Mathematics 498/523: Lie Algebras and Representation Theory

This is a very loose sketch of the topics which will be covered each lecture. The material will be stretched over twenty five classes to allow for some time dedicated to problem solving and discussion.

Lecture Schedule:

1. Introduce Lie algebras and their representations. In what mathematical contexts do they arise? Derivations.
2. Constructing some finite dimensional Lie algebras. Examples of Lie algebras.
3. Nilpotence and Engel's Theorem.
4. Solvable Lie algebras, Lie's Theorem, Jordan Decomposition
5. Cartan Criterion, Killing form, semisimplicity and simplicity
6. Complete reducibility of representations
7. Building representations: ideals, direct sums, duals, tensor products, quotients. Schur's Lemma.
8. Representation theory of $\mathfrak{sl}_2(\mathbb{F})$
9. Maximal toral subalgebras, their centralizers, roots
10. Root space decomposition: orthogonality and integrality properties
11. Root systems, Weyl group (chambers, base)
12. Simple roots, reduced expressions, fundamental weights
13. Cartan matrices, Dynkin diagrams, irreducible root systems
14. Universal enveloping algebra: Poincaré-Birkhoff-Witt Theorem, gradings
15. Cartan involution, Cartan subalgebras
16. Borel subalgebras
17. Representation theory of semisimple Lie algebras: weights, Verma modules, maximal vectors, finite-dimensional representations
18. Verma modules and realizations of finite-dimensional representations
19. Formal character, multiplicity formula, structure of Verma modules
20. infinitesimal characters and Harish-Chandra's Theorem
21. Weyl's formulas
22. Weyl's formulas cont'd