DEPARTMENT OF MATHEMATICS AND STATISTICS
U. of Windsor Theory of Interest 62-392 Test 2 M. Hlynka
Monday, March 21, 2011

Directions: You must show your work in ALL questions !

1. The amount of principal repaid in the 3rd payment of a 5-year level payment loan at 9% is 300. What is the original loan value?
SOLUTION:
Let L be loan amount and K be annual payment.
Repayment amount in third year is $300 = Kv^3 = K(1.09)^{-3}$. Thus $K = 300(1.09)^3 = 388.51$. The loan amount is $L = Ka_{\overline{5}|} = 1511.16$.

2. An investor makes a payment of \$200 immediately and a payment of \$264 at the end of two years in exchange for a payment of \$460 at the end of one year.
(a) Find the equation of value for the interest rate i for time 0.
(b) Find i.
SOLUTION:
The equation of value at time 0 is $200 + 264v^2 = 460v$ so $200(1+i)^2 + 264 = 460(1+i)$.
This is a quadratic in i.
$200 + 400i + 200i^2 + 264 = 460 + 460i$.
so $4 - 60i + 200i^2 = 0$ so $1 - 15i + 50i^2 = 0$. We get $i = .1$ and $i = .2$. So the interest rate is either 10% or 20%. We cannot eliminate either answer. They are both possible.

3. (SOA Practice problem # 54 :modified). Matt purchased a 20-year par value bond with semiannual coupons at a nominal annual rate of 6% convertible semiannually at a price of 1722.25. The bond can be called at par value X on any coupon date starting at the end of year 15 after the coupon is paid. The price guarantees that Matt will receive a nominal annual rate of interest convertible semiannually of at least 8%. Calculate X.
SOLUTION: Since bond rate is less than yield, the bond was bought at a discount. Thus the issuer wishes the coupon payments to continue as long as possible. So we must assume that the bond will be redeemed in 20 years=40 periods. The coupon rate is $r = .03$ per half year and the yield rate is $j = .04$ per half year. Thus $P = Xv_j^{40} + Xra_{\overline{40}|}$ so
$1722.25 = X(1.04^{-40} + .03\dfrac{1-1.04^{-40}}{.04}) = X(.2083 + .03(19.7928)) = X(.8021)$ so
$X = 1722.25/.8021 = 2147.18$.

4. (SOA FM Sample questions #24) A 20-year loan of 20000 may be repaid under the following two methods: i) amortization method with equal annual payments at an annual effective rate of 6.5% ii) sinking fund method in which the lender receives an annual payment of 1600 and a lump sum of 20000 after 20 years accumulated through a sinking fund which earns an annual effective rate of j. Both methods require a payment of X to be made at the end of each year for 20 years. Set up an equation involving j (and not X) and check which of the following values of j is best.
(A) .118 (B) .130 (C) .142
SOLUTION:
$20000 = Xa_{\overline{20}|.065} = X(11.01851)$ so $X = 1815.13$. Then
$20000 = (X - 1600)s_{\overline{20}|j} = (\dfrac{20000}{11.01851} - 1600)s_{\overline{20}|j}$.
Check the 3 values of j. For $j = .142$, RHS comes closest to 20000.

5. Society of Actuaries Sample Examination 140-83-94, Problem No. 10.(Ostaszewski)
Donald takes a loan to be paid with annual payments of 500 at the end of each year for 2n years. The annual effective interest rate is 4.94%.
The sum of the interest paid in year 1 plus the interest paid in year $n+1$ is equal to 720. Calculate the amount of interest paid in year 1.
SOLUTION
Both n and L (loan size) are unknown.
$500(1 - v^{2n}) + 500(1 - v^n) = 720$ (check formulas in the amortization table).

So $-500v^{2n} - 500v^n + 280 = 0$. This is a quadratic in v^n so from the quadratic formula, $v^n = .4$ (the other root is negative).
We want the amount of interest paid in year 1, namely $500(1 - v^{2n}) = 500(1 - (.4)^2) = 500(.84) = 420$.

6. (Broverman exercise 4.1.4) A 6% bond maturing in 8 years with semiannual coupons to yield 5% convertible semiannually is to be replaced by a 5.5% bond yielding the same return. In how many years should the new bond mature? (Both bonds have the same price, yield rate and face amount.)
SOLUTION:
Let P =price of bond 1 (= price of bond 2). 8 years means 16 periods. Bond 1 gives $P = Fv^{16} + Fra_{\overline{16}|} = F(1.025)^{-16} + F(.03)a_{\overline{16}|.025}$.
For bond 2, let n be the number of years. There are $2n$ periods. Bond 2 gives
$P = F(1.025)^{-2n} + F(.0275)a_{\overline{2n}|.025}$. Equating the two expressions and cancelling F gives
$1.025^{-16} + .03(1 - 1.025^{-16})/.025 = 1.025^{-2n} + .0275(1 - 1.025^{-2n})/.025$. Thus
$1.025^{-2n}/10 = 1.1 - (1.025^{-16} + .03(1 - 1.025^{-16})/.025) = 0.03472499$. Thus
$-2n(ln(1.025)) = ln(.3472499)$ so
$2n = -.5 * ln(.3472499)/ln(1.025)$ and $n = 21.41755$ years. We round this to 21.5 years to get the final coupon.

7. (Ostaszewski Exercise 52; May 1998 Course 140 Examination, Problem No. 9)
On January 1, 1999, Luciano deposits 90 into an investment account. On April 1, 1999, when the amount in Luciano's account in equal to X, a withdrawal of W is made. No further deposits or withdrawals are made to Luciano's account for the remainder of the year. On December 31, 1999, the amount in Luciano's account is 85. The dollar-weighted return over the 1-year period is 20%. The time-weighted return over the 1-year period is 16%. Calculate X.
A. 101.1 B. 103.6 C. 105.6 D. 107.6 E. 109.6
SOLUTION:
$.20 = \dfrac{B - (A + \sum C_i)}{A(1) + \sum C_i(1 - t_i)} = \dfrac{85 - (90 - W)}{90 + (-W)(.75)} = \dfrac{W - 5}{90 - .75W}$ so $W = 20$.
$1.16 = (\dfrac{X}{90})(\dfrac{85}{X - W}) = \dfrac{85X}{90(X - 20)}$
so $90(1.16)X - 1.16(90)(20) = 85X$ so $X = 107.63$

8. A loan of X has an effective annual rate 8%. It is to be repaid by the end of 10 years. If the loan is repaid with a single payment at time $t = 10$, the total interest paid is \$468.05 more than if the loan were repaid by ten equal annual payments. Find the value of X.
SOLUTION: $X = Ka_{\overline{10}|.08}$
TotalInterest1= $X(1.08)^{10} - X = X(1.08^{10} - 1) = 1.158925X$.
TotalInterest2= $\dfrac{X}{a_{\overline{10}|}}(10 - a_{\overline{10}|}) = 0.4902949X$.
$468.05 = 1.158925X - 0.4902949X = 0.6686301X$ so $X = 700.01$.