Dr. M. Hlynka. Math 62-392.01 Test 2. NAME ________________________________
Theory of Interest. April 2, 2007. 75 minutes.
Calculators allowed. ID ________________________________
Show all work. Justify all answers. All questions are worth 5 points.

1. (text 4.1.1) Find the price of a 12 year bond with \$100 face and redemption, with nominal annual rate 5% and semiannual coupons, which will give a nominal annual yield rate of 7%.
1. SOLUTION
There are $n = 24$ periods of length 6 months. The redemption value is 100. The coupon rate is $r = .025$ and the yield per period is $i = .035$. Let $v = 1/1.035$.The price is

$$P = Cv^{24} + (Fr)a_{\overline{24}|.035} = 100(1.035)^{-24} + 100(.025)(\frac{1 - 1.035^{-24}}{.035}) = 43.7957 + 40.1459 = 83.94$$

2. (MH) Write down the amortization table for a loan 5000 of which will be paid off in 3 years with 3 level payments at 5% annual interest.
2. SOLUTION:

$$5000 = Ka_{\overline{3}|.05} = K(v + v^2 + v^3) = K(2.723248) \text{ so } K = 5000/2.723248 = 1836.043$$

Yr	$Pymt$	Int	$PrincRepayd$	OB
0				5000
1	1836.04	250	1586.04	3413.96
2	1836.04	170.70	1665.34	1748.62
3	1836.04	87.43	1748.61	$0(APPROX)$

3. (Chap 3, modified from a Gorvett problem) A 20-year, \$100,000 loan has an effective annual interest rate of 8%. The interest is paid each year and a 6% sinking fund is used to accumulate the amount of the loan beginning with the first payment into the sinking fund at time 1. What is the amount accumulated in the sinking fund at time 6?
3. SOLUTION: The sinking fund accumulates to 100000 after 20 years at rate .06. Thus $Ks_{\overline{20}|.06} = 100000$ so $K = 100000/36.78559 = 2718.456$. The accumulated amount at time 6 is $2718.456s_{\overline{6}|.06} = 18962.09$.

4. (text 2.4.11) A purchaser pays 245000 for a mine which will be exhausted at the end of 18 years. What level annual revenue (received at the end of each year) is required in order for the purchaser to receive a 5% annual return on his investment if he can recover his principal in a sinking fund earning 3.5% per year?
4. SOLUTION:
Let K be the annual amount placed in the sinking fund. Then $Ks_{\overline{18}|.035} = 245000$ so $K = 245000/25 = 10000$. The 5% annual income requires .05(245000)=12250 each year. Total needed is 10000+12250=22250.

5. (Chap 4, from Nov. 2000 Actuarial Exam#30)
A 1000 par value 20-year bond with annual coupons and redeemable at maturity at 1050 is purchased for P to yield an annual effective rate of 8.25% . The first coupon is 75 . Each subsequent coupon is 3% greater than the preceding coupon. Determine P.
(A) 985 (B) 1000 (C) 1050 (D) 1075 (E) 1115
5. SOLUTION:

$$P = 75v + 75(1.03)v^2 + \cdots + 75v^{20}(1.03)^{19} + 1050v^{20} = \frac{75v(1 - (1.03v)^{20})}{1 - 1.03v} + 1050v^{20} = 1115. \text{ E}$$

6. (MH,similar to 3.1.6) A loan of L is paid off completely by 20 quarterly payments. The annual nominal interest rate is 9%. The payments are made at the end of each quarter and the jth payment is equal to $1000(.98)^j$ for $j = 1, \ldots, 20$. What is the outstanding balance

immediately after the 16th payment?
6. SOLUTION: Use prospective method. $v = (1 + .09/4)^{-1} = 1.225^{-1}$.

