Progress in Calculation of Three-Loop Radiative-Recoil Corrections to HFS in Muonium

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Outline

Experiment and Theory

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$$\Delta E = \left(-\frac{4}{3} \ln^3 \frac{M}{m}\right) \frac{\alpha^2(Z\alpha)}{\pi^3} \frac{m}{M} E_F$$

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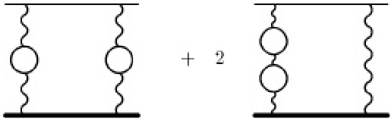

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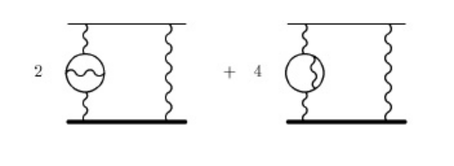

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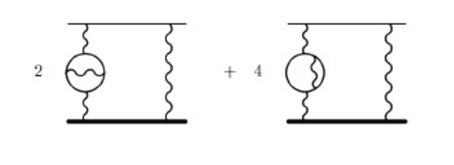

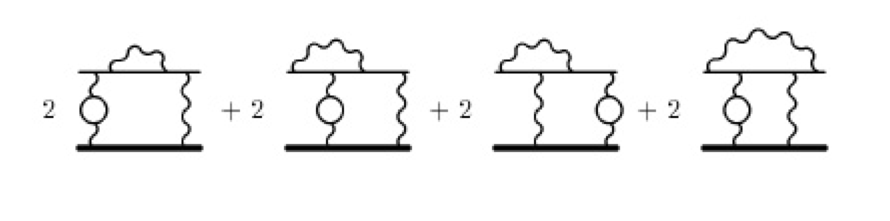

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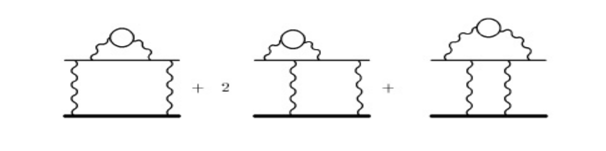

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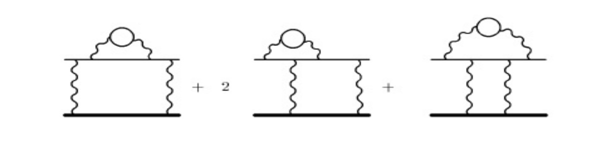

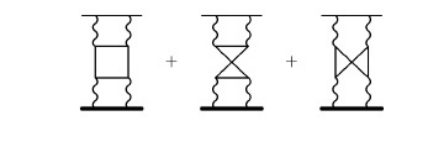

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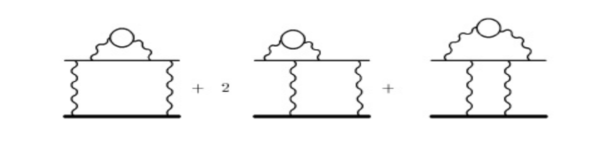

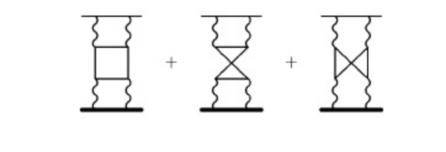

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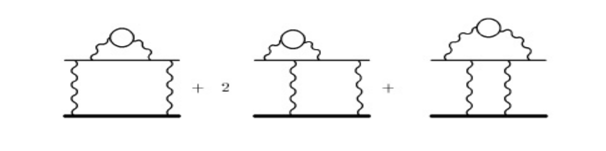

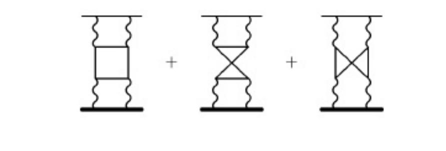

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All these terms were calculated long time ago (Eides, Karshenboim, Shelyuto)

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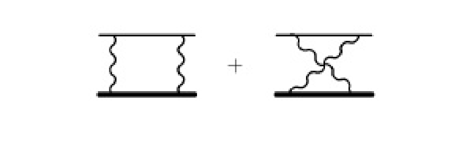

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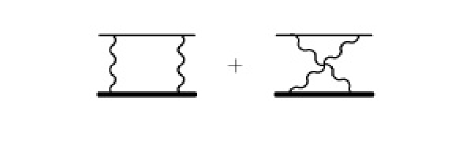

As a rule subleading terms are large and hard to extract

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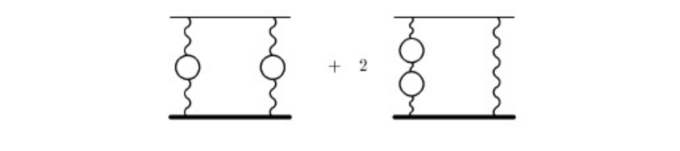

