A Queueing Model for Attention-Deficit / Hyperactivity Disorder

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ABSTRACT

A queueing model is presented to describe the treatment of Attention-Deficit / Hyperactivity Disorder (ADHD) by the stimulant Ritalin. The queueing model has two types of customers and uses a controller to affect traffic flow. When the controller works too slowly or the traffic is too heavy, there is an overflow. In human terms, if there are excessive distractions and/or a low ability to inhibit, then it is difficult for an ADHD individual to sustain concentration, to inhibit impulsivity, and to inhibit motoric restlessness.

Keywords: queueing, attention-deficit / hyperactivity disorder, hyperactivity, networks

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1. INTRODUCTION

Queueing models in medical, mental health, and clinical situations (other than patient flow in a hospital) are rare. Dansereau ([5]) looked at human attentional processes as a queueing network, but did not consider Attention-Deficit / Hyperactivity Disorder. A queueing model for drug dosages in a general setting (not specifically ADHD) was considered by Brill and Moon ([2]).

Attention-Deficit / Hyperactivity Disorder (ADHD) is defined according to the American Psychiatric Association Diagnostic Statistical Manual of Mental Disorders IV (DSM IV, 1994), ([1]) . According to this definition, individuals are diagnosed as having Attention-Deficit / Hyperactivity Disorder only if they have high levels of at least six of nine specified inattention symptoms, and/or at least six of nine specified hyperactivity/impulsivity symptoms. In addition to satisfying the symptoms already mentioned, the age of onset should be before seven years, the chronicity should be at least six months, and the symptoms must be present in more than one setting. Evidence of other disorders are to be ruled out for an ADHD diagnosis. According to the American Psychiatric Association web site www.psych.org, DSM V is not expected until 2010.

Attention Deficit / Hyperactivity Disorder is a common disorder among individuals in today's society. According to the DSM-IV ([1]), between three and five percent of school age children have the disorder. Phelan ([7]) claims that twenty million American children and adults, have the disorder.

Treatments may involve behavior modification and/or medication. One of the more popular treatment methods for ADHD, involves the use of the medication methylphenidate (better known by the brand name Ritalin). It is the medication of choice to which a majority of ADHD individuals will respond. Ritalin is a stimulant. It is curious that individuals who have hyperactive and impulsive symptoms would take a stimulant to slow them down. (The fact that a stimulant can result in certain characteristics of a system becoming slower is incorporated into the mathematical model to be presented in Section 2.)

Children and adolescents who exhibit ADHD are likely to have features of impulsivity, short attention span, and hyperactivity. Those individuals who have mostly features of attentional difficulties are said to have ADHD, Predominantly Inattentive Type, and may respond best to lower dosages of Ritalin. Those individuals who have mostly features of hyperactivity/impulsivity are said to be ADHD, Predominantly Hyperactive-Impulsive Type. If both features of inattention and hyperactivity-impulsivity are noted, then the individual is said to have ADHD, Combined Type.

Individuals who take ADHD medication likely will have tried dosages of Ritalin ranging from 5 mg to 60 mg daily. Ritalin is usually the first choice of medication for ADHD if the individual is at least six years of age. The stimulant Ritalin acts to increase the level of the neurotransmitter dopamine in the synapse between the brain cells of the frontal lobe. It does this by increasing the release of dopamine from the neurons (p.578, [4]). The neurotransmitter is released from the presynaptic axon side, diffuses across the synapse, and produces excitation in the postsynaptic dendrite side. Thus the individual is able to inhibit distracting noises, impulsivity and overactivity. One way of describing the reaction to Ritalin is through the use of a queueing model. Queueing theory is the study of waiting lines. It is used extensively in studies of traffic flow, in communications (especially telephone connections), and in studies of information flow in computer systems.

