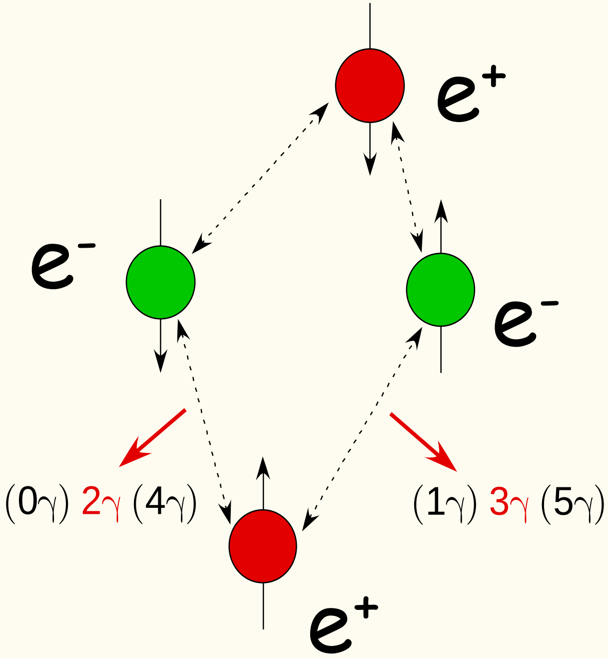
Positronium molecule



PSAS 2008
University of Windsor, July 26, 2008

Andrzej Czarnecki University of Alberta

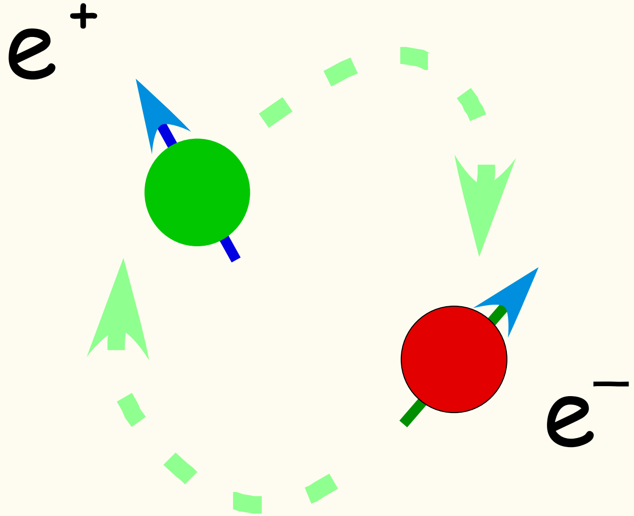
Outline

Positronium, its ion, and the molecule

1951 1981 2007

- Positronium atom
- Ion: theoretical prediction for the decay rate. Comparison with experiment.
- Molecule: spectrum. Dipole transition vs. annihilation.

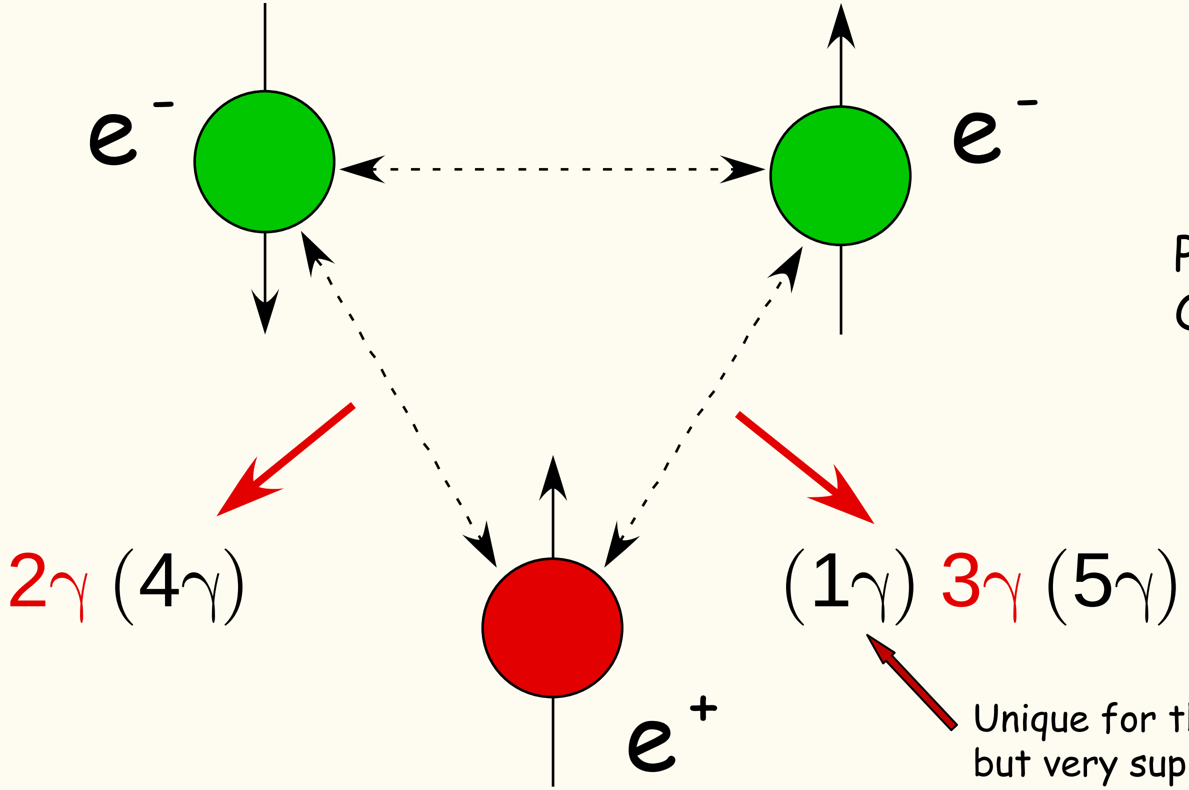
Positronium and its ion



Very similar to hydrogen, except
- no hadronic nucleus
- annihilation
- reduced mass reduced $m_e \to \frac{m_e}{2}$

Two spin states:
singlet (para-Ps)
triplet (ortho-Ps)

Positronium ion: a new test of bound-state QED



e^- e^- e^+

2γ (4γ)

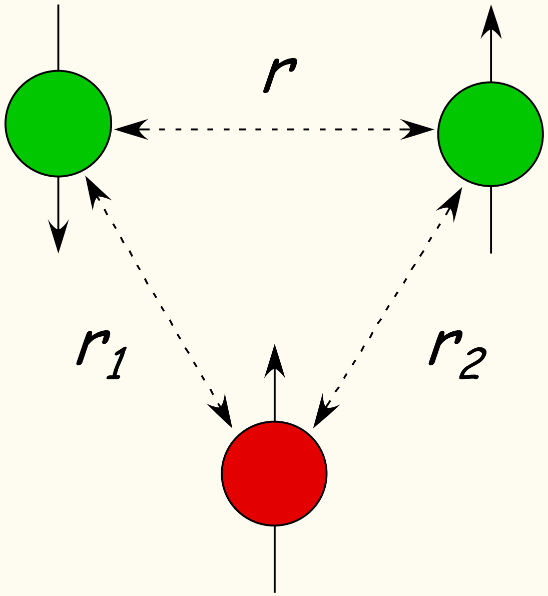
(1γ) 3γ (5γ)

Unique for the ion
but very suppressed

Predicted 1946 Wheeler
Observed 1981 Mills

New positronium-ion source
at the FRM II reactor in Munich:
measurements of branching ratios.

