MATH 62-498-15/523

Lie Algebras and Representation Theory Winter 2010

Instructor: Dr. W.L. Yee
Office: 9-111 Lambton Tower
E-mail: wlyee (at) uwindsor.ca

Lectures: TR 16:00-17:20, Dillon Hall 353
Office hours: MW 13:00-14:00, or by appointment

Text: *Introduction to Lie Algebras and Representation Theory* by J.E. Humphreys
Webpage: http://web2.uwindsor.ca/math/wlyee/LieAlgW10/index.html

Course content: Chapters I-VI of the textbook. From Wikipedia:
In mathematics, a **Lie algebra** (pronounced "lee", not "lye") is an algebraic structure whose main use is in studying geometric objects such as Lie groups and differentiable manifolds. Lie algebras were introduced to study the concept of infinitesimal transformations. The term "Lie algebra" (after Sophus Lie) was introduced by Hermann Weyl in the 1930s. In older texts, the name "infinitesimal group" is used.
Representation theory is a branch of mathematics that studies abstract algebraic structures by representing their elements as linear transformations of vector spaces. In essence, a representation makes an abstract algebraic object more concrete by describing its elements by matrices and the algebraic operations in terms of matrix addition and matrix multiplication. The algebraic objects amenable to such a description include groups, associative algebras and Lie algebras. The most prominent of these (and historically the first) is the representation theory of groups, in which elements of a group are represented by invertible matrices in such a way that the group operation is matrix multiplication. Representation theory is a powerful tool because it reduces problems in abstract algebra to problems in linear algebra, which is well understood. Furthermore, the vector space on which a group (for example) is represented can be infinite dimensional, and by allowing it to be, for instance, a Hilbert space, methods of analysis can be applied to the theory of groups. Representation theory is also important in physics because, for example, it describes how the symmetry group of a physical system affects the solutions of equations describing that system.
A striking feature of representation theory is its pervasiveness in mathematics. There are two sides to this. First, the applications of representation theory are diverse: in addition to its impact on algebra, representation theory illuminates and vastly generalizes Fourier analysis via harmonic analysis, is deeply connected to geometry via invariant theory and the Erlangen program, and has a profound impact in number theory via automorphic forms and the Langlands program. The second aspect is the diversity of approaches to representation theory. The same objects can be studied using methods from algebraic geometry, module theory, analytic number theory, differential geometry, operator theory and topology.

Evaluation: You may learn from one another and work on assignments together (the point of the assignments is to learn the material), but write them up on your own—do not copy someone else's assignment. **You may not consult with anybody but the instructor for the midterm and final exams.**

Assignments	Six, due throughout the term	40%
Midterm	Take home. Handout: February 25th. Handin: March 11th, in class.	20%
Final Exam	Take home. Handout: April 8th. Handin: my office, April 22nd 5pm.	40%

For each assignment and exam, students are asked to fill in and sign:
1. Statement of honesty for assignments
2. Confidentiality and statement of honesty for exams

The files are available: http://www.uwindsor.ca/academicintegrityoffice/templates

IF YOU ARE UNABLE TO WRITE THE MIDTERM OR FINAL EXAMINATION DUE TO ILLNESS, YOUR MAKEUP EXAMINATION WILL BE AN <u>ORAL</u> EXAM. WEIGHTS OF ASSIGNMENTS WILL NOT BE TRANSFERRED.

ANY ILLNESS MUST BE DOCUMENTED BY A PHYSICIAN USING THE UNIVERSITY'S MEDICAL FORM, WHICH IS AVAILABLE ON THE COURSE WEBPAGE.

Remarks:

1. Last day for late registration and course change: January 20th
2. Last day for reversal of incidental fees for course withdrawal: February 3rd
3. Last day for voluntary withdrawal from courses and partial tuition refund: March 17th
4. Students will be penalized for cheating. All cheating cases will be referred to the Dean's Office.
5. The Student Evaluation of Teaching will be administered during the last week of class.
6. Exam conflicts: Students scheduled to write three invigilated final examinations in one day and students who are unable to write a final examination due to a conflict with religious conviction may apply for an alternate examination date. Please download the appropriate form from http://www.uwindsor.ca/registrar and submit to the Office of the Registrar. Note: these applications must be submitted by the end of the fourth week of classes.
7. Assignments/examinations/quizzes in this course are protected by copyright. Reproduction or dissemination of examinations or the contents or format of examinations/quizzes in any manner whatsoever (e.g., sharing content with other students), without the express permission of the instructor, is strictly prohibited. Students who violate this rule or engage in any other form of academic dishonesty will be subject to disciplinary action under Senate Bylaw 31: Student Affairs and Integrity.
8. The raw final scores may be adjusted. The final letter grade from the final adjusted scores will be as follows:

Raw Score	Grade	Raw Score	Grade	Raw Score	Grade	Raw Score	Grade
93-100	A+	86-92.9	A	80-85.9	A-	77-79.9	B+
73-76.9	B	70-72.9	B-	67-69.9	C+	63.66.9	C
60-62.9	C-	57-59.9	D+	53-56.9	D	50-52.9	D-
35-49.9	F	0-34.9	F-				