We model Attention-Deficit / Hyperactivity Disorder by creating a hypothetical controller for a queueing system. Taylor ([8], p.25) makes the statement ``The hyperactive child has a faulty gatekeeper." In this analogy, the gatekeeper can block undesirable individuals from entering. However when the gatekeeper is busy, any individual, desirable or not, can enter unchallenged. If we think of undesirable individuals as corresponding to unfocused thoughts, and desirable individuals as properly focused thoughts, then we can see the relationship between our queueing model and our ADHD model. Although this is a simplification of an extremely complex human condition, Koziol and Stout ([6], p.48) "provide evidence that ADHD is an executive function disorder, operationally measured using frontal lobe evaluation." The frontal lobe plays the role of the controller in our model.

Comings ([4], p.395) states that "children with ADHD appear to have a deficiency of prefrontal lobe dopamine. As a result the subcortical structures such as the striatum and limbic system are disinhibited and these children present with hyperactivity and irritable behaviour."

This paper has several goals. The primary goal is to show how ADHD and Ritalin medication can be explained as a queueing system. This may help to explain ADHD more clearly to individuals who have an analytic background but who have little understanding of the treatment of clinical disorders.

A second goal is to suggest models that may be useful, if studied in more detail, for determining modifications of dosage levels. Too high a dosage of Ritalin may result in lethargy, while too low a dosage may result in no improvement. The shape of the response function derived can tell us much about the expected effects of medication. Several different dosages are often needed to find the correct dosage level for an individual. Of course, every individual has his or her own set of parameter values. Knowledge of the shape of the response function may help to find the correct dosage level faster than would be possible without such knowledge.

A final goal is allow the mathematical formulation to help focus attention on the factors that control an individual's behavior. Thus a number of possible solutions to the disorder can be suggested.

Section 2 of this paper is presented from the viewpoint of a probabilist rather than a psychologist. The primary task is the construction of a queueing model for ADHD and the determination of its properties.

2. THE QUEUEING MODEL

2.1: Model Assumptions and Derivation of the Response Function

Assume that we have a queueing network with flow $\rightarrow$[controller]$\rightarrow$[completion area] .

Assume there are two types of customer arriving to the system. Call these Type 1 and Type 2. Type 1 customers correspond to properly focused thoughts for an ADHD individual. Type 2 customers correspond to unfocused or improperly focused thoughts. Assume that the arrival rate of Type 1 customers is 8_1 and the arrival rate of Type 2 customers is 8_2. Assume exponential interarrival times. Assume that there is a controller with room for exactly one customer. If a customer arrives while the controller is busy, the arriving customer bypasses the controller and immediately enters the "completion" area behind the controller. If the controller is not busy, then an arriving customer must be serviced by the controller before proceeding to the completion area. Assume the controller has exponential service times with rate :. After a Type 2 customer is processed by the controller, it is discarded from the system. After a Type 1 customer is processed by the controller, it moves to the completion area.

We would like to know the proportion of Type 1 and Type 2 customers in the completion area under various values of 8_1, 8_2 , and :. Since Type 1 customers are properly focused thoughts, we would like to have a large proportion of Type 1 customers. The value of : corresponds to the level of neurotransmitters in the frontal lobe.

Our state space consists of the type of customer being served by the controller. Define the states 0,1,2 as:

0 means there is no customer being server by the controller.
1 means that there is a Type 1 customer being served by the controller.
2 means that there is a Type 2 customer being served by the controller.

Theorem 1: Let the limiting probabilities of states 0,1,2 be q_0, q_1, and q_2. The limiting probabilities are:

$$q_0 = \frac{\mu}{\mu + \lambda_1 + \lambda_2},\ q_1 = \frac{\lambda_1}{\mu + \lambda_1 + \lambda_2},$$
$$q_2 = \frac{\lambda_2}{\mu + \lambda_1 + \lambda_2} \qquad (1)$$

Proof : The balance equations are:

$$(\lambda_1 + \lambda_2) q_0 = \mu q_1 + \mu q_2,$$
$$\mu q_1 = \lambda_1 q_0,$$
$$\mu q_2 = \lambda_2 q_0. \qquad (2)$$

In matrix form, these equations can be written as $\underline{0} = \underline{q}\,\Lambda$ where $\underline{0}$ is a 3x1 column vector of zeros, $\underline{q}$ is a 3x1 column vector with entries q_0, q_1, q_2, and 7 is the infinitesimal rate matrix,

$$\Lambda = \begin{bmatrix} -(\lambda_1 + \lambda_2) & \lambda_1 & \lambda_2 \\ \mu & -\mu & 0 \\ \mu & 0 & -\mu \end{bmatrix}. \qquad (3)$$

The three equations in (2), together with the constraint $q_0 + q_1 + q_2 = 1$, are solved to get (1).

