

UNIVERSITY OF WINDSOR UNDERGRADUATE MATHEMATICS CONTEST
October 9th

Each of the following problems is worth 10 points.
Please explain your answers completely!

- (1) If $\sin x = 3 \cos x$, then what is $\sin x \cos x$?
- (2) A 10 metre long rope goes from the ground to the top of a flag pole, wrapping around it five times in a parallel spiral. If the circumference of the flag pole is 60 centimetres, what is the height of the flag pole?
- (3) The points $(0, 0)$, $(a, 11)$, and $(b, 37)$ are vertices of an equilateral triangle. Find the value of ab .
- (4) Find the last two digits of the product of the positive roots of

$$\sqrt{2007}x^{\log_{2007} x} = x^2.$$

- (5) Consider the number 123456789. In how many ways can you rearrange its digits to get a number divisible by 11?
- (6) Let n be a fixed positive integer. Show that for only non-negative integers k , the diophantine equation:

$$x_1^3 + x_2^3 + \cdots + x_n^3 = y^{3k+2}$$

has infinitely many solutions in positive integers x_i and y .