Theory of the Ps ion: the wavefunction



$$\psi\left(\vec{r}_1,\vec{r}_2\right)=\frac{\uparrow\downarrow-\downarrow\uparrow}{\sqrt{2}}\phi\left(r_1,r_2,r\right)$$

Ground state:
symmetric 1 ↔ 2

The wave function is not known analytically,
but can be found using the variational method.

Example of a variational calculation

Test function:

$$\phi(r_1, r_2, r) = \exp\left[-k\left(r_1 + r_2\right)\right]$$

$$\frac{\langle \phi | H | \phi \rangle}{\langle \phi | \phi \rangle} \geq E$$

With this very simple ansatz we do find a positive *k* but the binding energy is smaller than for positronium. The ion would not be stable.

A better fit to the wavefunction

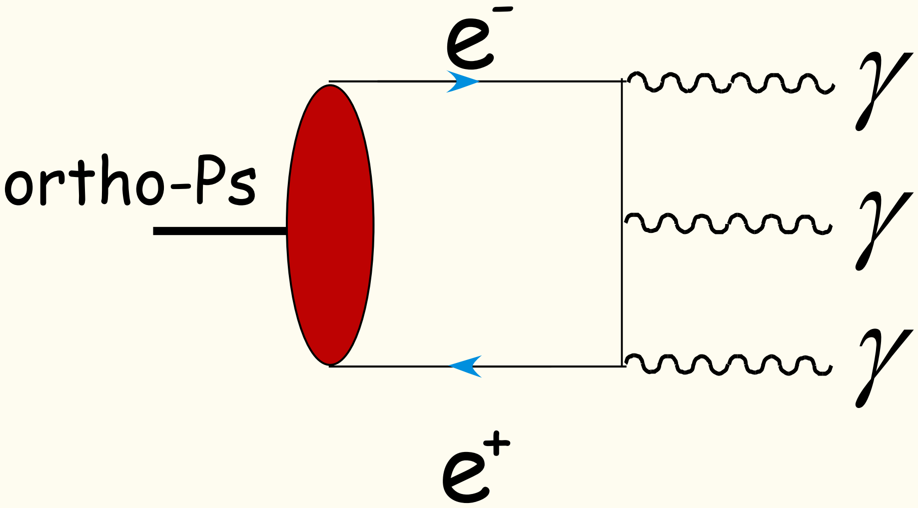
$$\phi(r_1, r_2, r) \sim \exp(-k_1 r_1 - k_2 r_2) + \exp(-k_2 r_1 - k_1 r_2)$$

One finds that $k_1 \sim 2k_2$

Drachman & Bhatia

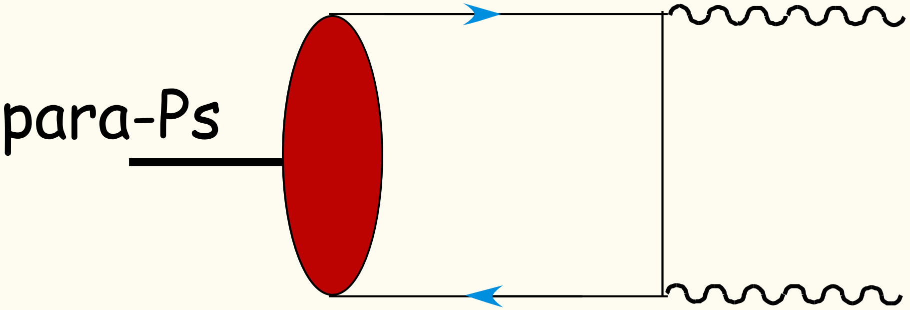
We can picture the ion as a positronium "shell" and a loosely-bound electron.
Now we can easily estimate the dominant decay channels.

For comparison: decays of positronium (atoms)



