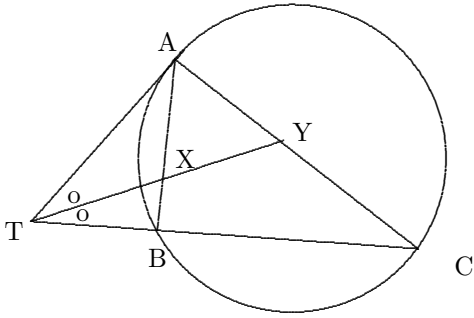
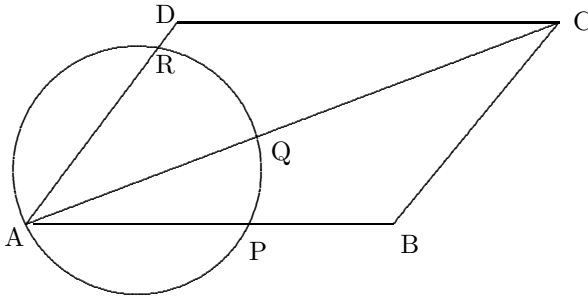


1.  $TA$  is tangent to the circle. Prove that for all choices of secant  $TBC$ , the bisector of  $\angle ATC$  cuts an isosceles triangle  $\triangle AXY$  from  $\triangle ABC$ .

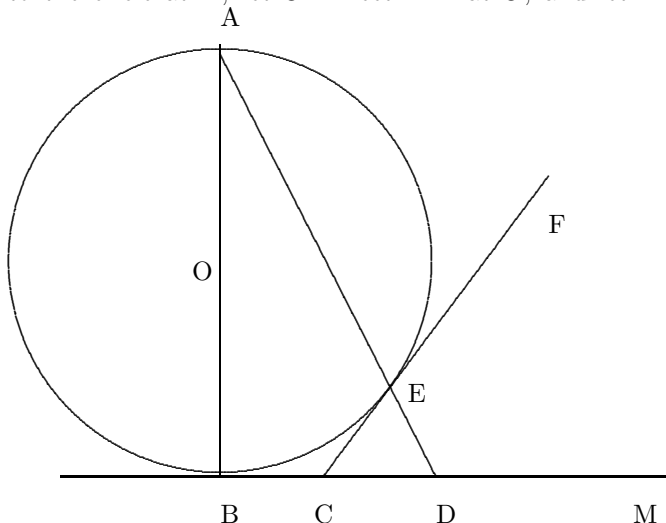


2. A captain finds his way to Treasure Island, which is circular in shape. He knows that there is treasure buried at the midpoint  $T$  of the segment joining the orthocentres  $\triangle ABC$  and  $\triangle DEF$ , where  $A, B, C, D, E, F$  are six palm trees on the shore of the island in some order unknown to the captain. What is the maximum number of points at which the captain has to dig in order to recover the treasure?
3. i) Prove that the figure formed by joining midpoints of consecutive sides of a quadrilateral is a parallelogram.  
ii) Prove: the diagonals of a quadrilateral are perpendicular if and only if the lengths of the lines joining the midpoints of opposite sides are equal.
4. Let  $\triangle ABC$  be an equilateral triangle and let  $P$  be a point in the same plane. Prove:  $PC + PA = PB$  if and only if  $P$  lies on the arc  $CA$  of the circumcircle of  $\triangle ABC$ .
5. A circle cuts two sides and a diagonal of a parallelogram  $ABCD$  at  $P, Q, R$  as shown:

