Family Name ________________________________ Given Name __________________________
ID. No. __________________________

DEPARTMENT OF MATHEMATICS AND STATISTICS
Theory of Interest 62-392 Test 2 M. Hlynka
Tuesday, March 23, 2010. Time allowed: 75 minutes.

Directions: You must show your work! You will be graded on your completeness as well as your correctness. Choose the closest answer. Calculators encouraged.

1. (Chap 5, class notes)
An investor makes a payment of \$100 immediately and a payment of \$132 at the end of two years in exchange for a payment of \$230 at the end of one year. Find the yield rate $i$.
SOLUTION:
The equation of value (at the end of year 2) is

$$100(1+i)^2 + 132 = 230(1+i).$$

This is a quadratic in $i$ and if we solve it, we get $i = .1$ and $i = .2$. So the interest rate is either 10% or 20%. We cannot eliminate either answer. They are both possible.

2. (DickLondon1.4) A 1000 face value 20-year 8% bond with semi-annual coupons is purchased for 1014. The redemption value is 1000. The coupons are reinvested at a nominal annual rate 6%, compounded semi-annually. Determine the purchaser's annual effective yield over the 20 year period.
(A) 6.9% (B) 7.0% (C) 7.1% (D) 7.2% (E) 7.3%
SOLUTION:
$1014(1+j)^{20} = 1000 + .04 * 1000 * s_{\overline{40}|.03}$ so $j = (\dfrac{1000 + 40 s_{\overline{40}|.03}}{1014})^{1/20} - 1 = .071243$ Answer =C

3. (notes) Maxine invests 1000 at the end of each year for 5 years at an effective rate of .10 per year, with interest payable annually. She reinvests the interest each year at rate .08 per year. Find the accumulated value at the end of 5 years.
SOLUTION: The time diagram is

| Time | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| | \| | \| | \| | \| | \| | \| |
| Deposit | | 1000 | 1000 | 1000 | 1000 | 1000 |
| CumDeposit | | 1000 | 2000 | 3000 | 4000 | 5000 |
| Interest | | | 100 | 200 | 300 | 400 |

Hence the accumulated amount is

$$5000 + 100(Is)_{\overline{4}|.08} = 5000 + 100\frac{\ddot{s}_{\overline{4}|.08} - 4}{.08} = 5000 + 100\frac{4.866601 - 4}{.08} = 6083.25$$

4. (DickLondon1.9)A 9% bond with a 1000 par value and coupons payable semi-annually is redeemable at maturity for 1100. At a purchase price of P, the bond yields a nominal annual interest rate of 8%, compounded semiannually, and the present value of the redemption amount is 190. Determine P.
(A) 1050 (B) 1085 (C) 1120 (D) 1165 (E) 1215
SOLUTION:
There are $2n$ half years at coupon rate .045 per half year and yield rate .04 per half year.
$1100v^{2n} = 190$ so $v^{2n} = 190/1100$.
$P = Cv^{2n} + 1000(.045)a_{\overline{2n}|.04} = 190 + 45\dfrac{1 - v^{2n}}{.04} = 190 + 45 * \dfrac{1 - 190/1100}{.04} = 1120.682$ C

5. (DickLondon2.13) A 35 year loan is to be repaid in equal annual installments. The amount of interest paid in the 8th installment is 135. The amount of interest paid in the 22nd installment is 108. Calculate the

amount of interest in the 29th installment.
(A) 72 (B) 73 (C) 74 (D)75 (E) 76
SOLUTION: $108 = K(1 - v^{14})$
$135 = K(1 - v^{28}) = K(1 - v^{14})(1 + v^{14}) = 108(1 + v^{14})$ so $1 + v^{14} = 135/108 = 1.25$ so $v^{14} = .25$.
Thus $K = 108/(1 - v^{14}) = 108/(1 - .25) = 144$
Interest in 29th installment is $K(1 - v^7) = 144(1 - \sqrt{.25}) = 72$.

6. May.2001.31. You are given the following information about an investment account:

| $Date$ | $Value immediately$ $before deposit$ | $Deposit$ |
|---|---|---|
| $January 1$ | 10 | |
| $July 1$ | 12 | $X$ |
| $December 31$ | $X$ | |

Over the year, the time-weighted return is 0%, and the dollar-weighted return is Y . Calculate Y .
(A) -25% (B) -10% (C) 0% (D) 10% (E) 25%
SOLUTION: $10(1 + i_1) = 12$; $(12 + X)(1 + i_2) = X$. Thus:
$1 = (1 + 0) = (1 + i_1)(1 + i_2) = (\frac{12}{10})\frac{X}{12 + X}$ so $120 + 10X = 12X$ so $X = 60$.
$$Y = \frac{B - (A + \sum C)}{A + \sum C(1 - t)} = \frac{X - (10 + X)}{10 + X(1 - .5)} = \frac{-10}{10 + 30} = -.25$$

7. (Dick London 1.12) A 10 year adjustable rate mortgage loan of 23115 is being repaid exactly with quarterly installments of 1000 based on an initial interest rate of 12% compounded quarterly. Immediately after the 12th payment, the interest rate of 12% is increased to 14% compounded quarterly. The quarterly installments remain at 1000, so the term of the loan must be different. Calculate the loan balance immediately after the 24th payment.
(A) 12,000 (B) 12,550 (C) 12,950 (D)13,350 (E) 13,750
SOLUTION: Prospectively $L* = 1000a_{\overline{28}|.03} = 1000(18.76411) = 18764.11$. is OB of loan after 12 payments.
Loan balance after the 24th payment (extra 12 payments) is (retrospectively)
$18764.11(1 + .035)^{12} - 1000s_{\overline{12}|.035} = 13751.90$.

8. (May 2000 Actuarial Exam #10.)
A bank customer borrows X at an annual effective rate of 12.5% and makes level payments at the end of each year for n years.
(i) The interest portion of the final payment is 153.86.
(ii) The total principal repaid as of time (n-1) is 6009.12.
(iii) The principal repaid in the first payment is Y.
Calculate Y.
(A) 470 (B) 480 (C) 490 (D) 500 (E) 510
SOLUTION:
Let $X$ be amount of the loan, let $K$ be annual payment, $i = .125 = 1/8$, so $v = (1 + i)^{-1} = 8/9$. Let $I_j$ be interest in the $j$ payment, let $P_j$ be principal paid in the $j$th payment, $j = 1, \ldots, n$. Then $153.60 = I_n = K(1 - v) = K/9$ so $K = 153.86(9) = 1384.74$. Hence $P_n = K - I_n = 1384.74 - 153.86 = 1230.88$.
Hence $X = Loan Amount = \sum P_j = 6009.12 + 1230.88 = 7240$.
Thus $I_1 = Xi = 7240(.125) = 905$. so $Y = P_1 = K - I_1 = 1384.74 - 905 = 479.74$. B