$$\Gamma_{\exp}\left(\text{o-Ps}\right)=7.0404\left(14\right)\mu\text{s}^{-1}$$

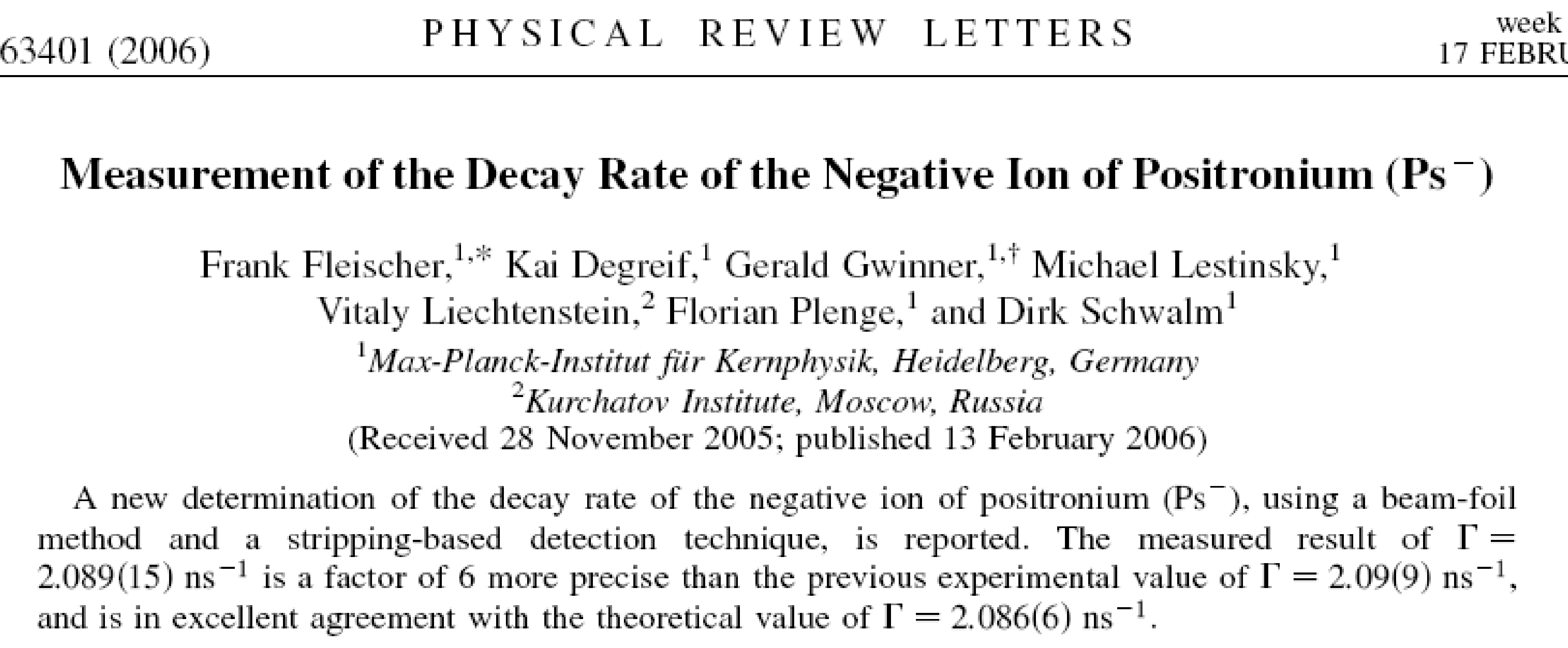
$$2\cdot 10^{-4}$$



$$\Gamma_{\exp}\left(\text{p-Ps}\right)=7.9909\left(17\right)\text{ns}^{-1}$$

$$2\cdot 10^{-4}$$

Recent measurement of the ion decay rate



63401 (2006) PHYSICAL REVIEW LETTERS week
17 FEBRU

Measurement of the Decay Rate of the Negative Ion of Positronium (Ps^-)

Frank Fleischer,[1,*] Kai Degreif,[1] Gerald Gwinner,[1,†] Michael Lestinsky,[1]
Vitaly Liechtenstein,[2] Florian Plenge,[1] and Dirk Schwalm[1]
[1]*Max-Planck-Institut für Kernphysik, Heidelberg, Germany*
[2]*Kurchatov Institute, Moscow, Russia*

(Received 28 November 2005; published 13 February 2006)

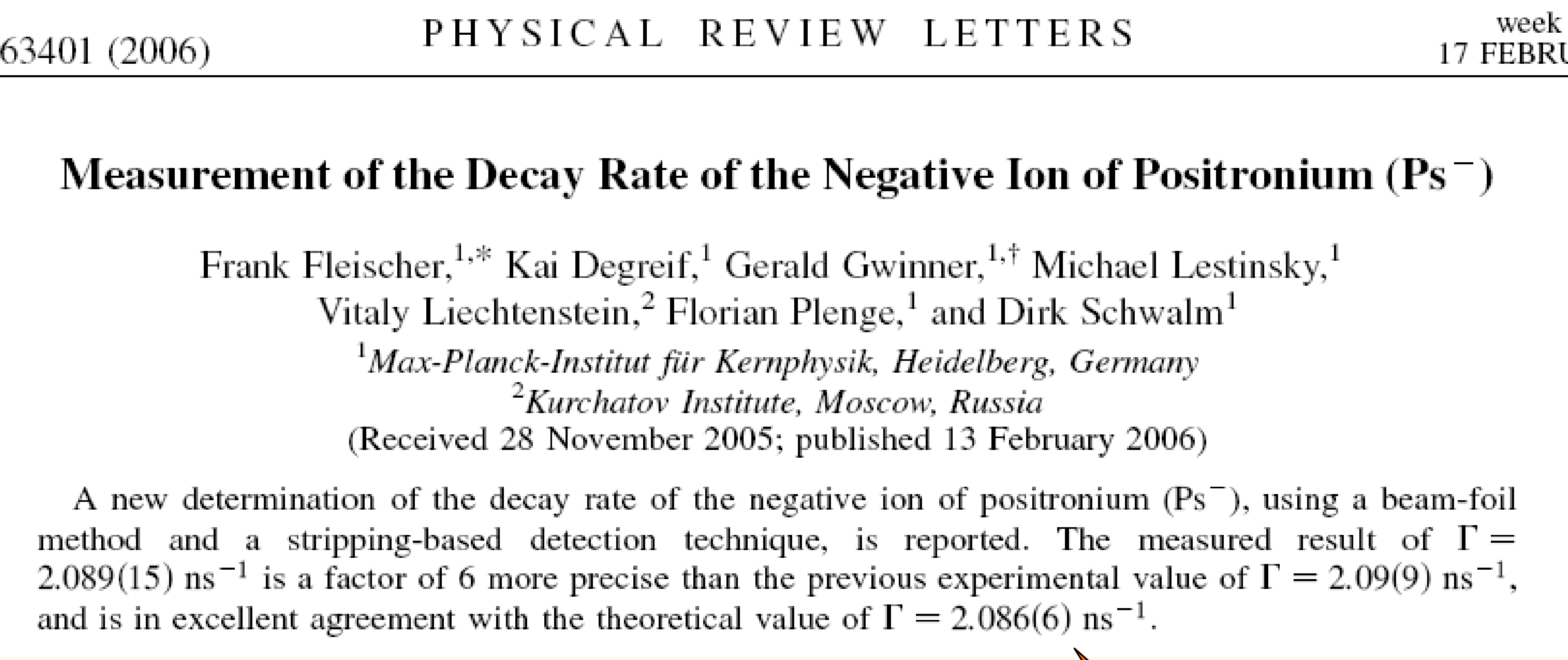
A new determination of the decay rate of the negative ion of positronium (Ps^-), using a beam-foil method and a stripping-based detection technique, is reported. The measured result of $\Gamma = 2.089(15)\ \mathrm{ns}^{-1}$ is a factor of 6 more precise than the previous experimental value of $\Gamma = 2.09(9)\ \mathrm{ns}^{-1}$, and is in excellent agreement with the theoretical value of $\Gamma = 2.086(6)\ \mathrm{ns}^{-1}$.

$$\Gamma_{\exp}\left(\mathrm{Ps}^-\right) = 2089\left(15\right)\mu s^{-1}$$

$$7 \cdot 10^{-3}$$

(factor 4-5
improvement
expected)

Recent measurement of the ion decay rate



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$$\Gamma_{\mathrm{exp}}\left(\mathrm{Ps}^-\right)=2089\left(15\right)\mu s^{-1}$$

We wanted to improve this

$$7\cdot 10^{-3}$$

(factor 4-5 improvement expected)

