Proton structure corrections to electronic and muonic hydrogen hyperfine splitting

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work done with Vahagn Nazaryan and Keith Griffioen

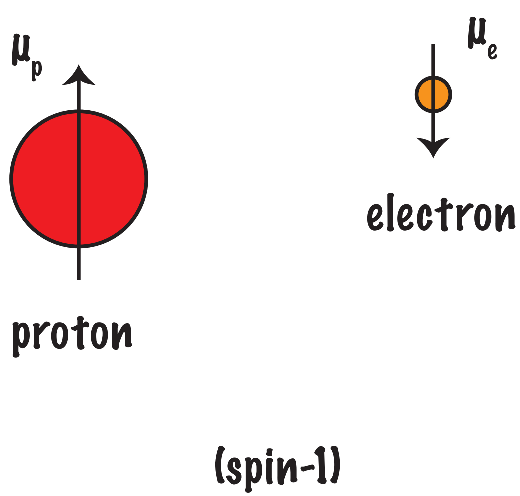
PRL 96, 163001 (2006) and 0805.2603 [atom-ph]

Introduction

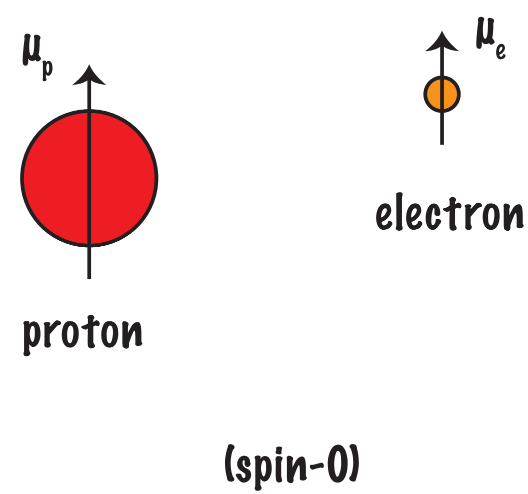
- Subject: hyperfine splitting line in ordinary and muonic hydrogen
- Mainly worry about one correction, that involves proton structure and hence nuclear/particle physics
- Outline
  - Introduction
  - Description of calculation
  - Results
  - Conclusions

Introduction

- Well known: spin-dependent interaction gives hyperfine splitting in hydrogen ground state (and in other states).



← slightly higher energy than →



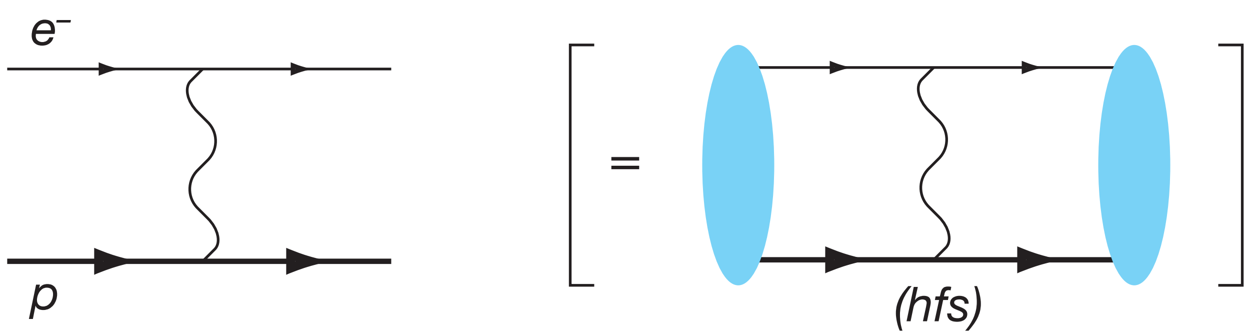
- Splitting known to 13 figures in frequency units,

$$E_{hfs}(e^- p) = 1\,420.405\,751\,766\,7\,(9)\ \mathrm{MHz}$$

- Goal: Calculate hfs to part per million (ppm)

Introduction

- Lowest order calculation often pesented in NR quantum mechanics course:



$$E_F^p = \frac{8\alpha^3 m_r^3}{3\pi}\mu_B\mu_p = \frac{16\alpha^2}{3}\frac{\mu_p}{\mu_B}\frac{R_\infty}{\left(1+m_e/m_p\right)^3}$$

- Convention: put in actual measured μ_p for proton, and Bohr magneton μ_B for electron.

- Constants well enough known to allow part in 10^8 calculation of "Fermi energy."

$$E_F^p = \frac{8\alpha^3 m_r^3}{3\pi}\mu_B\mu_p = \frac{16\alpha^2}{3}\frac{\mu_p}{\mu_B}\frac{R_\infty}{(1+m_e/m_p)^3}$$

- R_∞ is Rydberg constant in Hertz (6.6 ppt)
- m_e/m_p known to ppb
- α known to 1/2 ppb
- μ_p/μ_B known to 10 ppb
- Hence E_F^p calculated to 10 ppb

Introduction

- Re: part per million (ppm) goal,
  - Challenge ...
  - New physics?
    - Note: Hints of new physics in B-meson physics (BEACH 2008: Conference on Hyperons, Charm, and Beauty Hadrons)
- Pure QED systems (e.g., muonium) easily allow this and better. Problem is hadronic corrections --- proton structure.

corrections codified

$$E_{\rm hfs}(\ell^- p) = \left(1 + \Delta_{\rm QED} + \Delta^p_{\rm hvp} + \Delta^p_{\mu\rm vp} + \Delta^p_{\rm weak} + \Delta_{\rm S}\right) E_F$$

