Number Theory: Fermat's Little Theorem and the Chinese Remainder Theorem

(1) Let $f(n) = 5n^{13} + 13n^5 + 9an$. Find the smallest positive integer a such that $f(n)$ is divisible by 65 for every integer n.
(2) If p is a prime, show that $2^p + 3^p$ cannot be an n^{th} power where $n > 1$.
(3) Show that for any prime $p \neq 2, 5$, infinitely many numbers of the form $11\dots1$ are multiples of p.
(4) Find all integral solutions to $3^m - 1 = 2^n$.
(5) $a^n + b^n = p^k$ for positive integers a, b and k, where p is an odd prime and $n > 1$ is an odd integer. Show that n must be a power of p.
(6) Find all primes p for which $\frac{2^{p-1}-1}{p}$ is a perfect square.
(7) Find all primes p such that $2^p + p^2$ is also prime.
(8) Prove that each prime divisor of $2^p - 1$, where p is a prime, is greater than p. What famous result known for centuries does this immediately imply?
(9) Prove that for any positive integer n, there exists a power of 2 whose last n digits are all 1s and 2s.