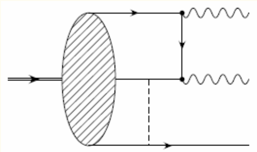
Positronium ion decay: refinements

Corrections O(α)

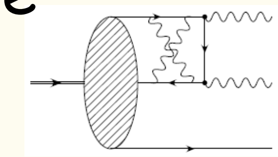
single hard photon loops

Corrections O(α^2); challenge: divergences → $\ln \alpha$

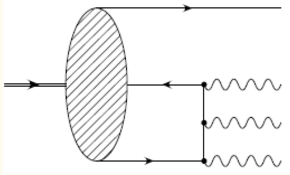
soft
- O(k^2) corrections to the amplitude M
- Breit hamiltonian → correction to ψ (r=0)



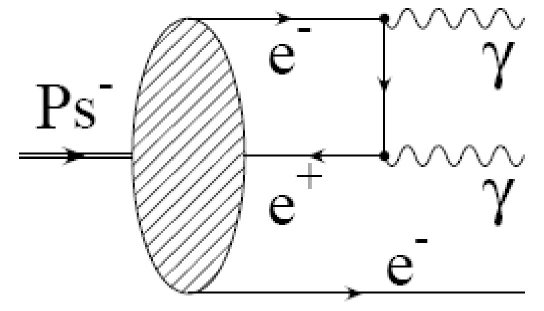
hard
- Short-distance two-loop photon exchange



- Real photon radiation



Decay rate prediction



+ corrections

$$\Gamma(\mathrm{Ps}^-) = 2\pi \frac{\alpha^5 m_e c^2}{\hbar}(1 + C)\left\langle \delta^3(r_{12}) \right\rangle$$

$$\boxed{\Gamma\left(\mathrm{Ps}^-\right) = 2.087\,963\left(12\right)\mathrm{ns}^{-1}}$$

with M. Puchalski and S. Karshenboim
PRL **99**, 203401 (2007)

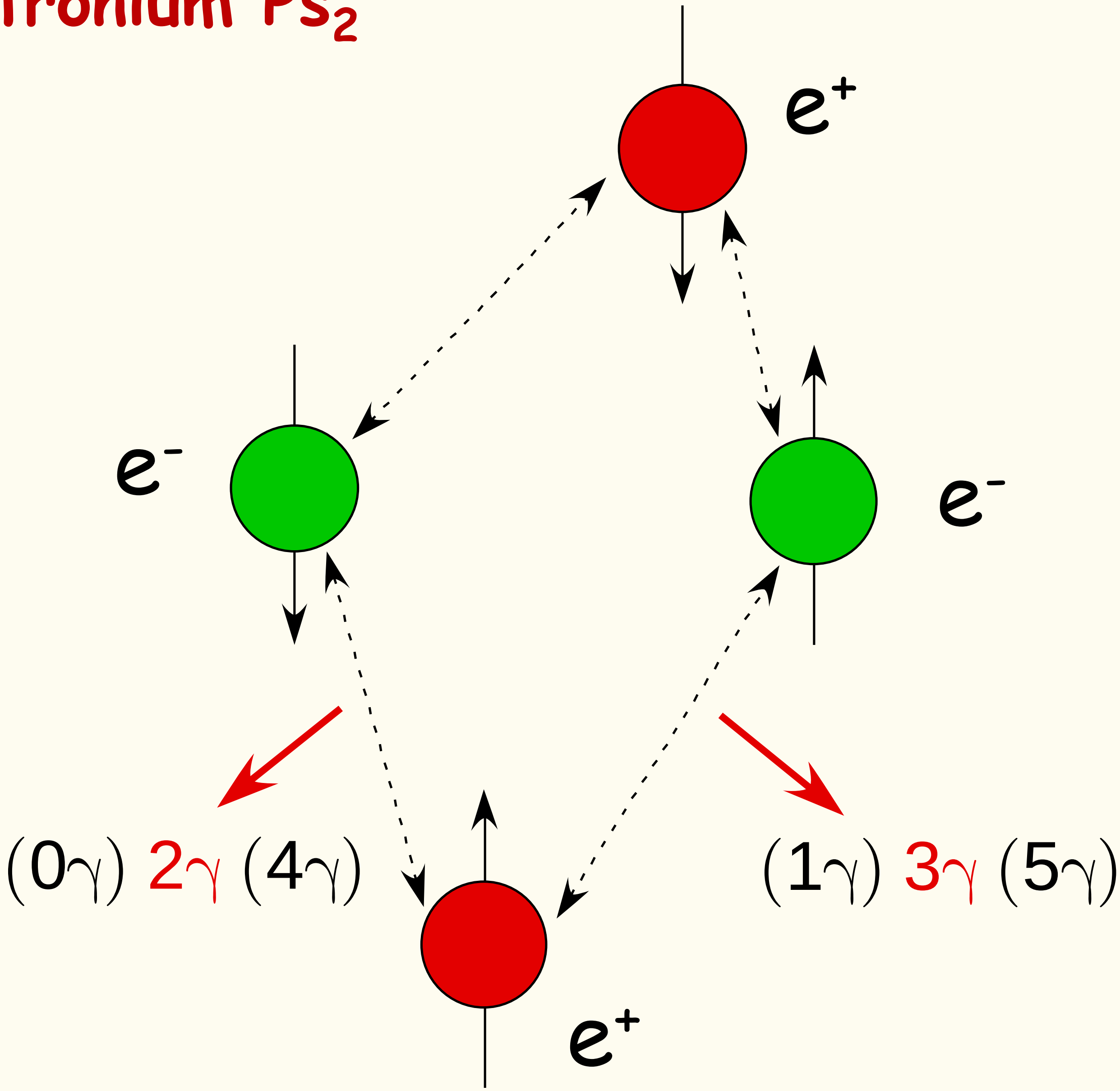
Ratio of three- to two-photon annihilation

$$\mathrm{BR}\,(\mathrm{Ps}^- \to \gamma\gamma\gamma) \equiv \frac{\Gamma(\mathrm{Ps}^- \to \gamma\gamma\gamma)}{\Gamma(\mathrm{Ps}^-)}$$

$$= \alpha\left[A^{3\gamma} + \alpha(B^{3\gamma} - AA^{3\gamma}) - \frac{7}{3}A^{3\gamma}\alpha^2 \ln\frac{1}{\alpha} + \ldots \right]$$