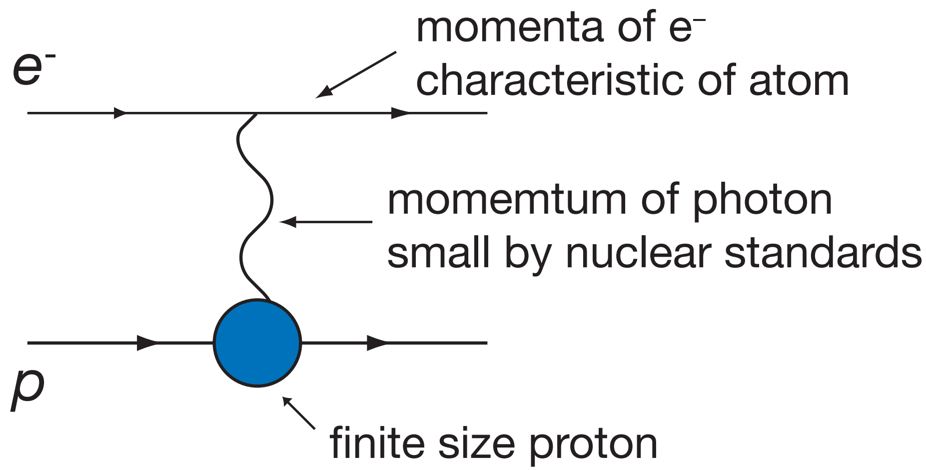
- $\Delta_{\rm QED}$: pure QED, well calculated
- $\Delta_{\rm hvp}$, $\Delta_{\mu\rm vp}$, $\Delta_{\rm weak}$: some vacuum polarization terms and Z-boson exchange: small, not a problem
- $\Delta_{\rm S} = \Delta_{\rm Z} + \Delta_{\rm R} + \Delta_{\rm pol}$
  - Proton structure dependent
  - Zemach, recoil, & polarizability terms
  - all 2-photon exchange

Introduction (cont.)

- Δ_S (total) will be about 40 ppm, so need ca. 2% accuracy
- What we do
  - Use data from electron scattering to measure proton structure
  - Use above measurements to calculate proton structure effects on hydrogen hyperfine splitting (hhfs)
- What we don't do
  - We don't start from scratch, using QCD Lagrangian, or facsimile, to calculate proton structure correction. Not yet possible to reach target precision.
  - Cf., Excellent chiral Lagrangian calculation by Pineda (2003) gets about 2/3 target Δ_S; or about 13 ppm accuracy
  - Also see preceding talk by Buchmann

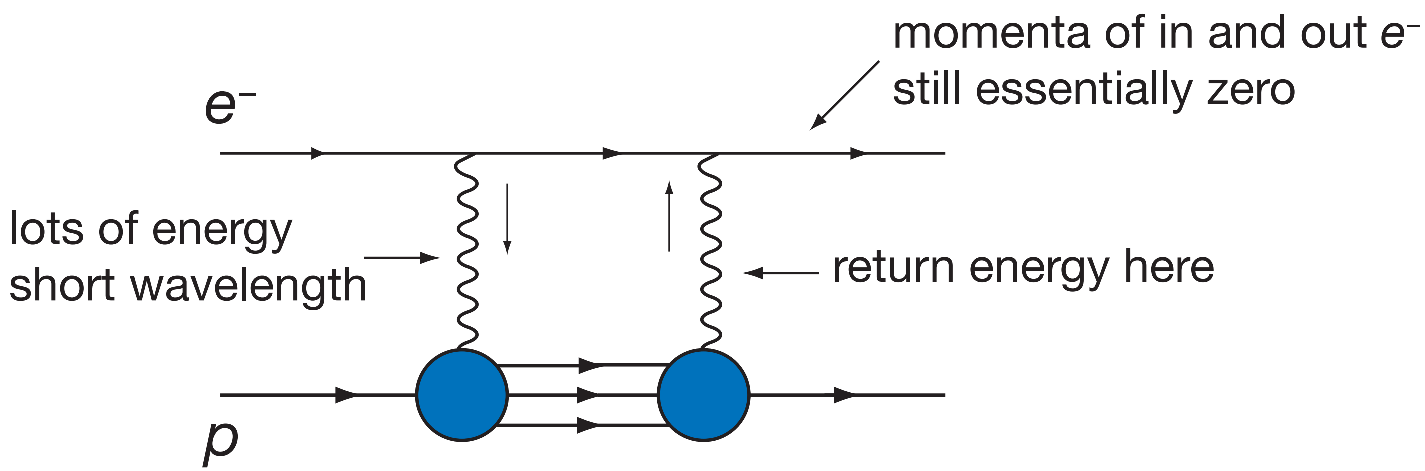
Calculation of proton structure corrections

- Proton size about 10^{-5} Ångström---enough to notice
- But not in lowest order:



- Photon sees whole proton: structure plays no role
- Hence not learned in "freshman" QM

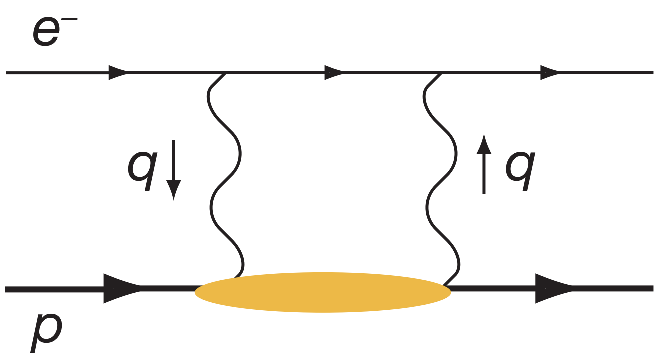
Calculation: Two-photon exchange



- short wavelength photon sees inside proton---need details on proton structure
- Inter-proton intermediate state may or may not still be a proton

more professionally ...