We are really interested in the proportions of the two types of customers in the completion area.

Theorem 2: The limiting proportion p_1 of Type 1 customers in the completion area is given by

$$p_1 = \frac{(\mu + \lambda_1 + \lambda_2)\lambda_1}{(\lambda_1 + \lambda_2)^2 + \mu\lambda_1}. \qquad (4)$$

The proportion p_2 of Type 2 customers exiting the system is given by $p_2 = 1 - p_1$.
Proof: Let T_i be the event that the next addition to the completion area of the queueing system is a Type i customer, i=1,2.

Let $P(T_1|0) = P_0(T_1)$ = probability of event T_1 given that the system is currently in state 0
Let $P(T_1|1) = P_1(T_1)$ = probability of event T_1 given that the system is currently in state 1.
Let $P(T_1|2) = P_2(T_1)$ = probability of event T_1 given that the system is currently in state 2.

Then

$$P_0(T_1) = \frac{\lambda_1}{\lambda_1 + \lambda_2} P_1(T_1) + \frac{\lambda_2}{\lambda_1 + \lambda_2} P_2(T_1),$$
$$P(T_1 | 1) = \frac{\mu + \lambda_1}{\mu + \lambda_1 + \lambda_2},$$
$$P_2(T_1) = \frac{\lambda_1}{\mu + \lambda_1 + \lambda_2} + \frac{\mu}{\mu + \lambda_1 + \lambda_2} P_0(T_1) \qquad (5)$$

Solving the three equations in (5) gives

$$P(T_1 | 0) = \frac{\lambda_1(\mu + \lambda_1 + \lambda_2)}{(\lambda_1 + \lambda_2)^2 + \mu\lambda_1},$$
$$P(T_1 | 1) = \frac{\mu + \lambda_1}{\mu + \lambda_1 + \lambda_2},$$
$$P(T_1 | 2) = \frac{A}{B} \qquad (6)$$

where $A = \lambda_1(\lambda_1 + \lambda_2)\lambda_1(\mu + \lambda_1 + \lambda_2) + \mu\lambda_1(\mu + \lambda_1)$

and $B = (\mu + \lambda_1 + \lambda_2)[\mu\lambda_1 + (\lambda_1 + \lambda_2)^2]$.

Thus, $p_1 = q_0 P(T_1 | 0) + q_1 P(T | 1) + q_2 P(T | 2).$
Using (1), and (6), we obtain (4).

Note: For our analysis, we have assumed that the interarrivals and service times are both exponentially distributed. In fact, our final result (Theorem 2) is true for general service time distributions with mean service time 1/:. This follows because all of the type 1 customers eventually enter the system, either immediately upon arriving or after a service period. Further, the only type 2 customers who enter the system are those who encounter a busy period. Thus our desired proportions depend only on the proportions of type 1 and 2 customers in the arrival stream and on the probability that the server is busy.

The assumption of exponential interarrival times is equivalent to assuming that the arrivals come from a Poisson process. This would true if the customers are arriving randomly and independent of each other. In terms of ADHD, if the interruptions are from random sources, then the assumptions are reasonable. If not, there is sufficient robustness in the model to give results that are good approximations to reality.

2.2: Graphical study of the model and ADHD interpretations

Recall that Type 1 customers correspond to properly focused thoughts of an ADHD individual. Type 2 customers correspond to unfocused or improperly focused thoughts of the individual. We have three independent variables of interest, namely 8_1, 8_2, and :, together with one dependent variable p_1. In order to reduce the number of variables that we have to consider, we let $x=8_1/:$ and $y=8_2/:$.

From (4), we obtain $p_1 = \dfrac{(1+x+y)x}{(x+y)^2+x}$.

