University of Windsor Mathematics Contest Practice Problems
Principle of Inclusion-Exclusion and Pigeonhole Principle

1. Suppose there are 200 first-year students, 100 of which are taking Calculus and 70 of which are taking Algebra. If there are 50 first-year students that are taking both Calculus and Algebra, how many first-year students are taking neither course?

2. Determine the number of integers between 1 and 10000 that are not divisible by 6, 7, or 8.

3. In how many ways can a poker hand (5 cards) be selected from a regular deck (52 cards) such that the hand contains at least one card in each suit?

4. How many permutations of the 26 letters do not contain any of the following sequences:
$PUTNAM, EXAM, DEC, FIRST$

The Pigeonhole Principle
If m pigeons occupy n pigeonholes with $m > n$ then at least one pigeonhole has 2 or more pigeons.

5. 5 friends run a race everyday for four months (excluding February). If no race ends in a tie, show that there are at least 2 races with identical outcomes.

6. Given any 5 points in a unit square, show that two of these points must be within $\frac{\sqrt{2}}{2}$ of each other.

7. The capacity of an arena is 800 people. How many people must there be to ensure that at least 2 people have the same first and last initial?

8. A bag contains 100 apples, 100 oranges, 100 bananas and 100 pears. Every minute you choose one fruit from the bag. How long will it take to ensure that you have at least a dozen fruit of the same kind?