- Lower part of diagram is forward Compton scattering amplitude:



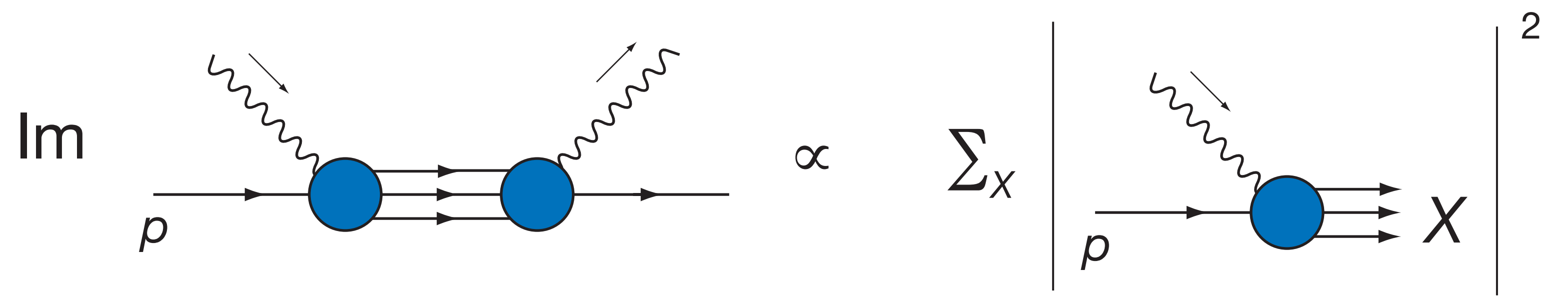
$$T_{\mu\nu} \quad = \quad \frac{i}{2\pi m_p} \int d^4\xi \, e^{iq\cdot\xi} \, \langle pS| \, T \left\{ j_\mu(\xi), j_\nu(0) \right\} |pS\rangle$$

- Want spin dependent part, which is antisymmetric,

$$T^A_{\mu\nu} = \frac{i}{m_p \nu} \, \epsilon_{\mu\nu\alpha\beta} \, q^\alpha \left[\Big(H_1(\nu, q^2) + H_2(\nu, q^2) \Big) \, S^\beta - H_2(\nu, q^2) \frac{S\cdot q \; p^\beta}{p\cdot q} \right]$$

- S^β is proton spin vector
- $H_{1,2}$ are functions of photon lab energy ν and photon "mass" Q^2

Optical theorem



Im {forward scattering amplitude} ∝ total cross section

- RHS is cross section for $e + p \to e' + X$
- Measured at SLAC, DESY, JLab, Mainz, ...

Optical theorem (cont.)

- Standard definitions for Im parts of $H_{1,2}$:

$$\begin{aligned} \mathrm{Im}\, H_1(\nu, q^2) &= \frac{1}{\nu}\, g_1(\nu, q^2) \\ \mathrm{Im}\, H_2(\nu, q^2) &= \frac{m_p}{\nu^2}\, g_2(\nu, q^2) \end{aligned}$$

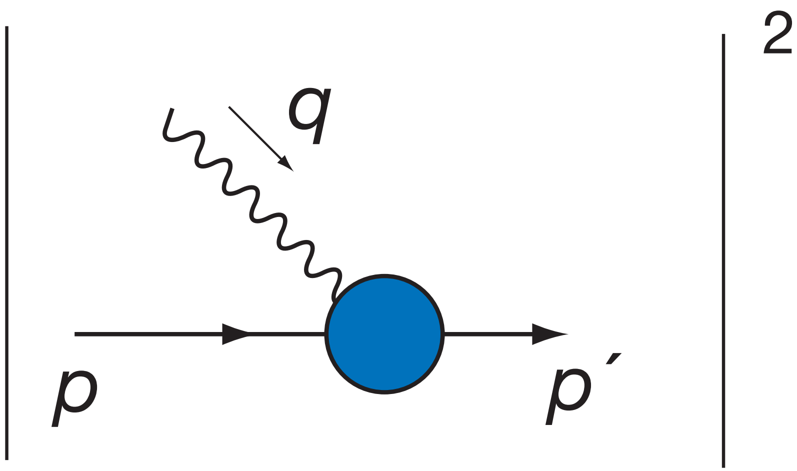
- Problem: can measure g_i, but that means we only know Im H, but we need all of H.

- Solution: dispersion relations, or the Cauchy integral theorem

$$H_1(\nu_1, q^2) = \frac{1}{\pi} \int_{\nu_{th}^2}^{\infty} d\nu\, \frac{\mathrm{Im}\, H_1(\nu, q^2)}{\nu^2 - \nu_1^2} + \text{elastic part}$$

Elastic part

- The part of the total cross section that has only final state proton, i.e., is elastic scattering



- Vertex (real p') given by

$$\Gamma_\mu = \gamma_\mu F_1(q^2) + \frac{i}{2m_p}\sigma_{\mu\nu}q^\nu F_2(q^2)$$

where $F_1(q^2), F_2(q^2)$ are measured in elastic scattering

- Gives the original NR Zemach term + the recoil term

- Often $G_M = F_1 + F_2;\; G_E = F_1 - \frac{q^2}{4m_p^2}F_2$

Most recent useful experiment for $g_{1,2}$ (from JLab)

- Jefferson Lab (Newport News, VA, USA) experiment EG1 measured spin-dependent inelastic electron-proton scattering
- $Q^2 > 0.045\ \mathrm{GeV}^2$ (earlier SLAC expt. had $Q^2 > 0.15\ \mathrm{GeV}^2$)
- gave results in terms of structure functions g_i
- for reference,

Aerial view of accelerator and experimental halls

$$\frac{d\sigma_{\rightarrow\rightarrow}}{dE'\,d\Omega} - \frac{d\sigma_{\rightarrow\leftarrow}}{dE'\,d\Omega} = \frac{8\alpha^2 E'}{m_p Q^2 E}\left(\frac{E + E'\cos\theta}{m_p \nu} g_1 + \frac{Q^2}{m_p \nu^2} g_2\right)$$

$$\frac{d\sigma_{\rightarrow\uparrow}}{dE'\,d\Omega} - \frac{d\sigma_{\rightarrow\downarrow}}{dE'\,d\Omega} = \frac{8\alpha^2 E'^2}{m_p^2 Q^2 E \nu}\sin\theta\left(g_1 - \frac{2E}{\nu} g_2\right)$$

