Mathematics 410/510: Real Analysis I

Instructor: Tim Traynor

Text: Tim Traynor, *Real Analysis, part I — Basic Measure and Integration.* This will be provided.

Office: 9117 Lambton Tower

Office hours: Yet to be determined

The course will follow a near minimal path through those aspects of the theory of Lebesgue measure and integral used in virtually all applications in analysis and probability theory, illustrating major techniques: area and volume, Lebesgue outer measure, the Caratheodory process, Borel sets, sigma-rings and the Unique Extension Theorem, measurable functions (= random variables), image measures (= distributions), Lebesgue integral, Monotone Convergence Theorem, Fatou's Lemma, Dominated Convergence Theorem, differentiation under the integral, change of variable, connection with the Riemann and Riemann-Stieltjes integrals, Radon-Nikodým theorem, Lebesgue differentiation theorem Fubini Theorem. We also define the classical $\mathcal{L}^p$ spaces, prove that they are complete seminormed spaces, and give the connections between various notions of convergence of sequences of functions (almost everywhere, almost uniform, in measure, and in $\mathcal{L}^p$).

Grades will be assigned subjectively, based on

class participation,

approximately weekly assignments,

a test on 1 November,

and a final examination Thursday Dec 13, 12:00 noon.

Graduate students will be expected to do a some more challenging problems and present some material.

There will be an opportunity to evaluate the instructor and the course in the last 2 weeks of classes

References

Warning. These references have different approaches and notation from mine and from each other. If you decide to use them, be sure you know what each thing means. When it comes to deducing results from the definitions, of course, the ones given in class will be the standard.

P.R. Halmos, *Measure Theory*, Springer (or Van Nostrand).

Bruckner, Bruckner, Thomson, *Real Analysis*, Prentice Hall ISBN 0-13-458886-x.

H.L. Royden, *Real Analysis, Third Edition*, Macmillan.
M.E. Monroe, *Measure and Integration*, Addison Wesley, 1971.
R.G. Bartle, *The Elements of Integration*, Wiley.
Billingsley, *Probability and measure*, Wiley.
R. Ash, *Real Analysis and Probability*, Academic Press.
Krishna B. Athreyna and Somendra N. Lahiri, *Measure Theory and Probability Theory*, Springer.
W. Rudin, *Principles of Mathematical Analysis*, McGraw-Hill.
W. Rudin, *Real and Complex Analysis*, McGraw-hill.

Halmos and Monroe are Measure Theory standards, many people cite the former,[1] though I think most prefer a simpler construction of the integral.

Rao includes more advanced and more modern topics. It is very expensive.

Bartle is an easily accessible, stripped presentation, unfortunately out of print.

Bruckner, Bruckner, and Thomson is now out of print, but available free on line.

Billingsley intertwines probability and measure, and includes lots of material.

Ash does a Real Analysis course, a little Functional Analysis and a strong graduate Probability course.

The first book by Rudin is really a compact 314/315 text, but has a nice little chapter on measure. The second is considerably more advanced.

[1]It also makes a dandy office decoration