Thus we have reduced our expression for p_1 from an expression in three variables (8_1, 8_2, :) to an expression in two variables (x, y). This will be useful for simplifying our graphical analysis.
The variables x and y give the relative proportion of arriving focused and unfocused thoughts relative to the service rate of the controller.

Our goal is to have a high value of p_1, indicating a high proportion of properly focused thoughts (or desirable customers) in the completion area stream. This corresponds to a high level of concentration ability. As we increase the service rate : of the controller, perhaps by using drugs such as Ritalin, the values of $x=8_1/:$ and $y=8_2/:$ decrease.

We now consider contour plots of the function p_1 in terms of x and y. We look at three plots - one for values of the independent variables x and y between 1 and 10, the second for values of the independent variables between .1 and 1, and the third for values of the independent variables between .01 and .1.

The following MAPLE program will generate 4 plots which we denote Diagram 1, Diagram 2, Diagram 3, Diagram 4.

```
with(plots);
contourplot((1+x+y)*x/(x+(x+y)^2),x=1..10,y=1..10,
 contours=[.9,.8,.7,.6,.5,.4,.3,.2]);
contourplot((1+x+y)*x/(x+(x+y)^2),x=.1..1,y=.1..1,
 contours=[.9,.8,.7,.6,.5,.4,.3,.2]);
contourplot((1+x+y)*x/(x+(x+y)^2),x=.01..0.1,
 y=.01..0.1,grid=[50,50],
 contours=[.9,.8,.7,.6,.5,.4,.3,.2]);
plot([(1+1.1*x)*x/(x+(1.1*x)^2),(1+1.5*x)*x/(x+(1.5*x)
^2), (1+2*x)*x/(x+(2*x)^2),(1+3*x)*x/(x+(3*x)^2)],
 x=0..4,p1=0..1);
```

In Diagram 1, x,y both lie between 1 and 10. The contours correspond to values of p_1=.2, .3, .4, .5, .6, .7, .8, .9, where p=.9 is the lowest curve. The contours look very close to straight lines. The section of the region {(x,y):1<x< 10, 1 < y < 10} with values of p_1 greater than or equal to .9 consists of the lower right hand corner. The range of values implies that the controller is often busy and that most arrivals proceed directly to the completion area, bypassing the controller. Thus the proportion of Type I customers in the completion area is not very different from the proportion in the input.

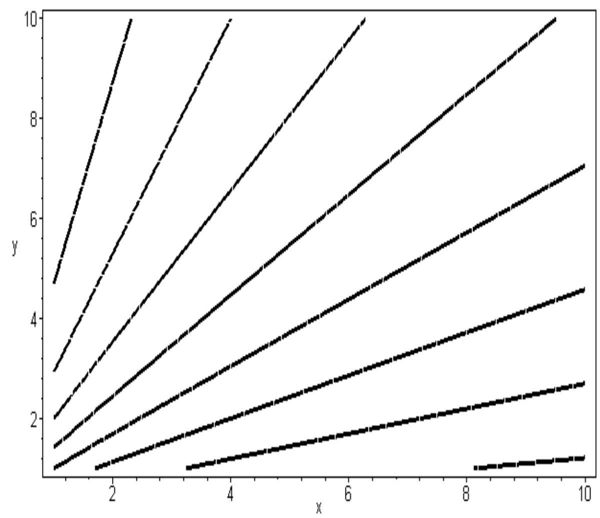

Diagram 1: y vs x (1<x<10, 1<y<10)

We wish to have high values of p_1. The lower right hand corner corresponds to high values of x and low values of y. These occur when 8_1 is large and 8_2 is small. Thus we wish to increase the properly focused signals and decrease the distractions. For a classroom setting, we could increase the value of 8_1 by having more teacher student interaction, by having more interesting multimedia presentations, by having lower student-teacher ratios, or by having more "hands-on" activities. We could decrease the value of 8_2 by having windowless classrooms, stricter enforcement of "no talking rules" or by shutting classroom doors to external noise.