Formulaic results for parts of Δ_S

- NR part of elastic contribution: Zemach term

$$\Delta_Z = \frac{8\alpha m_r}{\pi} \int_0^\infty \frac{dQ}{Q^2} \left[G_E(Q^2) \frac{G_M(Q^2)}{1+\kappa_p} - 1 \right] \equiv -2\alpha m_r r_Z$$

- Charles Zemach, 1956

More formula results

- Recoil term: relativistic part of elastic contrib.

$$\Delta_R^p = \frac{2\alpha m_r}{\pi m_p^2}\int_0^\infty dQ\, F_2(Q^2)\frac{G_M(Q^2)}{1+\kappa_p}$$

$$+ \frac{\alpha m_\ell m_p}{2(1+\kappa_p)\pi(m_p^2-m_\ell^2)}\left\{\int_0^\infty \frac{dQ^2}{Q^2}\left(\frac{\beta_1(\tau_p)-4\sqrt{\tau_p}}{\tau_p}-\frac{\beta_1(\tau_\ell)-4\sqrt{\tau_\ell}}{\tau_\ell}\right)F_1(Q^2)G_M(Q^2)\right.$$

$$\left. +3\int_0^\infty \frac{dQ^2}{Q^2}\Big(\beta_2(\tau_p)-\beta_2(\tau_\ell)\Big)F_2(Q^2)G_M(Q^2)\right\}$$

$$- \frac{\alpha m_\ell}{2(1+\kappa_p)\pi m_p}\int_0^\infty \frac{dQ^2}{Q^2}\beta_1(\tau_\ell)F_2^2(Q^2)$$

- $\beta_{1,2}$ soon defined; $\tau_i \equiv Q^2/4m_i^2$
- Bodwin & Yennie 1988; Faustov et al. 1970
- Memorize the last term

Polarizability (inelastic) terms

$$\Delta_{\text{pol}} = \frac{\alpha m_\ell}{2(1+\kappa_p)\pi m_p}(\Delta_1 + \Delta_2)$$

the prefactor is about 1/4 ppm for electrons,

$$\begin{aligned}
\Delta_1 &= \int_0^\infty \frac{dQ^2}{Q^2} \Bigg\{ \beta_1(\tau_\ell) F_2^2(Q^2) \\
&\quad + \frac{8m_p^2}{Q^2} \int_0^{x_{th}} dx \, \frac{x^2 \beta_1(\tau) - (m_\ell^2/m_p^2)\beta_1(\tau_\ell)}{x^2 - m_\ell^2/m_p^2} g_1(x,Q^2) \Bigg\} \\
\Delta_2 &= -24m_p^2 \int_0^\infty \frac{dQ^2}{Q^4} \int_0^{x_{th}} dx \, \frac{x^2\left[\beta_2(\tau) - \beta_2(\tau_\ell)\right]}{x^2 - m_\ell^2/m_p^2} g_2(x,Q^2)
\end{aligned}$$

with

$$\begin{aligned}
\tau &= \nu^2/Q^2 \\
\beta_1(\tau) &= -3\tau + 2\tau^2 + 2(2-\tau)\sqrt{\tau(\tau+1)} \\
\beta_2(\tau) &= 1 + 2\tau - 2\sqrt{\tau(\tau+1)}
\end{aligned}$$

- Massless lepton: Drell and Sullivan and others, 1960 and early 1970s
- Massive lepton: Faustov, Cherednikova, and Martynenko, 2003; Us, 2008.

Comments

- Formulas (esp. for electron case) old.
- New since 2000:
  - Data good enough to give non-zero Δ_{pol}
- New since 2006:
  - Final data from JLab EG1 expt. published
    - systematic errors! (Prok et al., 0802.2232)
  - New low-Q^2 G_E data from Mainz (J. Bernauer, unpub.,
    - shown at conferences, e.g. Walcher, ECT*, May 2008)
- BTW: where did the inelastic (?) $F_2{}^2$ term come from? ..... (It makes Δ_{pol} finite in the massless lepton limit.)
- Leads to rant: Do not take Δ_{pol} and recoil corrections from different sources (without checking compatability).

Results for Δ_{pol} 2008

Term	Q^2 (GeV^2)	From	Value w/AMT F_2
Δ_1	[0, 0.0452]	F_2 & g_1	1.35(0.22)(0.87) ( )
	[0.0452, 20]	F_2	7.54 ( ) (0.23) ( )
		g_1	−0.14(0.21)(1.78)(0.68)
	[20, ∞]	F_2	0.00 ( ) (0.00) ( )
		g_1	0.11 ( ) ( ) (0.01)
total Δ_1			8.85(0.30)(2.67)(0.70)
Δ_2	[0, 0.0452]	g_2	−0.22 ( ) ( ) (0.22)
	[0.0452, 20]	g_2	−0.35 ( ) ( ) (0.35)
	[20, ∞]	g_2	0.00 ( ) ( ) (0.00)
total Δ_2			−0.57 ( ) ( ) (0.57)
$\Delta_1 + \Delta_2$			8.28(0.30)(2.67)(0.90)
Δ_{pol} (ppm)			1.88(0.07)(0.60)(0.20)

- errors (statistical)(systematic from data)(modeling)
- AMT = Form factors fit by Arrington, Melnitchouk, Tjon (2007)
- Quote polarizability correction as 1.88 ± 0.64 ppm
- compatible with Faustov-Martynenko.

Results for Δ_{pol} 2008 --- muonic hydrogen

Term	Q^2 (GeV^2)	From	Value w/AMT F_2
Δ_1	[0, 0.0452]	F_2 and g_1	0.86(0.17)(0.67) ( )
	[0.0452, 20]	F_2	6.77 ( ) (0.21) ( )
		g_1	0.18(0.18)(1.62)(0.64)
	[20, ∞]	F_2	0.00 ( ) (0.00) ( )
		g_1	0.11 ( ) ( ) (0.01)
total Δ_1			7.92(0.25)(2.30)(0.66)
Δ_2	[0, 0.0452]	g_2	−0.12 ( ) ( ) (0.12)
	[0.0452, 20]	g_2	−0.29 ( ) ( ) (0.29)
	[20, ∞]	g_2	−0.00 ( ) ( ) (0.00)
total Δ_2			−0.41 ( ) ( ) (0.41)
$\Delta_1 + \Delta_2$			7.51(0.25)(2.30)(0.77)
Δ_{pol} (ppm)			351.(12.)(107.)(36.)

