University of Windsor Mathematics Contest Training Questions

1. The function f has the property that for each real number x,

$$f(x) + f(x-1) = x^2$$

If $f(19) = 94$ what is the remainder when $f(94)$ is divided by 1000?

2. Let $f(x) = \frac{9^x}{9^x+3}$. Evaluate the sum

$$f(\frac{1}{2007}) + f(\frac{2}{2007}) + f(\frac{3}{2007}) + ... + f(\frac{2006}{2007}).$$

3. Let a, b and c be positive real numbers. Prove that

$$a^a b^b c^c \geq (abc)^{\frac{a+b+c}{3}}$$

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