In Diagram 2, x,y both lie between .1 and 1.0. The portion of the region {(x,y): 0.1< x < 1, 0.1 < y < 1} for which p_1 is greater than or equal to 0.9 again consists of the lower right hand corner. The contours correspond to values of p_1=.2,.3,.4,.5,.6,.7,.8,.9. The proportion is now slightly larger than in Diagram 1. In fact, near x=0.1, y=0.1, the value of p_1 is clearly higher than 0.5, which would be the result if there were no controller. The regions that appear in Diagram 2 would be more likely to appear than those of Diagram 1 if we increase the value of :, perhaps by giving a small dosage of Ritalin.

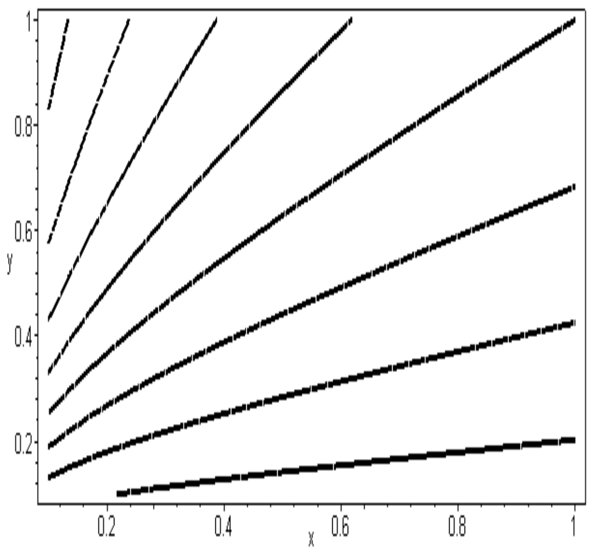

Diagram 2: y vs x (.1<x<1.0, .1<y<1.0)

Next we consider Diagram 3. In Diagram 3, x,y both lie between .01 and .1, The contours correspond to values of p1=.6,.7,.8,.9, with p_1=.9 being the lowest curve. Here the effect of the controller is very evident and the vast majority of the region {(x,y): .01< x < 0.1, .01 < y < .1 } has p_1 greater than or equal to 0.8. Diagram 3 is the diagram that would be expected of an ADHD individual who is taking a larger dosage of Ritalin.

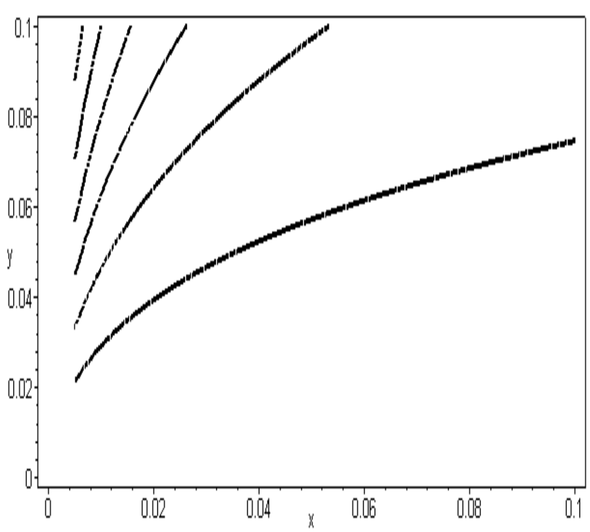

Diagram 3: y vs x (.01<x<0.1, .01<y<0.1)

Finally, in Diagram 4, we look at plots of p_1 versus $x=8_1/:$ for 4 different levels of $8_1/8_2$, namely.1, .5, 1, 2. We choose 0 < x < 4. The upper curve corresponds to $8_2/8_1$ =.1 and the lower curve to $8_2/8_1$ =2. We would like to see high values of p_1, which correspond to a high percentage of ``correctly focused" thoughts (or high work efficiency). We see from the plot that we obtain high values of p_1 when $8_2/8_1$ is small (for example, when $8_2/8_1$ =.1), or when $x=8_1/:$ is small (for fixed $8_2/8_1$).

