Family Name _____ Given Name _____
ID. No. _____

DEPARTMENT OF MATHEMATICS AND STATISTICS
Theory of Interest 62-392 Test 2 M. Hlynka
Thursday, March 19, 2009. Time allowed:75 minutes.

Directions: You must show your work! You will be graded on your completeness as well as your correctness. Calculators encouraged. 6 points each.

1. (Chapter 4, Broverman 4.1.1) Find the price of a 12 year bond with $100 face and redemption, with nominal annual rate 5% and semiannual coupons, which will give a nominal annual yield rate of 7%.
1. SOLUTION
There are $n = 24$ periods of length 6 months. The redemption value is 100. The coupon rate is $r = .025$ and the yield per period is $i = .035$. Let $v = 1/1.035$.The price is

$$P = Cv^{24} + (Fr)a_{\overline{24}|.035} = 100(1.035)^{-24} + 100(.025)(\frac{1 - 1.035^{-24}}{.035}) = 43.7957 + 40.1459 = 83.94$$

2. (Chap 3, MH) Assume an annual interest rate of 8%. Consider a loan of 10000 to be repaid by level payments over 10 years. Write down the amortization table for years 1,2,3 showing for each year AmountPd, InterestPd, PrinciplePd OutstandingBalanceAtEndOfYear. Your answer should consist of numbers, not formulas.
SOLUTION:
Let K be level payment. $10000 = Ka_{\overline{10}|}$ so $K = 10000i/(1 - v^n) = 10000 * .08/(1 - 1.08^{-10}) = 1490.295$

Yr	$AmtPd$	$IntPd$	$PrincPd$	OB
0				10000
1	1490.295	$.08 * 10000 = 800$	$1490.295 - 800 = 690.30$	$10000 - 690.30 = 9309.71$
2	1490.295	$.08 * 9309.61 = 744.78$	$1490.295 - 744.78 = 745.52$	$9309.71 - 745.52 = 8564.19$
3	1490.295	$.08 * 8564.19 = 685.13$	$1490.295 - 685.13 = 805.16$	$8564.19 - 805.16 = 7759.03$

3. (Chapter 2, May.2003.15) John borrows 1000 for 10 years at an annual effective interest rate of 10%. He can repay this loan using the amortization method with payments of P at the end of each year. Instead, John repays the 1000 using a sinking fund that pays an annual effective rate of 14%. The deposits to the sinking fund are equal to P minus the interest on the loan and are made at the end of each year for 10 years. Determine the balance in the sinking fund immediately after repayment of the loan.
(A) 213 (B) 218 (C) 223 (D) 230 (E) 237
SOLUTION: $1000 = Pa_{\overline{10}|.10} = P\frac{1 - 1.1^{-10}}{.10}$ so $P = \frac{.10(1000)}{1 - 1.1^{-10}} = 162.7454$.
Amount in sinking fund after payment of loan is
$(P - 100)s_{\overline{10}|.14} - 1000 = 62.7454\frac{1.14^{10} - 1}{.14} - 1000 = 213.3263$. A.

4. (Chap 5, class notes)
An investor makes a payment of $100 immediately and a payment of $132 at the end of two years in exchange for a payment of $230 at the end of one year. Find the yield rate i.
SOLUTION:
The equation of value (at the end of year 2) is

$$100(1 + i)^2 + 132 = 230(1 + i).$$

This is a quadratic in i and if we solve it, we get $i = .1$ and $i = .2$. So the interest rate is either 10% or 20%. We cannot eliminate either answer. They are both possible.

5. (Chap 5, SOA Practice question #7) An association had a fund balance of 75 on January 1 and 60 on December 31. At the end of every month during the year, the association deposited 10 from membership

fees. There were withdrawals of 5 on February 28, 25 on June 30, 80 on October 15, and 35 on October 31. Calculate the dollar-weighted (money-weighted) rate of return for the year.
(A) 9.0% (B) 9.5% (C) 10.0% (D) 10.5% (E) 11.0%
SOLUTION:
$I = B - (A + \sum C_i) = 60 - (75 + 12(10) - 5 - 25 - 80 - 35) = 60 - (75 + 120 - 145) = 205 - 195 = 10.$

$$
\begin{aligned}
&Average Amount For Year \\
&\quad = A + 10(11/12 + 10/12 + 9/12 + 8/12 + 7/12 + 6/12 + 5/12 + 5/12 + 4/12 + 3/12 + 2/12 + 1/12) \\
&\quad - 5(10/12) - 25(6/12) - 80(2.5/12) - 35(2/12)) \\
&\quad = 75 + (10/12) * 11 * 12/2 - 50/12 - 150/12 - 200/12 - 70/12 \\
&\quad = 90.83333
\end{aligned}
$$

Thus $i_D = 10/90.83333 = .11009$. E

6. (Chap 3, May.2001.37. Seth borrows X for four years at an annual effective interest rate of 8%, to be repaid with equal payments at the end of each year. The outstanding loan balance at the end of the second year is 1076.82 and at the end of the third year is 559.12. Calculate the principle repaid in the first payment.
(A) 444 (B) 454 (C) 464 (D) 474 (E) 484
SOLUTION: Let the level payment be K.
$OB_3 = 559.12 = Ka_{\overline{1}|} = Kv$
$PrincRepaid_1 = Kv^4 = (Kv)v^3 = 559.12(1.08)^{-3} = 443.85$ A

7. (Chap 3, from May 2000 Actuarial Exam #10.)
A bank customer borrows X at an annual effective rate of 12.5% and makes level payments at the end of each year for n years.
(i) The interest portion of the final payment is 153.86.
(ii) The total principal repaid as of time (n-1) is 6009.12.
(iii) The principal repaid in the first payment is Y.
Calculate Y.
(A) 470 (B) 480 (C) 490 (D) 500 (E) 510
SOLUTION:
Let K be annual payment. Here $v = (1+i)^{-1} = 8/9$. Then $153.60 = I_n = K(1-v) = K/9$ so $K = 153.86(9)$. Next $PR_n = K - I_n = 9(153.86) - 153.86 = 8(153.86)$. Then $X = \sum_{i=1}^{n} PR_i = PR_n + 6009.12 = 8(153.86) + 6009.12$
Thus $Y = K - Xi = 9(153.86) - [8(153.86) + 6009.12(9)](1/8) = 479.6235$. B

8. (Chap 4, SOA Practice question #55) Mary purchased a 10-year \$1100 par value bond with semiannual coupons at a nominal annual rate of 4% convertible semiannually at a price of 1022.50. The bond can be called at par value on any coupon date starting at the end of year 5. What is the minimum yield that Mary could receive, expressed as a nominal annual rate of interest convertible semiannually? (To keep things fair, the easiest way to find the rate is to check both answers A), (B) in the appropriate equation in order to see what rate Mary earns.)
(A) 4.8% (B) 4.9%
SOLUTION: If we take $C = 1100$ then the bond is purchased at a discount relative to C so coupons are relatively low and the issuer wants to keep the bond active as long as possible. To get Mary's minimum yield, take $n = 2(10) = 20$. $P = Cv^{20} + Fra_{\overline{20}|i}$ or

$$1022.50 = 1100(1+i)^{-20} + (1100)(.02)(1 - (1+i)^{-20})/i \qquad (*)$$

As the rate i is nominal, we need to look at cases $i = .024$, $i = .0245$.
Case 1: $i = .024$ The RHS of the above equation are 1030.76
Case 2: $i = .0245$ The RHS of the above equation is $1022.47 = LHS$.
So the answer is B.