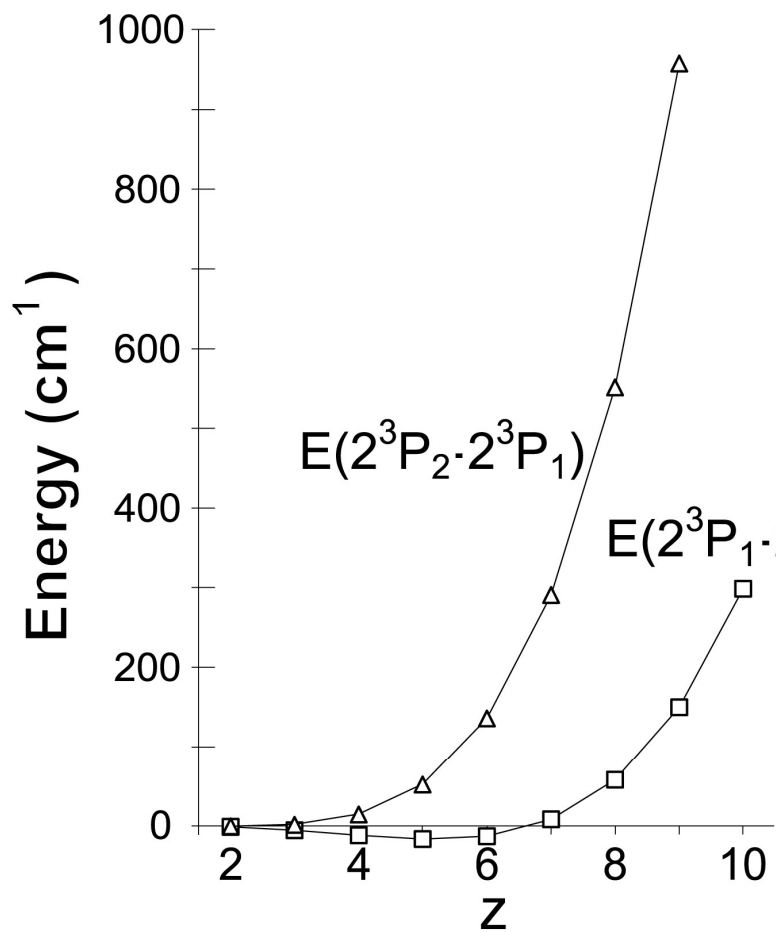
QED Sensitivity of $2^3S - 2^3P_{0,2}$

“QED Sensitivity”