Overall results for ordinary hydrogen 2008

Quantity	value (ppm)	uncertainty (ppm)
$(E_{\mathrm{hfs}}(e^- p)/E_F^p) - 1$	1 103.48	0.01
Δ_{QED}	1 136.19	0.00
$\Delta^p_{\mu\mathrm{vp}} + \Delta^p_{\mathrm{hvp}} + \Delta^p_{\mathrm{weak}}$	0.14	
Δ_Z (using AMT)	−41.43	0.44
Δ^p_R (using AMT)	5.85	0.07
Δ_{pol} (this work, using AMT)	1.88	0.64
Total	1102.63	0.78
Deficit	0.85	0.78

Ending and outlook

- Our 2008 result using 2001 EG1 data (final data out in '08, Prok et al., 0802.2232):

$$\Delta_{\text{pol}} = 1.88 \pm 0.64 \text{ ppm}$$

- Table of non-zero results

Authors	Δ_{pol} (ppm)
Faustov & Martynenko (2002)	1.4 ± 0.6
Us (2006)	1.3 ± 0.3
Faustov, Gorbacheva, & Martynenko (2006)	2.2 ± 0.8
Us (2008)	1.88 ± 0.64

- (Faustov et al. don't use JLab data)

- Sum of all corrections now just under 1 ppm, or about 1 standard deviation, from data

Outlook

- Have come a long way since my 1987 QM course notes claim that best calculations had 30 ppm accuracy.
- Some diffidence: there was a 1988 paper that claimed 1 ppm calculations were possible
- More accuracy ?
  - Uncertainties in Zemach term not now trivial. Low Q^2 elastic FF important. New data from Mainz. And better analysis of existing data may be possible.
  - Largest uncertainty is from systematic error in g_1. Already exists unanalyzed EG4 data (Q^2 > 0.015 GeV^2).
  - Hfs less sensitive to g_2, but there are proposals for better g_2 measurements at JLab (e.g., "Semi-SANE").

Outlook

- Current best charge radius measurements come from Lamb shift, error 1% vs. 2% from electron scattering. Experiment “imminent” to do muonic hydrogen Lamb shift, with possible 0.1% accurate charge radius!