$$= 0.002\,635\,8(8).$$

Dipositronium Ps_2



e^+

e^-

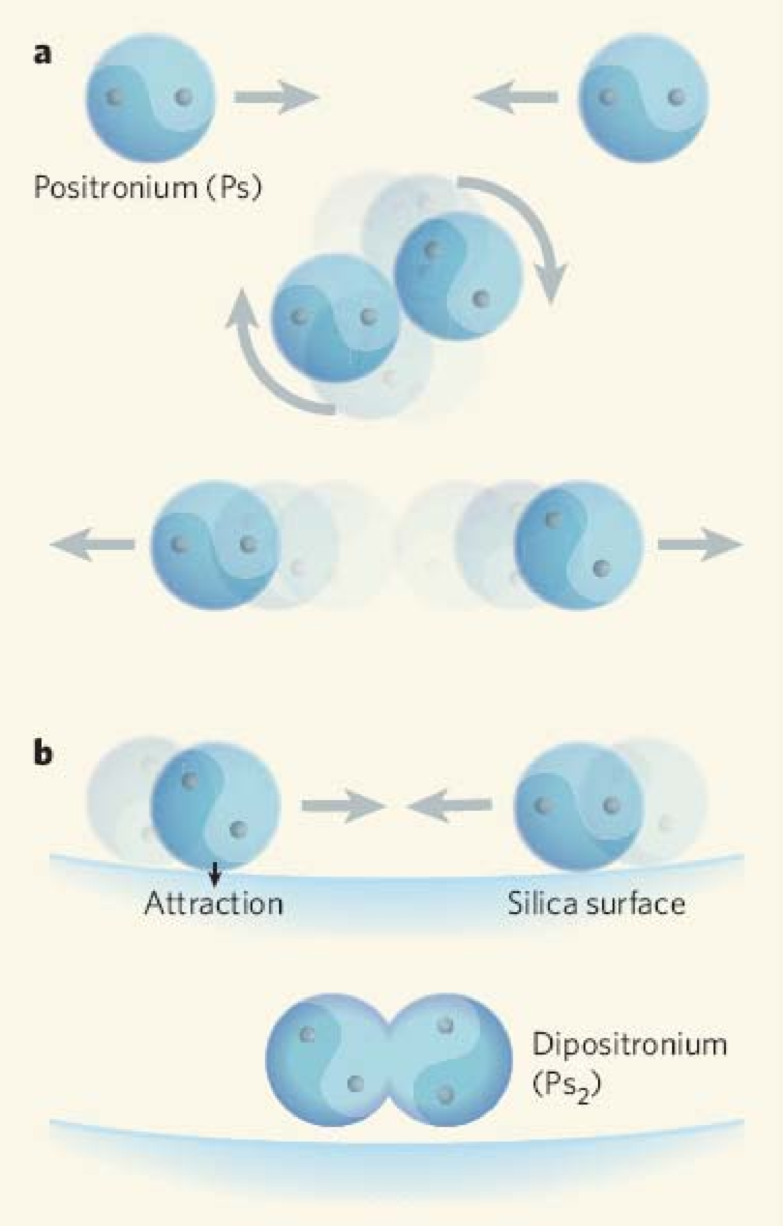
e^-

(0γ) 2γ (4γ)

(1γ) 3γ (5γ)

e^+

Discovery of dipositronium 2007



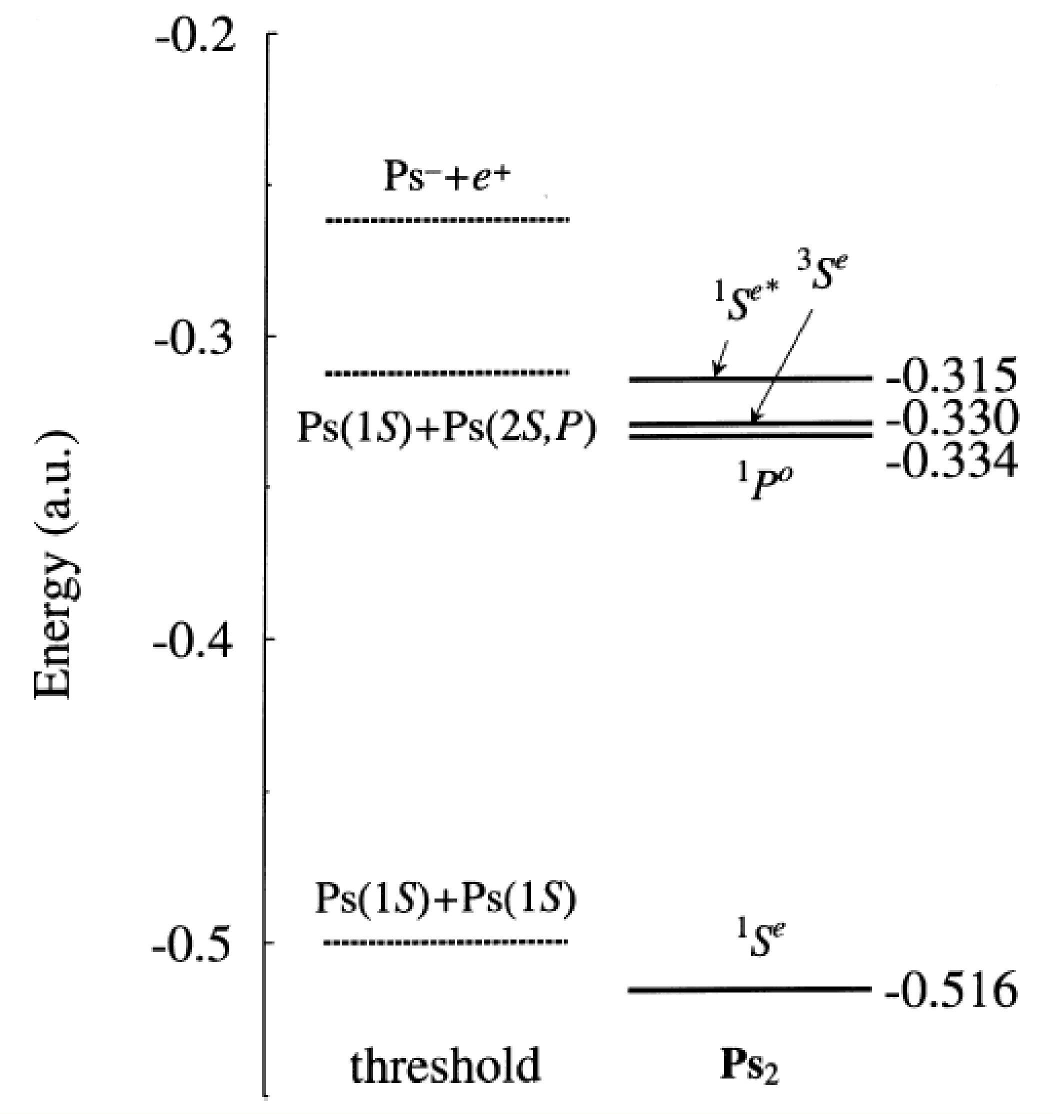
Molecule formation kills long-lived positronia.