$$\frac{\Delta E_{\text{QED}}}{\hbar\Gamma}$$

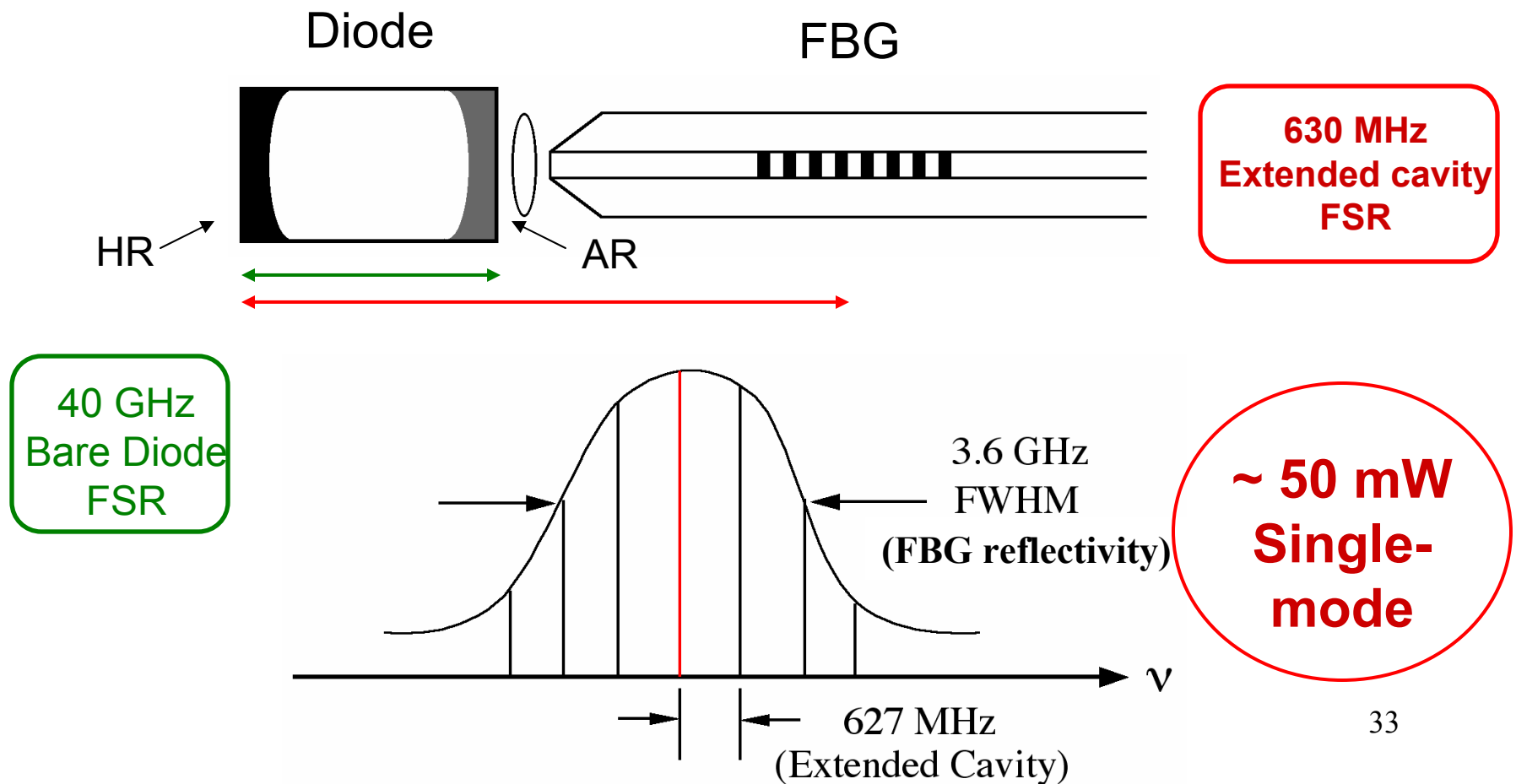


*E.G. Myers, PSAS 2000
“Hydrogen Atom”*



Enforcing single-frequency operation & lasing at 1,450 nm (*Shiner, Khademian*)