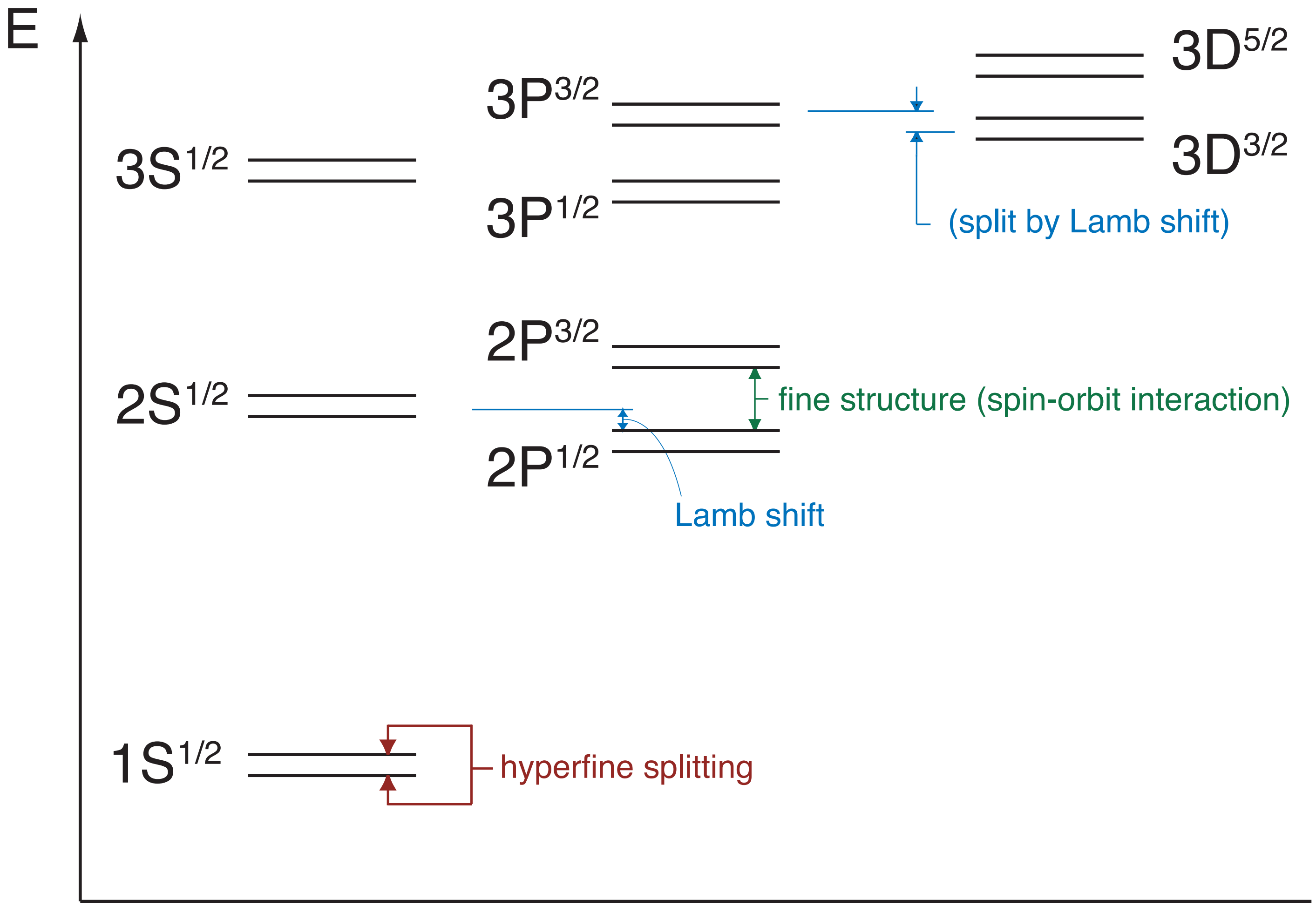
End

Extras

Just in case

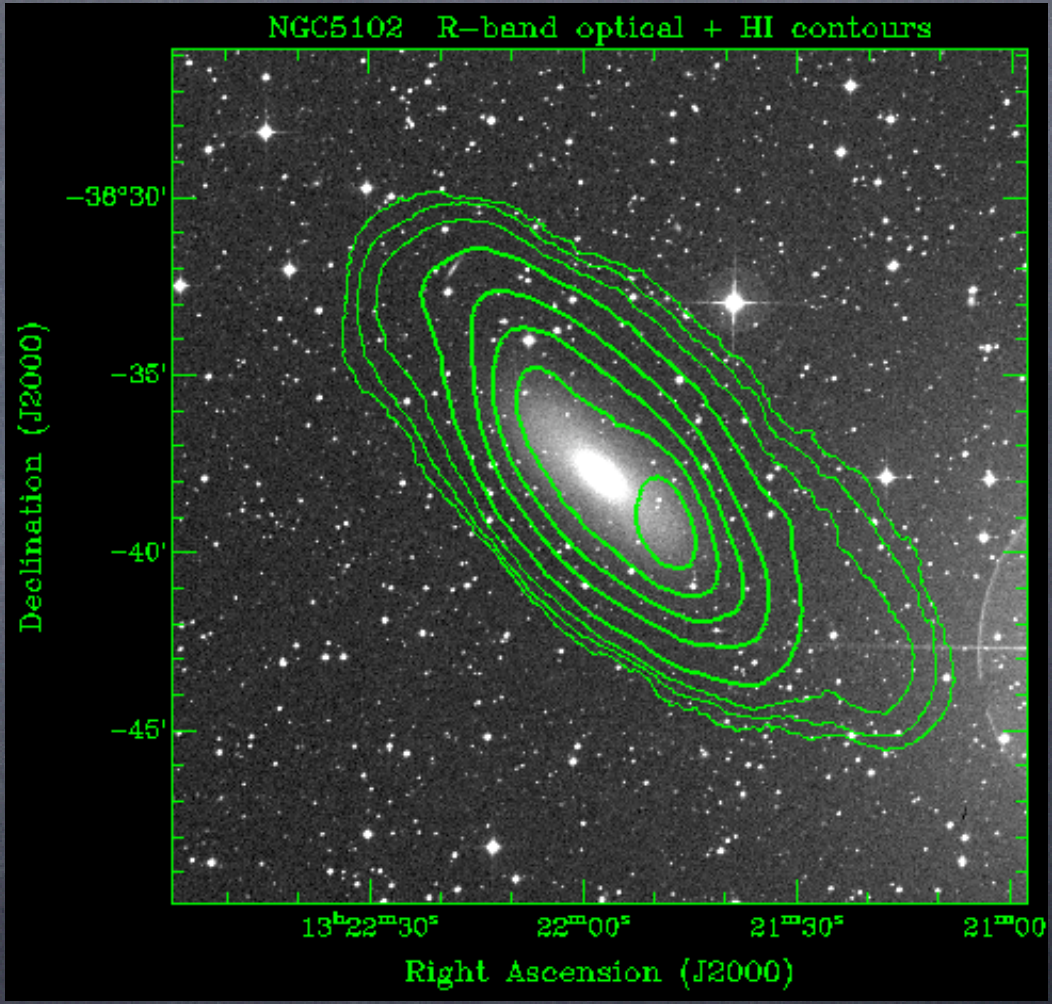

Canada
British Columbia
Alberta
Saskatchewan
Manitoba
Ontario
Quebec
New Brunswick
Nova Scotia
Washington
Oregon
Idaho
Montana
North Dakota
South Dakota
Minnesota
Wisconsin
Michigan
Wyoming
Nevada
California
Utah
Colorado
Nebraska
Iowa
Illinois
Indiana
Ohio
Pennsylvania
New York
Vermont
New Hampshire
Maine
Massachusetts
Connecticut
Rhode Island
New Jersey
Delaware
Maryland
West Virginia
Virginia
Williamsburg
(or Newport News)
Kansas
Missouri
Kentucky
U. S. A.
Arizona
New Mexico
Oklahoma
Arkansas
Tennessee
North Carolina
South Carolina
Georgia
Alabama
Mississippi
Louisiana
Texas
Florida
Russia
Alaska
Canada
Hawaii
Mexico
The Bahamas

