2007 International Tournament of the Towns
Senior O Level Paper

Your three highest scoring solutions determine your total score.
Please provide full explanations for your answers!

[3] **1.** There are 100 pictures, and each of them includes images of an adult and of a child of smaller height (all 200 persons in the pictures are different). These pictures must be combined into a single large picture. We are allowed to change the scale of each picture by diminishing its size by some integer factor (independently for different pictures). Prove that we can do this so that on the combined picture all the adults are taller than all the children.

2. The integer 1 and two positive numbers x and y are written on a sheet of paper. At each step we can write down the sum or the difference of some two numbers already written or write down the inverse of some number already written ($\frac{1}{a}$ is the inverse of a if $a \neq 0$). Is it possible to obtain at some step

   [2] **a)** the number x^2?

   [2] **b)** the number xy?

[4] **3.** A straight line and two points A and B on the same side of it and at the same distance from it are given. By compass and straightedge, find the point C on the line such that the product $AC \cdot BC$ is the minimal possible.

[4] **4.** A magician is blindfolded and gives 29 cards with numbers from 1 to 29 to a spectator. The spectator hides two cards and returns the remaining cards to the magician's assistant. The assistant chooses two of them, and then the spectator communicates the numbers chosen by the assistant to the magician (in any order he prefers). After that, the magician guesses the numbers of the cards hidden by the spectator. How can the magician and the assistant arrange to succeed in performing this trick?

5. A square of side 1 cm on side is cut into three convex polygons. Is it possible that the diameter of each of them does not exceed

   [1] **a)** 1 cm;

   [2] **b)** 1.01 cm;

   [2] **c)** 1.001 cm?

   (The diameter of a polygon is the maximal distance between its vertices).