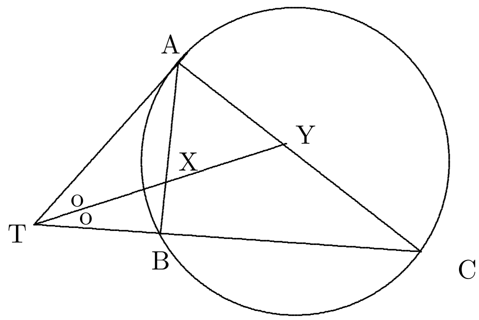
1. TA is tangent to the circle. Prove that for all choices of secant TBC, the bisector of $\angle ATC$ cuts an isosceles triangle $\triangle AXY$ from $\triangle ABC$.

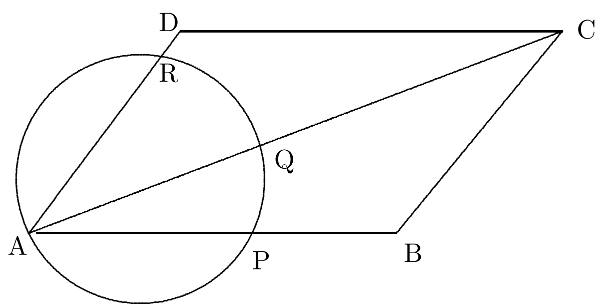


2. A captain finds his way to Treasure Island, which is circular in shape. He knows that there is treasure buried at the midpoint T of the segment joining the orthocentres $\triangle ABC$ and $\triangle DEF$, where A, B, C, D, E, F are six palm trees on the shore of the island in some order unknown to the captain. What is the maximum number of points at which the captain has to dig in order to recover the treasure?

3. i) Prove that the figure formed by joining midpoints of consecutive sides of a quadrilateral is a parallelogram.
ii) Prove: the diagonals of a quadrilateral are perpendicular if and only if the lengths of the lines joining the midpoints of opposite sides are equal.

4. Let $\triangle ABC$ be an equilateral triangle and let P be a point in the same plane. Prove: $PC + PA = PB$ if and only if P lies on the arc CA of the circumcircle of $\triangle ABC$.

5. A circle cuts two sides and a diagonal of a parallelogram $ABCD$ at P, Q, R as shown:



i) Show: $\triangle PQR$ is similar to $\triangle CBA$.
ii) Show: $AP \cdot AB + AR \cdot AD = AQ \cdot AC$.

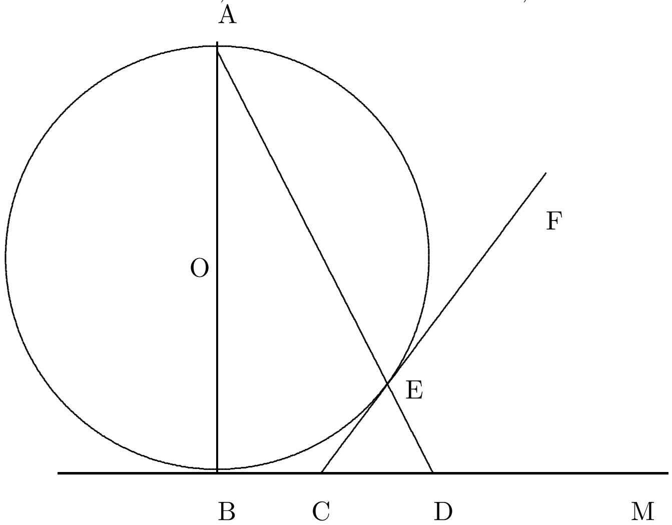
6. Let D, E, F be the feet of the altitudes from A, B, C, respectively. As usual, let H denote the orthocentre of $\triangle ABC$. Show that if D', E', F' are the intersections of the altitudes from A, B, C, respectively, with the circumcircle of $\triangle ABC$ then D bisects HD', E bisects HE', and F bisects HF'.

7. From the vertex A of $\triangle ABC$ three segments are drawn: the bisectors AM and AN of its interior and exterior angles and the tangent AK to the circumcircle of the triangle. The points M, K, and N are on the line BC. Prove that $MK = KN$. Hint: first show that one of $A/2 + B$ and $A/2 + C$ must be greater than or equal to 90 degrees. Draw the diagram according to this and look for isosceles triangles.