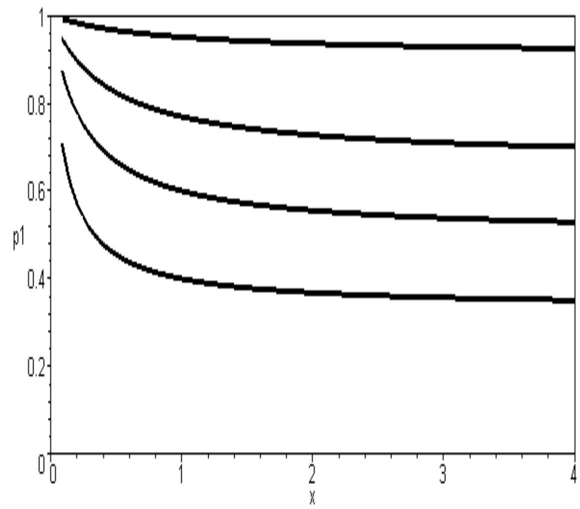

Diagram 4: p1 vs x

We can make $8_2/8_1$ small by lowering 8_2 (i.e. by eliminating distractions). We can also make $8_2/8_1$ small by increasing 8_1. Multimedia presentations, or ``hands-on" activities which hold one's attention, would be examples of increased 8_1. However, we have already mentioned the desirability of large 8_1 and small 8_2 in connection with Diagrams 1, 2 and 3. We can make $x=8_1/:$ small by increasing : through the use of medication like Ritalin. It is possible that : can be affected by exercise, particular foods, or sufficient sleep. We can also make $x=8_1/:$ small by lowering 8_1, while keeping $8_2/8_1$ fixed. This means the arrival rate of correct focused information and the arrival rate of all information are BOTH kept low. In practice, this would translate into giving only a few tasks to an ADHD individual at a time rather than all together.

3. CONCLUSIONS

In this paper, we have attempted to apply queueing theory to Attention Deficit / Hyperactivity Disorder to help explain the effects of a stimulant such as Ritalin. An understanding of the mathematical effects could potentially help to predict effects and to set dosage levels. If there existed a test which could measure the attention level relative to some expected optimal level (depending on the individual), for each dosage level of Ritalin, then knowing the convexity or concavity of the response curve could help to find an improved dosage level. It should be emphasized that individuals are different and have their own parameter values in the model presented. In practice, individuals receive several exploratory medication dosages in an attempt to find the smallest dosage which is effective.

There are medications used in the treatment of ADHD other than the stimulant Ritalin. Other stimulants are Dexedrine and Cylert. Anti-depressants are also used. However, this paper is restricted to a queueing model for stimulant medications for ADHD. Our graphical analysis has helped to focus on the various factors that affect ADHD behaviour. This focus helps to suggest what kind of solutions might be effective in reducing existing problems.

4. REFERENCES

[1] American Psychiatric Association Diagnostic and Statistical Manual of Mental Disorders (Fourth Edition) DSM-IV. 1994. American Psychiatric Association. Washington, D.C.

[2] P.H. Brill and R.E. Moon, "Application of Queueing Theory to Pharmacokinetics", Journal of Pharmaceutical Sciences , V. 69, 1980, pp. 558-560.

[3] CHADD website: http://www.chadd.org/

[4] D.E. Comings, Tourette Syndrome and Human Behaviour, Duarte, California: Hope Press, 1990.

[5] M. Dansereau, M. A Queueing Network Modeling Approach to the Analysis of Human Attentional Processes, Ph.D. Dissertation, Industrial Engineering, University of Toronto, 1983.

[6] L.F. Koziol and C.E. Stout, "Conceptualization of A.D.H.D. as an Executive Function Disorder",
in L.F. Koziol, C.E. Stout, and D.H. Ruben (Editors), Handbook of Childhood Impulse Disorders and A..D.H.D: Theory and Practice. Springfield, Illinois: Charles Thomas Publisher, 1993.

[7] T.W. Phelan, All About Attention Deficit Disorder: Part I, Video cassette, Glen Ellyn, Illinois: Child Management, 1990.

[8] J.F. Taylor, The Hyperactive Child and the Family, New York: Everest House Publishers, 1980.