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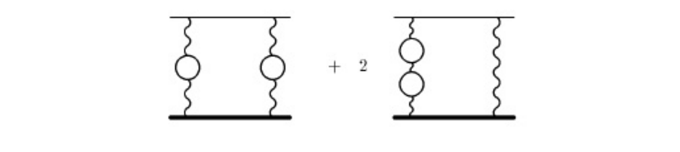

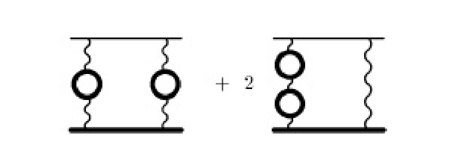

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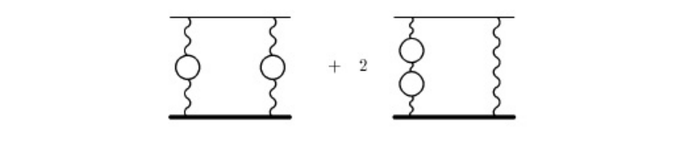

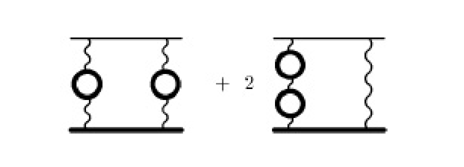

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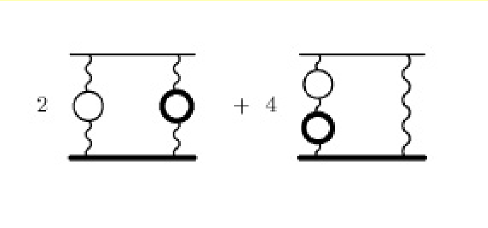

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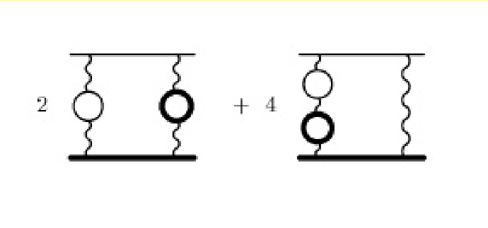

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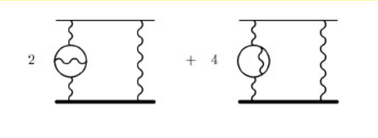

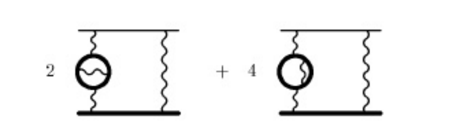

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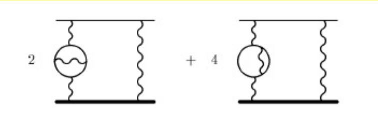

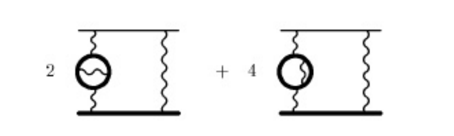

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Radiative Electron Factor and Electron Polarization

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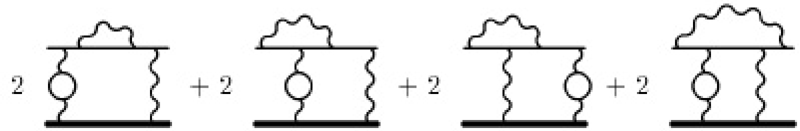

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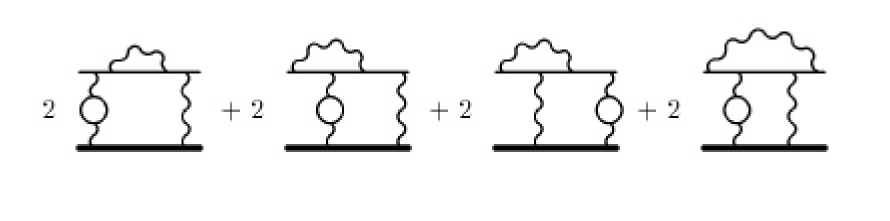

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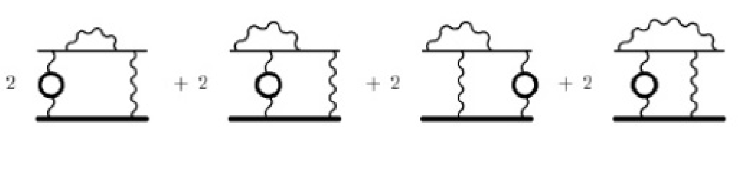