8. $PQRS$ is a cyclic quadrilateral. Diagonals PR and SQ intersect at X. From X, perpendiculars XK, XL, XM, XN are drawn to the sides PQ, QR, RS, SP, respectively.
i) Prove $\angle XKN = \angle XKL$.
ii) Prove that X is the centre of a circle that touches the sides of $KLMN$.

9. In quadrilateral $ABCD$, the points P, Q, R, S bisect the sides AB, BC, CD, DA, respectively. Show that PR and QS bisect each other.

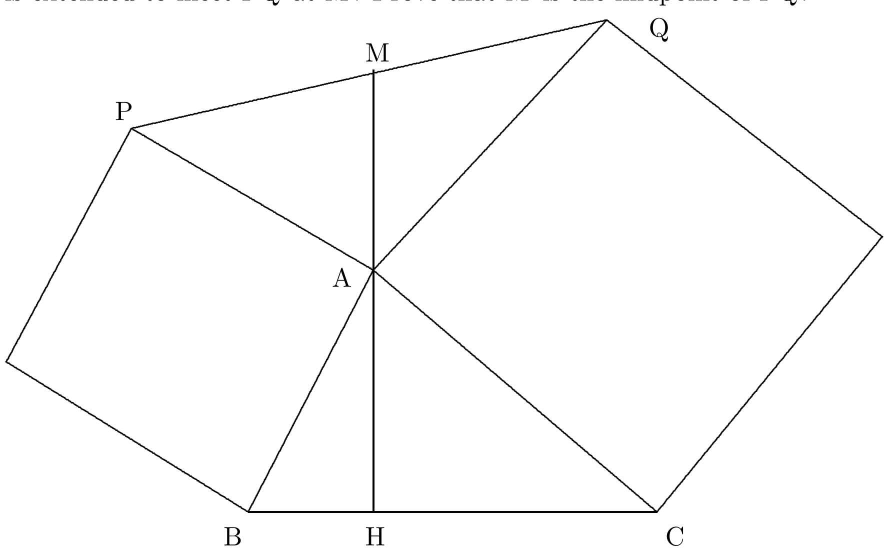
10. Let AOB be a diameter of a circle centered at O, BM be tangent to the circle at B, CF be tangent to the circle at E, let CF meet BM at C, and let AE meet BM at D. Prove that $BC = CD$.



11. Let A, B, C, D be four consecutive points on the circumference of a circle and P, Q, R, S be the points on the circumference which are the midpoints of the arcs AB, BC, CD, DA, respectively. Prove that PR is perpendicular to QS.

12. A square $PQRS$ has sides of length 12 cm and M is on PS such that $PM = 2$ cm. RQ is extended to T such that $QT = 4RQ$. PR and QM intersect X and XT cuts PQ at Y. Find the ratios $PX : XR$ and $PY : YQ$.

13. On sides AB and AC of $\triangle ABC$, squares are constructed as shown. Altitude AH is constructed and is extended to meet PQ at M. Prove that M is the midpoint of PQ.



14. Prove that for any points D, E, F taken on the interiors of the sides BC, CA, AB, respectively, of $\triangle ABC$, the circumcircles of $\triangle AEF$, $\triangle BDF$, and $\triangle CDE$ intersect at one point.

15. Prove the Pythagorean Theorem: if $\triangle ABC$ is a right triangle with hypotenuse BC, then $AB^2 + AC^2 = BC^2$.

16. Equilateral triangles are constructed externally on the sides of $\triangle ABC$ (that is, two of the vertices of the first equilateral triangle are AB, and so on). Prove that the centers of the equilateral triangles form an equilateral triangle. (This is called the outer Napoleon triangle.)

17. Prove that the line joining one vertex of a parallelogram to the midpoint of the opposite side trisects the diagonal.

18. Suppose that P is an interior point of $\triangle ABC$ and AP, BP, CP meet the opposite sides at D, E, F, respectively. Show that

$$\frac{AF}{FB} + \frac{AE}{EC} = \frac{AP}{PD}.$$

19. If A and B are fixed points on a given circle such that AB is not a diameter and if XY is a variable diameter, find the locus of P, the intersection of the line AX and the line BY.

20. $ABCD$ is a trapezoid with sides AB and CD parallel. Prove that if $AB + AC = CD$, then the angle bisector of $\angle ACD$ bisects the diagonal BD.