Sums

November 4^{th}

(1) Compute $\sum_{k=1}^{n} k!k$.

(2) Evaluate
$$\sum_{k=1}^{\infty} \frac{6^k}{(3^k - 2^k)(3^{k+1} - 2^{k+1})}.$$

(3) Compute
$$\sum_{k=1}^{n} k!(k^2 + k + 1).$$

(4) Let $a_1, a_2, \ldots, a_n$ be an arithmetic progression with common difference d. Compute
$$\sum_{k=1}^{n} \frac{1}{a_k a_{k+1}}.$$

(5) Prove that
$$\prod_{n=2}^{\infty} \frac{n^3 - 1}{n^3 + 1} = \frac{2}{3}.$$

(6) Sum the series:
$$\sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{m^2 n}{3^m (n3^m + m3^n)}.$$

(7) Compute
$$\tan 1 \tan 2 + \tan 2 \tan 3 + \cdots + \tan 2007 \tan 2008.$$

(8) Evaluate
$$\sum_{k=0}^{n} \tan^{-1} \frac{1}{k^2 + k + 1}.$$

(9) Compute
$$\frac{1}{\cos a - \cos 3a} + \frac{1}{\cos a - \cos 5a} + \cdots + \frac{1}{\cos a - \cos(2n+1)a}.$$

(10) Prove that:
$$\frac{1}{\cos 0^\circ \cos 1^\circ} + \frac{1}{\cos 1^\circ \cos 2^\circ} + \cdots + \frac{1}{\cos 88^\circ \cos 89^\circ} = \frac{\cos 1^\circ}{\sin^2 1^\circ}.$$

(11) Prove that for every positive integer n and for every real number $x \neq \frac{k\pi}{2^m}$ where k is an integer and $m = 0, 1, \ldots, n$,
$$\frac{1}{\sin 2x} + \frac{1}{\sin 4x} + \cdots + \frac{1}{\sin 2^n x} = \cot x - \cot 2^n x.$$

(12) Compute
$$\frac{\tan 1}{\cos 2} + \frac{\tan 2}{\cos 4} + \cdots + \frac{\tan 2^n}{\cos 2^{n+1}}.$$