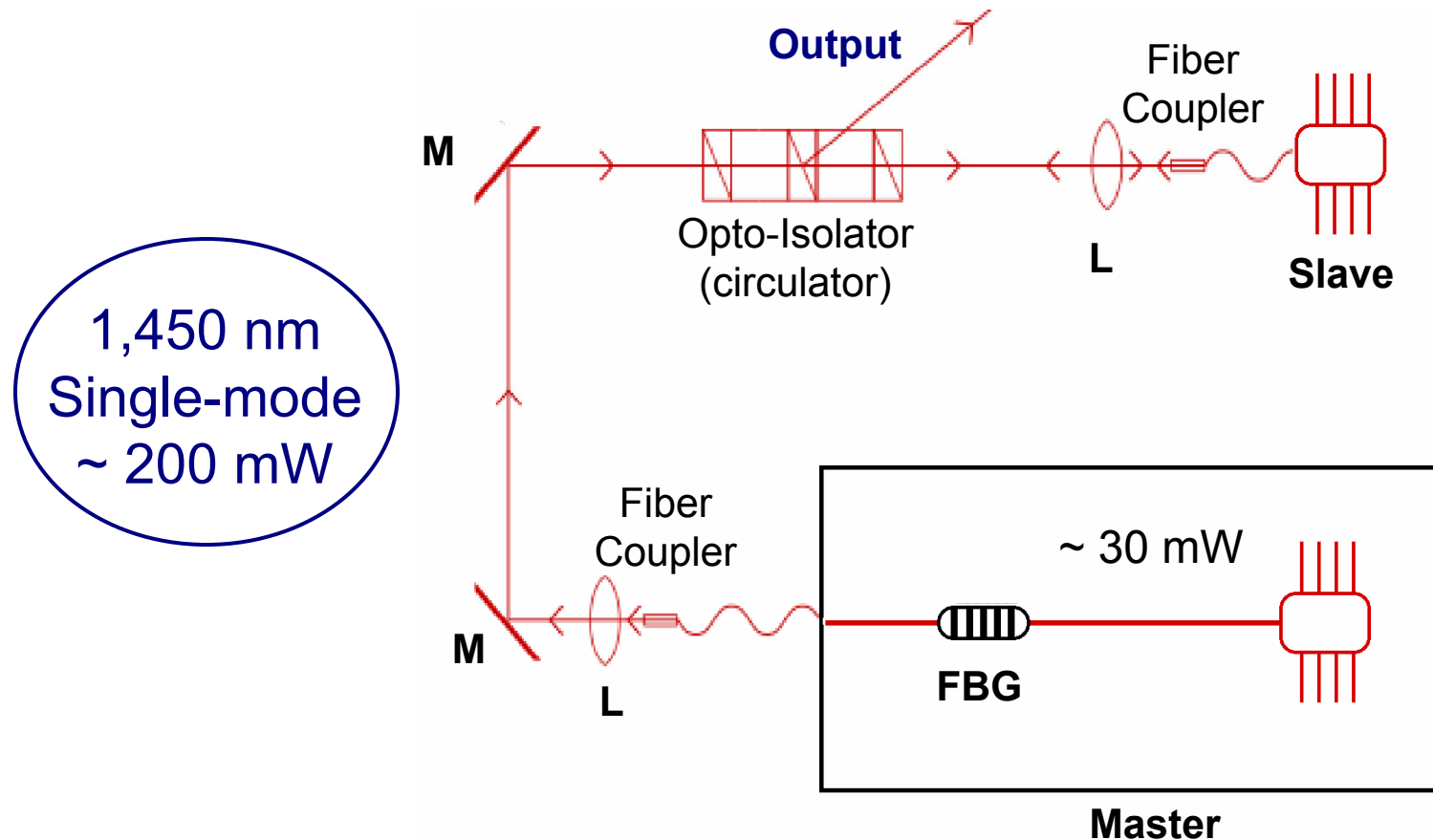
Fiber-Bragg grating: narrow-band, wavelength selective reflector



Power amplification: Injection-locking

Master laser: stable, single-frequency, low power (~ 30 mW)

Slave laser: less stable, similar λ , higher power (> 200 mW)



Why study energy levels in Helium-like ions?