Hydrogen energy levels



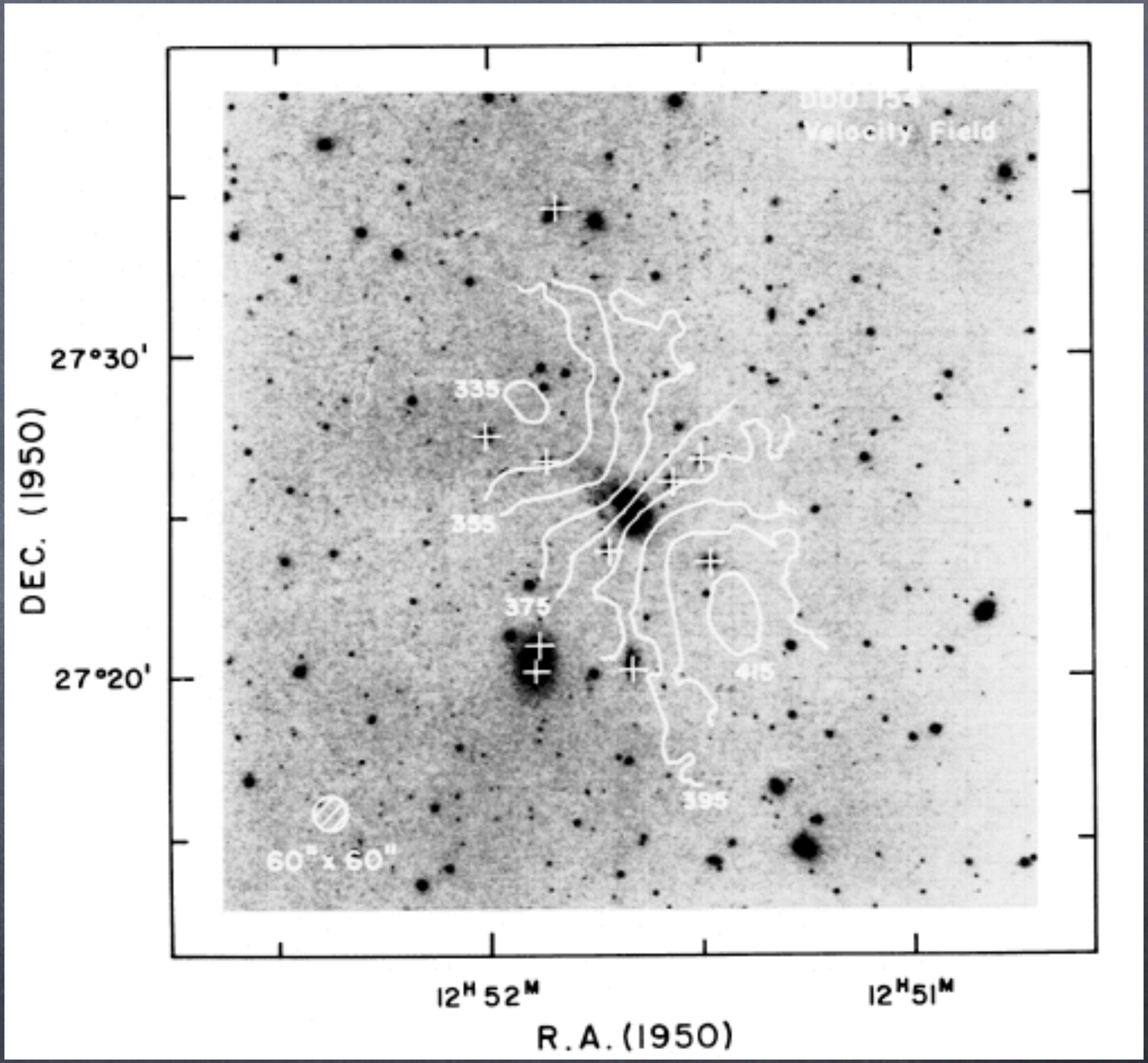
Extent of galaxies, seen in 21 cm radio light

- NGC 5102, Local Volume HI Survey
- Radio observations laid over optical photo
- 3X bigger in radio light

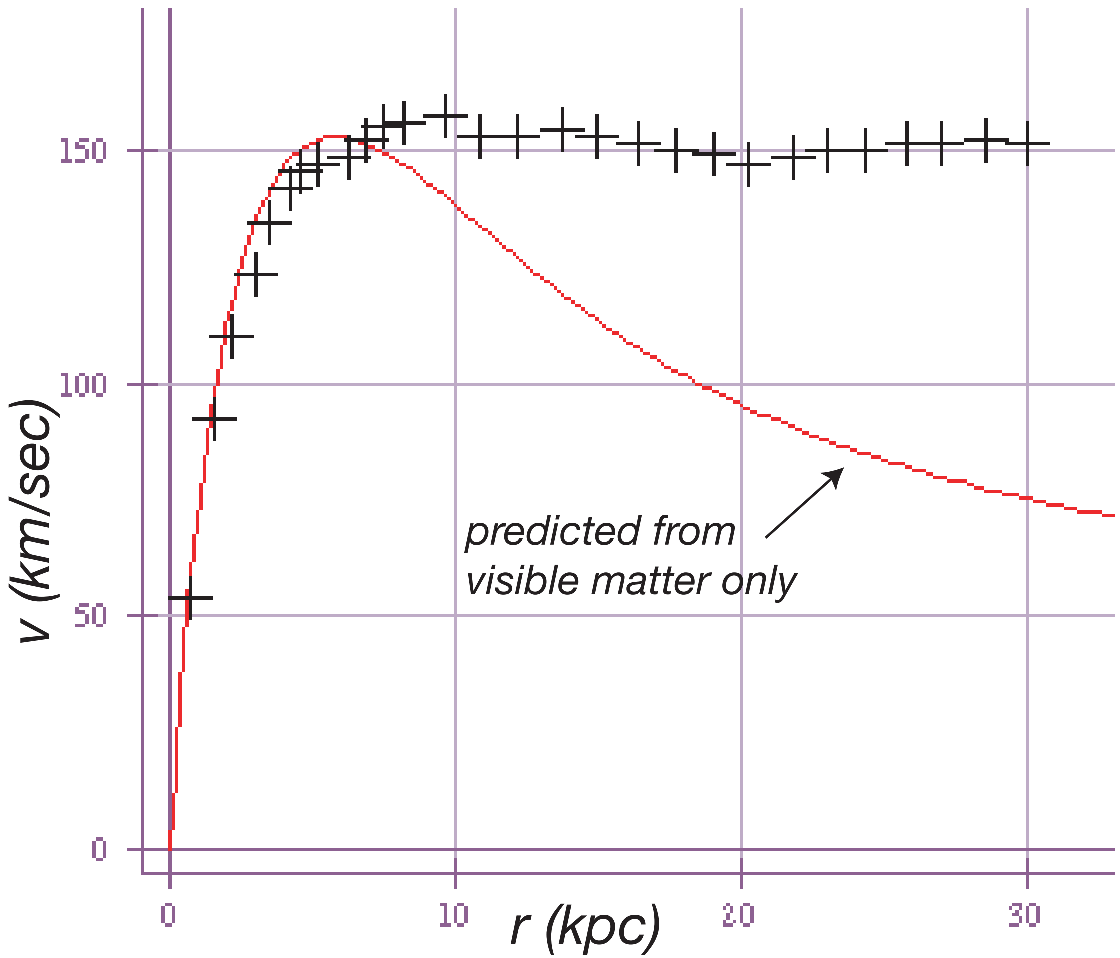


Velocity of H-gas, seen with 21 cm line

- DDO 154, Carignan et al.
- Numbers give velocities, in km/sec, from Doppler shift
- rotation curve