At higher temperature, fewer atoms on the surface, fewer molecules formed.

Indeed: at high-T, more long-lived positronia observed.

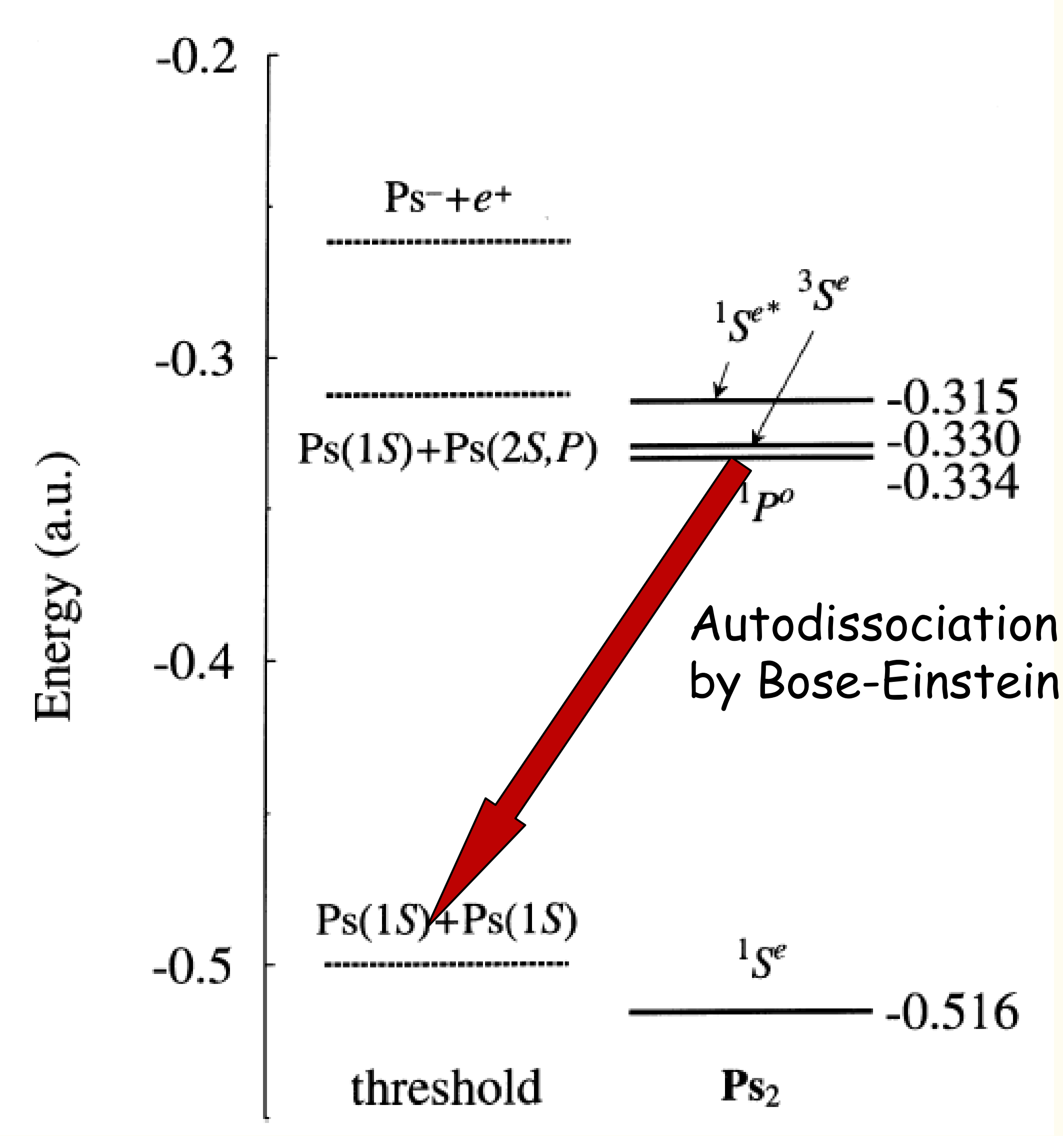
Cassidy & Mills, Nature 2007

Spectrum of the molecule Ps_2



From Suzuki & Usukura, 2000

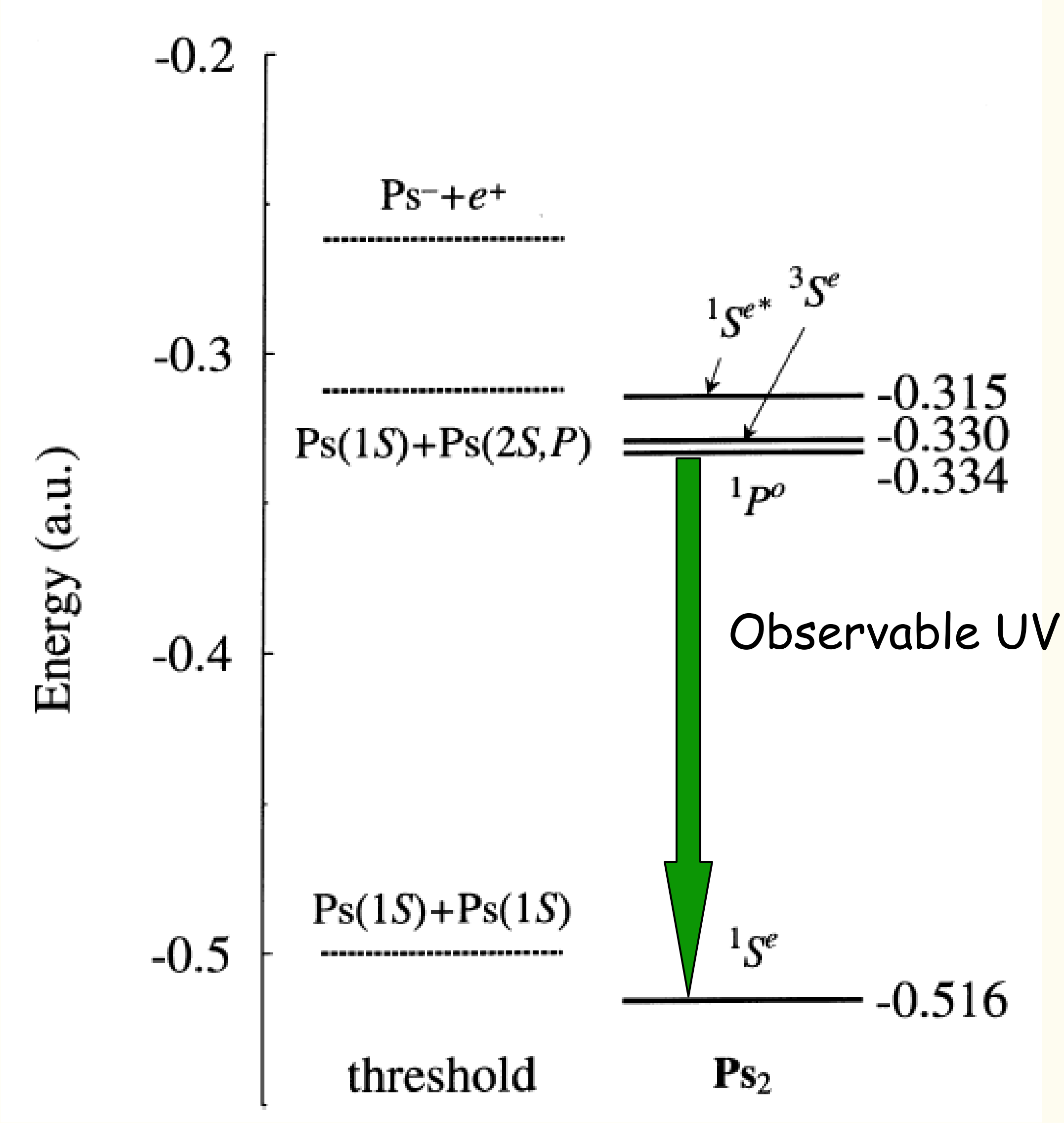
A direct signal of the molecule: transition line.