Helium-like ions vs Helium

Contributions to Energy levels vary with powers of Z

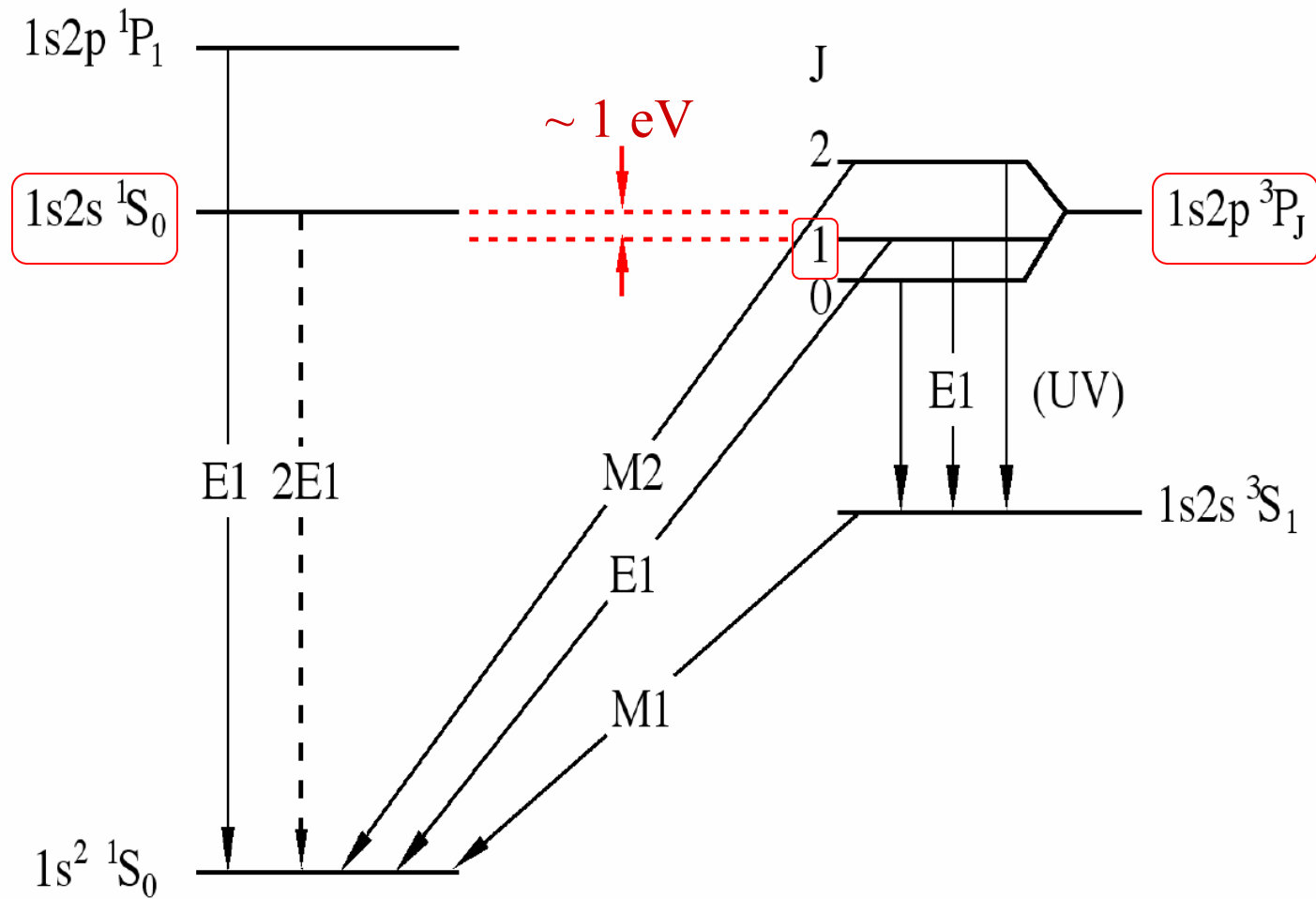
Relativistic effects: $\sim (Z\alpha)^4 mc^2, (Z\alpha)^6 mc^2 \dots$

