Recursion

(1) Find a general formula for the n^{th} Fibonacci number.
(2) Show that $\lfloor(4+\sqrt{11})^n\rfloor$ is odd for any positive integer n.
(3) For the Fibonacci sequence, $\{F_n\}$, where $F_1 = F_2 = 1$, $F_n = F_{n-1} + F_{n-2}, n \geq 3$, show that F_5, F_{10}, $F_{15}, \ldots$ are multiples of 5.
(4) Let $T_0 = 2$, $T_1 = 3$, $T_2 = 6$ and for $n \geq 3$

$$T_n = (n+4)T_{n-1} - 4nT_{n-2} + (4n-8)T_{n-3}.$$

Find, with proof, a formula for T_n of the form $T_n = A_n + B_n$ where A_n and B_n are well known sequences.
(5) Let F_n denote the n^{th} Fibonacci number. Express $F_{1999}^2 + F_{1998}^2$ in the form F_n for some n.
(6) Suppose that $a_1 = 3$ and $a_{n+1} = a_n(a_n + 2)$. Find a general formula for a_n.
(7) Show that if $(2+\sqrt{3})^k = 1 + m + n\sqrt{3}$ for positive integers m, n, k with k odd, then m is a perfect square.
(8) The sequence $\{a_n\}_{n\geq 1}$ is defined by $a_1 = 1$, $a_2 = 2$, $a_3 = 24$, and, for $n \geq 4$,

$$a_n = \frac{6a_{n-1}^2 a_{n-3} - 8a_{n-1}a_{n-2}^2}{a_{n-2}a_{n-3}}$$