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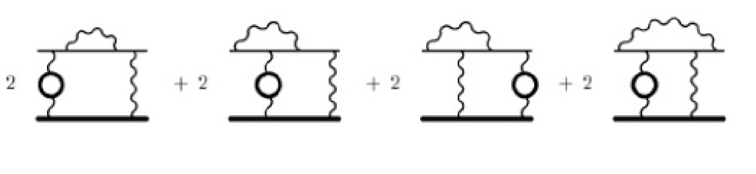
$$\Delta E = \left(-\frac{5\pi^2}{12} + \frac{1}{18}\right)\frac{\alpha(Z^2\alpha)(Z\alpha)}{\pi^3}\frac{m}{M}E_F$$



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Radiative Muon Factor and Muon Polarization

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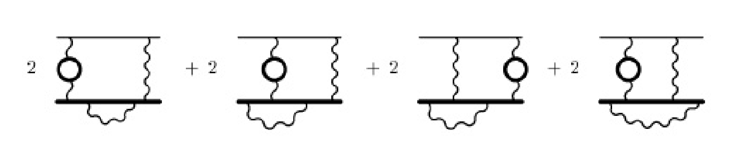

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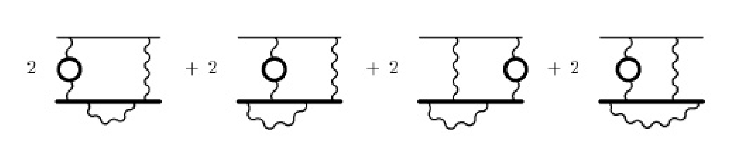

Radiative Muon Factor and Electron Polarization

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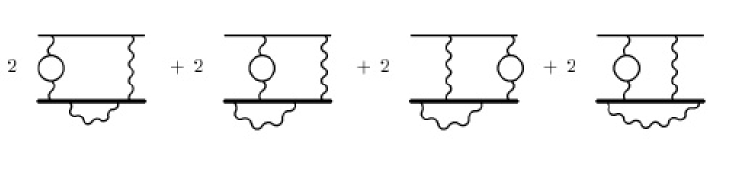

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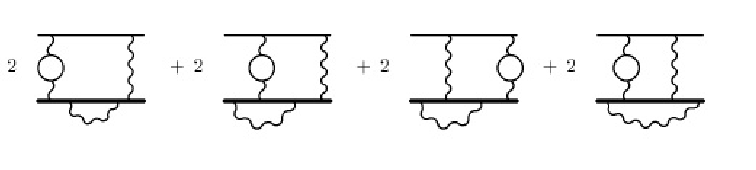

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Two One-Loop Fermion Factors

Diagrams with two fermion factors give only nonlogarithmic contribution

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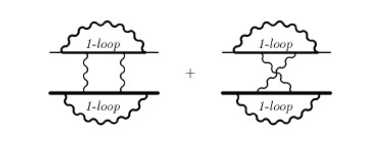

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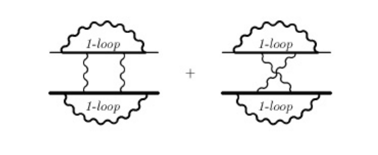

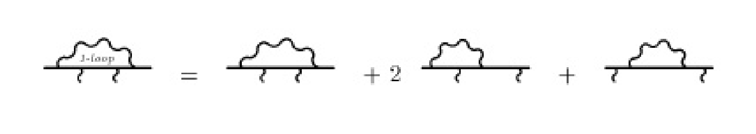

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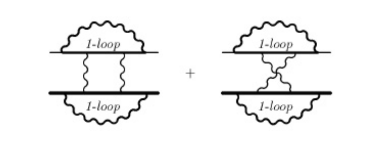

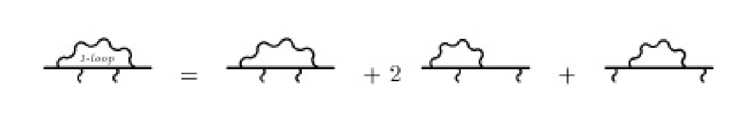

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$$\Delta E = -\frac{3}{8}\frac{(Z\alpha)mM}{\pi}E_F\int\frac{d^4k}{i\pi^2(k^2+i0)^2}\left[L^{(e)}_{\mu\nu}(k)+L^{(e)}_{\nu\mu}(-k)\right]L^{(\mu)}_{\mu\nu}(-k)$$

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Result for these diagrams is obtained analytically

$$\Delta E = \left[-\frac{15}{8}\zeta(3)+\frac{15\pi^2}{4}\ln 2+\frac{27\pi^2}{16}-\frac{147}{32}\right]\frac{\alpha(Z^2\alpha)(Z\alpha)}{\pi^3}\frac{m}{M}\ \widetilde{E}_F$$

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Electron Polarization in Electron Factor

Insertion of electron polarization in the electron factor produces single-logarithmic contribution

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$$\Delta E = \left[\left(\pi^2 - \frac{53}{6} \right) \ln \frac{M}{m} + 7.081 \right] \frac{\alpha^2 (Z\alpha)}{\pi^3} \frac{m}{M} E_F$$