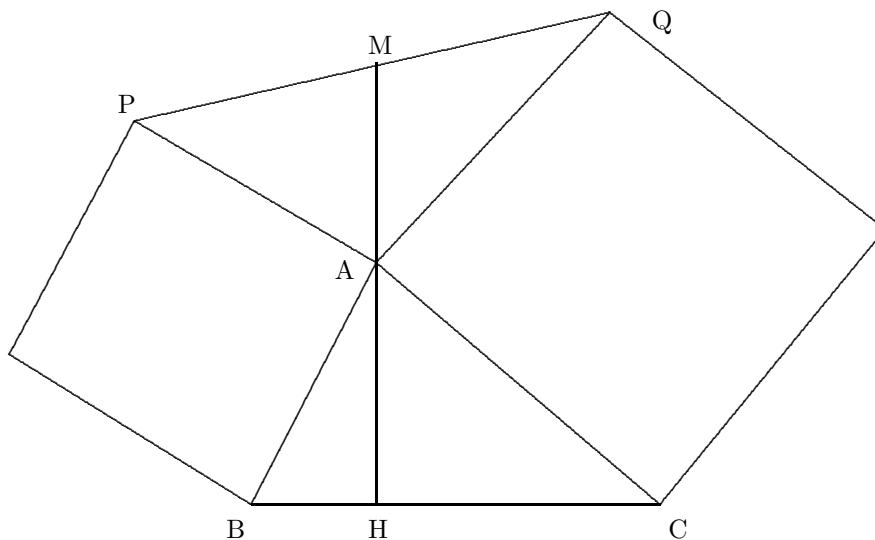


- i) Show:  $\triangle PQR$  is similar to  $\triangle CBA$ .
- ii) Show:  $AP \cdot AB + AR \cdot AD = AQ \cdot AC$ .
6. Let  $D, E, F$  be the feet of the altitudes from  $A, B, C$ , respectively. As usual, let  $H$  denote the orthocentre of  $\triangle ABC$ . Show that if  $D', E', F'$  are the intersections of the altitudes from  $A, B, C$ , respectively, with the circumcircle of  $\triangle ABC$  then  $D$  bisects  $HD'$ ,  $E$  bisects  $HE'$ , and  $F$  bisects  $HF'$ .
7. From the vertex  $A$  of  $\triangle ABC$  three segments are drawn: the bisectors  $AM$  and  $AN$  of its interior and exterior angles and the tangent  $AK$  to the circumcircle of the triangle. The points  $M, K$ , and  $N$  are on the line  $BC$ . Prove that  $MK = KN$ . Hint: first show that one of  $A/2 + B$  and  $A/2 + C$  must be greater than or equal to 90 degrees. Draw the diagram according to this and look for isosceles triangles.
8.  $PQRS$  is a cyclic quadrilateral. Diagonals  $PR$  and  $SQ$  intersect at  $X$ . From  $X$ , perpendiculars  $XK, XL, XM, XN$  are drawn to the sides  $PQ, QR, RS, SP$ , respectively.
  - i) Prove  $\angle XKN = \angle XKL$ .
  - ii) Prove that  $X$  is the centre of a circle that touches the sides of  $KLMN$ .
9. In quadrilateral  $ABCD$ , the points  $P, Q, R, S$  bisect the sides  $AB, BC, CD, DA$ , respectively. Show that  $PR$  and  $QS$  bisect each other.

10. Let  $AOB$  be a diameter of a circle centered at  $O$ ,  $BM$  be tangent to the circle at  $B$ ,  $CF$  be tangent to the circle at  $E$ , let  $CF$  meet  $BM$  at  $C$ , and let  $AE$  meet  $BM$  at  $D$ . Prove that  $BC = CD$ .



11. Let  $A, B, C, D$  be four consecutive points on the circumference of a circle and  $P, Q, R, S$  be the points on the circumference which are the midpoints of the arcs  $AB, BC, CD, DA$ , respectively. Prove that  $PR$  is perpendicular to  $QS$ .
12. A square  $PQRS$  has sides of length 12 cm and  $M$  is on  $PS$  such that  $PM = 2$  cm.  $RQ$  is extended to  $T$  such that  $QT = 4RQ$ .  $PR$  and  $QM$  intersect  $X$  and  $XT$  cuts  $PQ$  at  $Y$ . Find the ratios  $PX : XR$  and  $PY : YQ$ .
13. On sides  $AB$  and  $AC$  of  $\triangle ABC$ , squares are constructed as shown. Altitude  $AH$  is constructed and is extended to meet  $PQ$  at  $M$ . Prove that  $M$  is the midpoint of  $PQ$ .



14. Prove that for any points  $D, E, F$  taken on the interiors of the sides  $BC, CA, AB$ , respectively, of  $\triangle ABC$ , the circumcircles of  $\triangle AEF$ ,  $\triangle BDF$ , and  $\triangle CDE$  intersect at one point.
15. Prove the Pythagorean Theorem: if  $\triangle ABC$  is a right triangle with hypotenuse  $BC$ , then  $AB^2 + AC^2 = BC^2$ .
16. Equilateral triangles are constructed externally on the sides of  $\triangle ABC$  (that is, two of the vertices of the first equilateral triangle are  $AB$ , and so on). Prove that the centers of the equilateral triangles form an equilateral triangle. (This is called the outer Napoleon triangle.)

17. Prove that the line joining one vertex of a parallelogram to the midpoint of the opposite side trisects the diagonal.

18. Suppose that  $P$  is an interior point of  $\triangle ABC$  and  $AP, BP, CP$  meet the opposite sides at  $D, E, F$ , respectively. Show that

$$\frac{AF}{FB} + \frac{AE}{EC} = \frac{AP}{PD}.$$

19. If  $A$  and  $B$  are fixed points on a given circle such that  $AB$  is not a diameter and if  $XY$  is a variable diameter, find the locus of  $P$ , the intersection of the line  $AX$  and the line  $BY$ .

20.  $ABCD$  is a trapezoid with sides  $AB$  and  $CD$  parallel. Prove that if  $AB + AC = CD$ , then the angle bisector of  $\angle ACD$  bisects the diagonal  $BD$ .