Squares and Beyond
October 21st

(1) Find all integer solutions to $x^2 + xy + y^2 = x^2y^2$.
(2) Find all triples of positive integers (x, y, z) such that

$$x^2 + y^2 + z^2 + 2xy + 2xz + 2yz = -2x + 2y.$$

(3) Find all positive integer solutions to $2^x + 1 = y^2$.
(4) Find all integer solutions to $(x + 2)^4 - x^4 = y^3$.
(5) Find all integer solutions to $(2^x)^{2^x} - 1 = y^{z+1}$.