Sample rotation curve

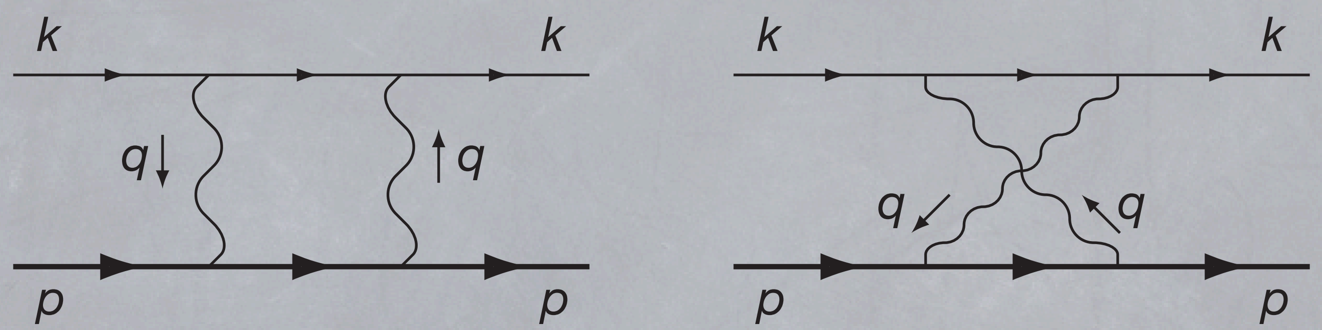


The visible NGC 3198

- Long history.
- Zemach (1956) calculates hfs from elastic contributions in terms of proton form factors.
- Iddings (1965), Drell and Sullivan (1967), deRafael (1971) calculate inelastic (polarizability) contribution to hydrogen hfs.
- Faustov and Martynenko (2002), using SLAC data, estimate numerically the polarizability contribution to hydrogen hfs. First to get result inconsistent with zero.
- Friar and Sick (2004) determine the Zemach radius [to be defined] using world form factor data.
- Dupays et al. (2003), Volotka et al. (2005), Brodsky et al. (2005) infer Zemach radius from hfs data using polarizability results of Faustov and Martynenko.
- Inconsistencies between last two called for a review of corrections.
- Newer data from JLab, esp. at lower Q^2, crucial for this purpose.

Final indelicate point

- Can we use the dispersion relation? Depends.
- E.g., do elastic box calculation



$$H_1^{el} = -\frac{2m_p}{\pi}\left(\frac{q^2 F_1(q^2) G_M(q^2)}{(q^2+i\epsilon)^2 - 4m_p^2\nu^2} + \frac{F_2^2(q^2)}{4m_p^2}\right)$$

- Pole at value of photon energy that makes the intermediate proton "real":

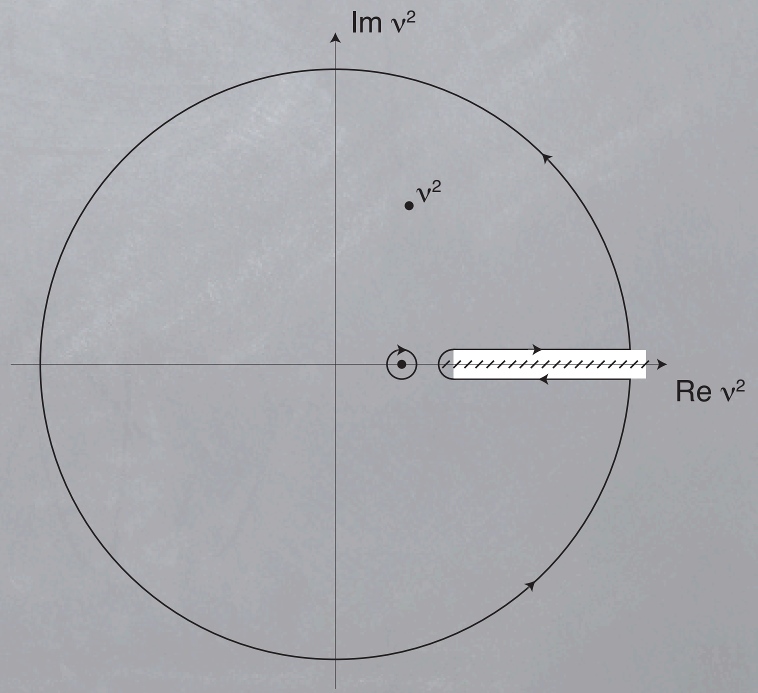
$$\nu = Q^2/(2m_p)$$

FIP

- Inelastic case similar. If total mass of intermediate state is W, pole at

$$\nu = (W^2 - m_p^2 + Q^2/(2m_p)$$

- W is continuously varying from threshold & up. Hence H_1 has elastic pole in ν plus cut,



pole/cut structure of H_1 in complex ν^2-plane

FIP

- In using Cauchy formula, pole and cut have been kept
- Infinite contour discarded: Legitimate if function falls to zero fast enough
- Fails for H_1^{el} alone, but we are dealing with composite particle
- QED models say it is o.k.
- Regge models say it is o.k.