Autodissociation forbidden
by Bose-Einstein statistics

From Suzuki & Usukura, 2000

A direct signal of the molecule: transition line.



Observable UV transition

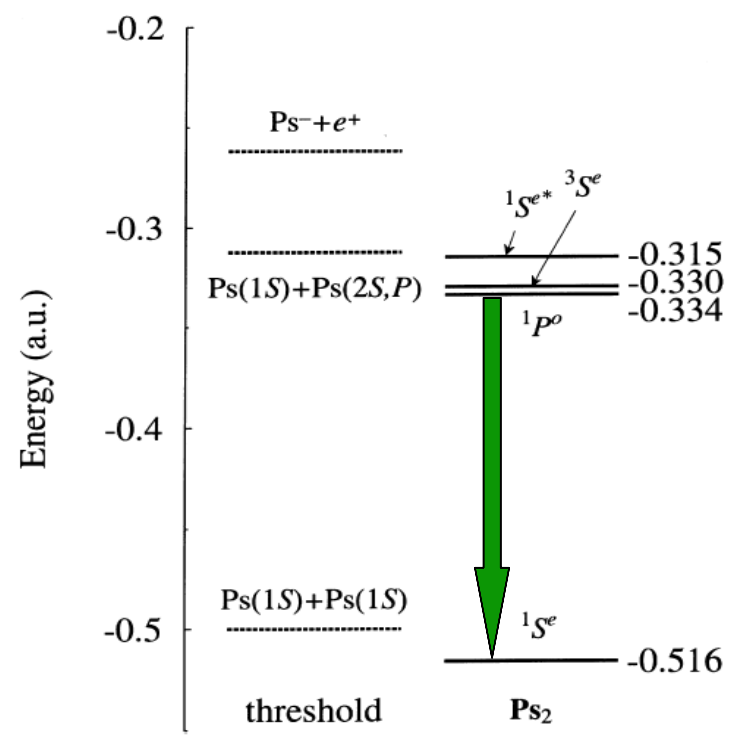
From Suzuki & Usukura, 2000

Two questions about this transition:

What is its accurate energy?

How often does it appear (before annihilation)?

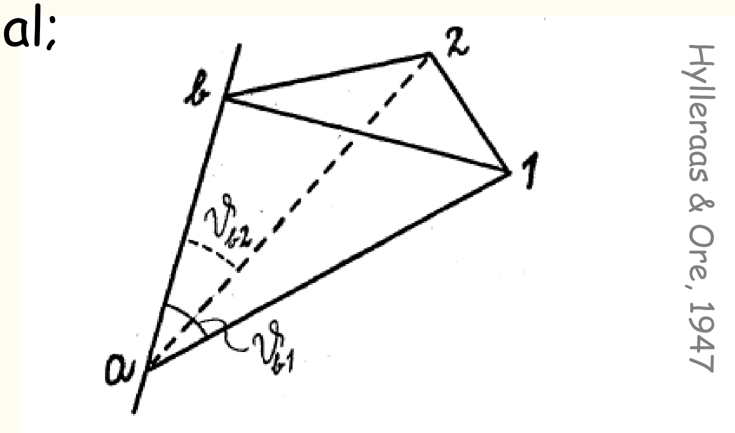
Results: arXiv:0810.0013
in press in PRL; with M. Puchalski



Energy levels: ground state and P-excitation

Wave function determined variationally, using Coulomb potential;

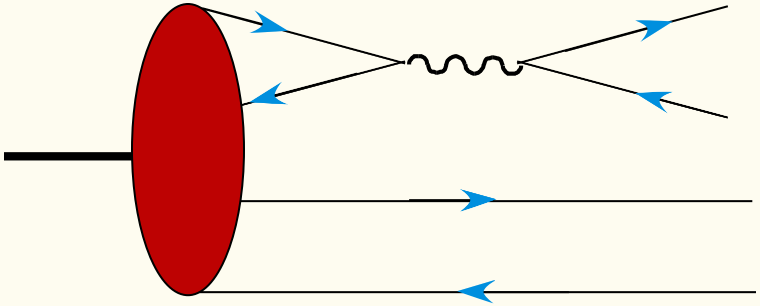
M. Puchalski



Hylleraas & Ore, 1947

Coordinate system for the positronium molecule

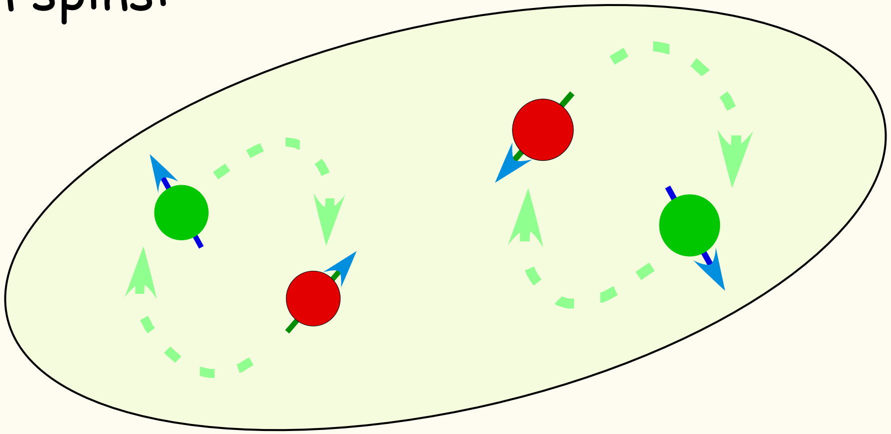
Relativistic corrections: perturbations.
Annihilation dominates.