QED effects: $\sim \alpha \ln[(Z\alpha)^{-2}] (Z\alpha)^4 mc^2 \dots$

Helium-like ions vs Hydrogen-like ions

1. No equivalent Dirac equation
2. QED corrections much more complicated

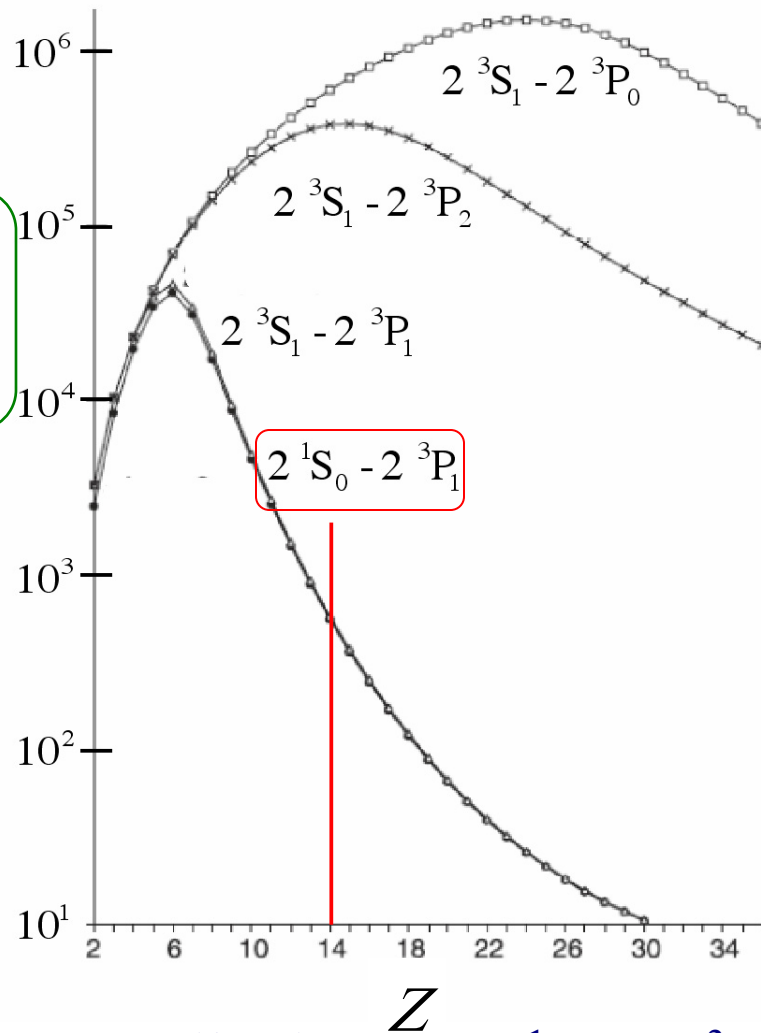
He-Like ions, $n = 1, 2$



Why helium-like silicon?

QED sensitivity

$$\frac{\Delta E_{\text{QED}}}{\hbar\Gamma}$$



Want highest Z possible,
BUT

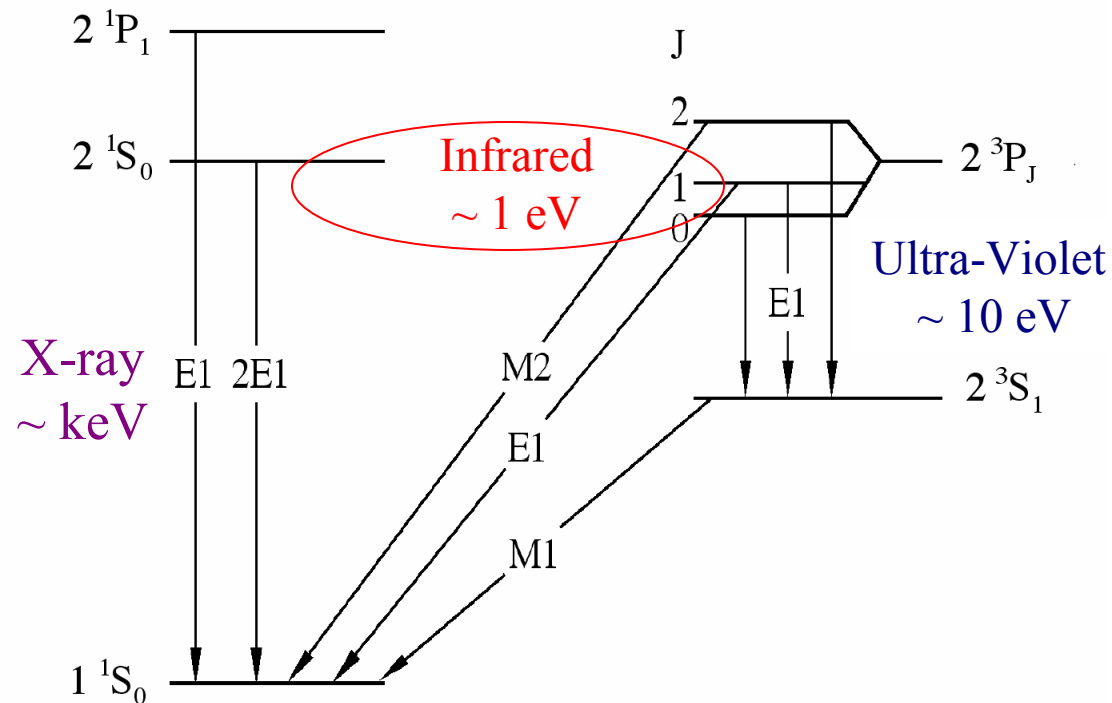
- $\tau(2^3P_1)$ falls rapidly $\sim Z^{-10}$
- laser - induced $\frac{d\text{Prob}}{dt}$ falls $\sim Z^{-6}$
(for a given laser intensity)

QED contributions to $2^1S_0 - 2^3P_1$ in silicon $\sim \underline{6\%}$

Why Si^{12+} $1s2s\ ^1S_0 - 1s2p\ ^3P_1$?

- QED and relativistic effects increase with Z
- But, theory is more accurate than **all** UV, X-Ray spectroscopy!
- High absolute precision \Rightarrow **small interval + laser spectroscopy**

Si^{12+} is highest Z for “precise” laser spectroscopy



Our Measurement

$1s2s\ ^1S_0 - 1s2p\ ^3P_1$ interval in helium-like silicon, $^{28}\text{Si}^{12+}$

