

2007 International Tournament of the Towns Junior O Level Paper

Your three highest scoring solutions determine your total score.

Please provide full explanations for your answers!

- [3] 1. What is the maximal number of black and white chips that can be placed on an 8×8 chess-board so that each horizontal and each vertical line contains twice as many white chips as black ones? (Each chip occupies a separate cell.)
- [4] 2. The number 1 and of some non-integer number x are written on a sheet of paper. At each step we can write down the sum or the difference of some two numbers already written or write down the inverse of some number already written ($\frac{1}{a}$ is the inverse of a if $a \neq 0$). Is it possible to obtain x^2 at some step?
- [4] 3. The midpoint of some side of a triangle and the bases of the altitudes drawn to two other sides form an equilateral triangle. Is it true that the original triangle is equilateral as well?
- [5] 4. A 29×29 table contains the integers $1, 2, 3, \dots, 29$, each of them 29 times. It turns out that the sum of the numbers above the principal diagonal is three times greater than the sum of the numbers below this diagonal. Determine the number in the central cell of the table.
- [5] 5. A magician is blindfolded and gives five cards with numbers from 1 to 5 to a spectator. The spectator hides two cards and returns the remaining three to the magician's assistant. The assistant chooses two of them, and then the spectator communicates the numbers chosen by the assistant to the magician (in any order he prefers). After that, the magician guesses the numbers of the cards hidden by the spectator. How can the magician and the assistant arrange to succeed in performing this trick?