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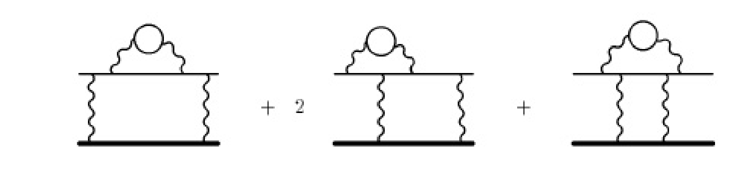

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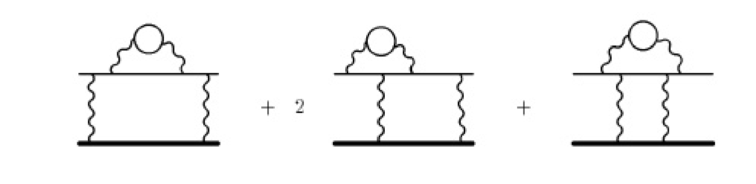

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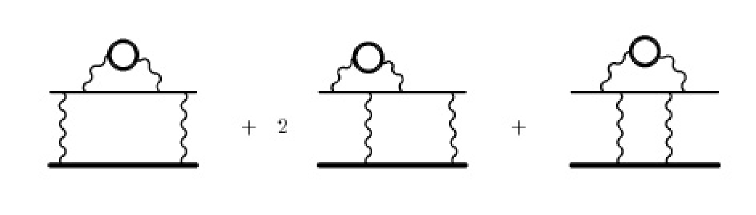

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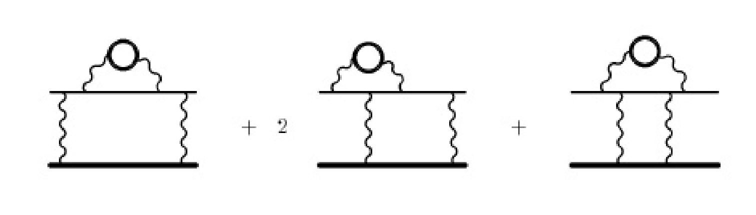

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$$\Delta E = \left[\left(3\,\zeta(3) - 2\pi^2 \ln 2 + \frac{13}{4} \right) \ln \frac{M}{m} + 12.227 \right] \frac{\alpha(Z^2\alpha)(Z\alpha)}{\pi^3} \frac{m}{M} E_F$$

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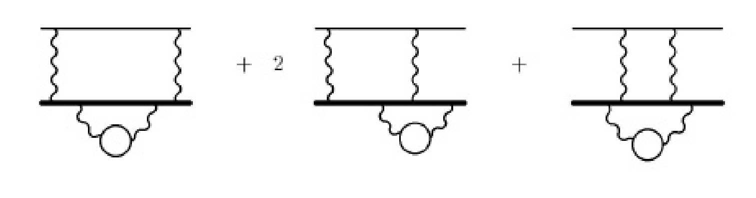

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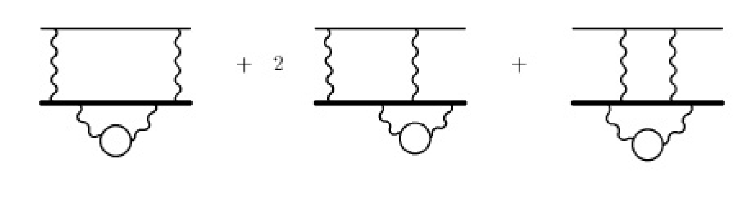

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Insertion of muon polarization in the muon factor produces nonlogarithmic contribution

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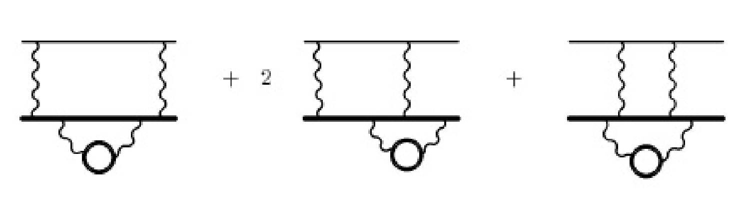

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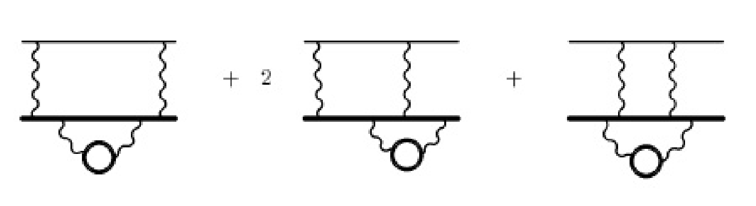

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- Work on these corrections is in progress now

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