Interval P-S determined with 5×10^{-6} accuracy (slightly smaller than in Ps, “dielectric effect").

Competition: dipole transition vs. annihilation

S-state annihilates quite rapidly.
Assume it consists of two weakly-interacting Ps atoms,
with random spins:



There is a para-positronium pair with probability $2 \cdot \frac{1}{4} = \frac{1}{2}$

The decay rate:

$$\Gamma_S \simeq \frac{1}{2} \cdot \frac{1}{\tau_{\mathrm{pPs}}} = \frac{1}{0.25\,\mathrm{ns}}$$

P-state: half of this rate.

Competition: dipole transition vs. annihilation

Quiz: if an isolated P-excited Ps atom lives ~3.2 ns,
what do we expect in Ps_2?
Let's assume the atoms interact very weakly in the molecule.

Competition: dipole transition vs. annihilation

Quiz: if an isolated P-excited Ps atom lives ~3.2 ns,
what do we expect in Ps_2?
Let's assume the atoms interact very weakly in the molecule.

$$d_{\text{atom}} = \left\langle \mathrm{S} \middle| \vec{d} \middle| \mathrm{P} \right\rangle$$

$$d_{\text{molecule}} = \left\langle \mathrm{SS} \middle| \vec{d} \middle| \frac{\mathrm{PS}+\mathrm{PS}}{\sqrt{2}} \right\rangle = \sqrt{2} d_{\text{atom}}$$

Transition in the molecule is twice faster.
Branching ratio

$$BR\left(\mathrm{P} \to \mathrm{S}\right) = 19\% \sim \frac{\frac{1}{1.6}}{\frac{1}{1.6} + \frac{1}{0.5}}$$

with M. Puchalski

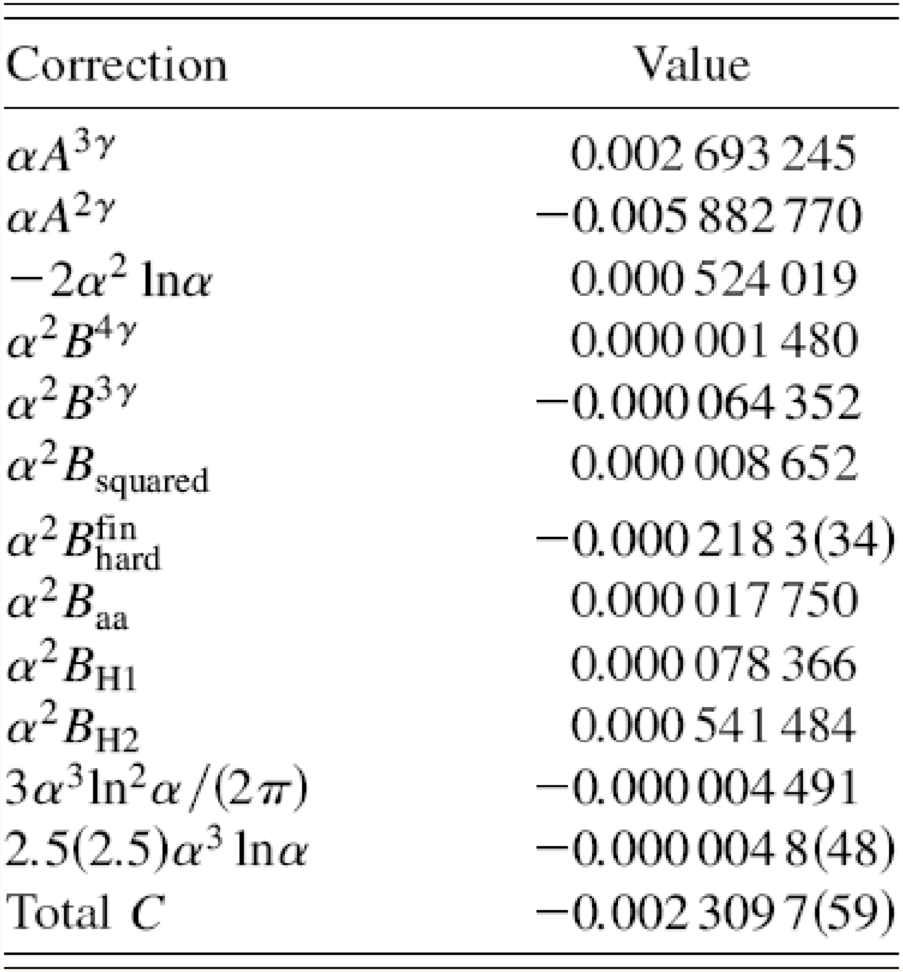
Summary

New states of positronium-like systems, ions and molecules: opportunity to test three- and four-body QED.

The Coulomb wave functions not known analytically for $e^+ e^- e^-$ and $e^+ e^- e^+ e^-$.

Variational methods very accurate. O(alpha^2) corrections computed for the ion decay rate and the Ps_2 P-S interval. Both are being measured.

Breakdown of corrections to the Ps ion width



Correction	Value
$\alpha A^{3\gamma}$	0.002 693 245
$\alpha A^{2\gamma}$	−0.005 882 770
$-2\alpha^2 \ln\alpha$	0.000 524 019
$\alpha^2 B^{4\gamma}$	0.000 001 480
$\alpha^2 B^{3\gamma}$	−0.000 064 352
$\alpha^2 B_{\mathrm{squared}}$	0.000 008 652
$\alpha^2 B_{\mathrm{hard}}^{\mathrm{fin}}$	−0.000 218 3(34)
$\alpha^2 B_{\mathrm{aa}}$	0.000 017 750
$\alpha^2 B_{\mathrm{H1}}$	0.000 078 366
$\alpha^2 B_{\mathrm{H2}}$	0.000 541 484
$3\alpha^3 \ln^2\alpha/(2\pi)$	−0.000 004 491
$2.5(2.5)\alpha^3 \ln\alpha$	−0.000 004 8(48)
Total C	−0.002 309 7(59)

$$\Gamma(\mathrm{Ps}^-) = 2\pi \frac{\alpha^5 m_e c^2}{\hbar}(1+C)\left\langle \delta^3(r_{12}) \right\rangle$$