$$\begin{aligned} OB &= 1000(.98)^{17}v + 1000(.98)^{18}v^2 + 1000(.98)^{19}v^3 + 1000(.98)^{20}v^4 \\ &= 1000(.98)^{17}v(1 + .98v + (.98v)^2 + (.98v)^3) = 1000(.98)^{17}v\frac{1-(.98v)^4}{1-.98v} \\ &= 2606.59. \end{aligned}$$

7. (text 2.3.3) Mike buys a perpetuity immediate with varying annual payments. During the first 5 years, the payment is constant and equal to 10. Beginning in a year 6, the payments start to increase. For year 6 and all future years, the current years payment is K% larger than the previous years payment. At an effective annual interest rate of 9.2%, the perpetuity has a present value of 167.50. Calculate K, given $K < 9.2$.
7. SOLUTION:

$$\begin{aligned} 167.50 &= 10(v + v^2 + v^3 + v^4 + v^5) + 10(1 + K/100)v^6 + 10(1 + K/100)^2v^7 + \ldots \\ &= 10a_{\overline{5}|.092} + 10(1 + K/100)v^6/(1 - v(1 + K/100)) \\ &= 38.6995 + 5.8974(1 + .01K)/(.08425 - .0091575K) \end{aligned}$$

Solve for K to get $((167.50 - 38.6995)/5.8974)(.08425 - .0091575K) = (1 + .01K)$
or $1.840038 - 0.20000K = 1 + .01K$ so $.21K = .840038$ and $K = 4$.

8. (Chap 3, specimen exam 30-3-72#34) We have a 3% annual effective rate on a loan which is being repaid over 17 years by monthly installments of 75 each one at the beginning of each month. The total principal repaid in the twelve installments of the eighth year is 400. What is the total interest paid in the twelve installments of the tenth year? (Answer to the nearest 25).
(A)400 (B)425 (C)450 (D)475 (E) 500
8. SOLUTION: There are 12*17=204 months.
Let j be the monthly interest rate. Let $v = 1/(1 + j)$. Then $(1 + j)^{12} = 1.03$. The payments are at the beginning of each month which confuses things. However the principal repaid each month is a power of v times the monthly level payment. The powers are decreasing so if the principal repaid in year 8 is 400, then the principal paid in year 10 is a lower power of v times 400, namely $PR(year10) = PR(year8)v^{-24} = 400(1 + j)^24 = 400(1.03)^2 = 424.36$. Thus $Interest(year10) = Payment(Year10) - PR(year10) = 75(12) - 424.36 = 475.64$.

9. (Chap 2/3, 30-4-72 exam,#7) A 1000 loan is to be repaid by a series of increasing payments, one at the end of each year, the first (following due one year hence) being \$10, the second \$20, the third \$30, and so on, until only a balance payment is due which is less than its position in the series would otherwise prescribe. If the loan interest rate is 5% per annum, at the end of what year will this balanced payment be made? use the multiple cjhoice answers to help you find the correct answer.
(A) 17th year (B) 18th year (C) 19th year (D) 20th year (E) 21st year
9. SOLUTION:
We want the smallest n such that $1000 \leq 10v + 20v^2 + 30v^3 + \cdots + 10nv^n = 10(Ia)_{\overline{n}|}$.
Try $n = 17$. Then $10(Ia)_{\overline{17}|.05} = 10\frac{\ddot{a}_{\overline{n}|} - nv^n}{i} = 884.1451742$ so $10(Ia)_{\overline{18}|} = 884.145 + 10 * 18 * (1.05)^{-18} =$ 958.94 so $10(Ia)_{\overline{19}|} = 958.94 + +10 * 19 * (1.05)^{-19} = 1034.13$.
Thus $n = 19$.

10. (Chap 3, from May 2000 Actuarial Exam #10.)
A bank customer borrows X at an annual effective rate of 12.5% and makes level payments

at the end of each year for n years.
(i) The interest portion of the final payment is 153.86.
(ii) The total principal repaid as of time (n-1) is 6009.12.
(iii) The principal repaid in the first payment is Y.
Calculate Y.
(A) 470 (B) 480 (C) 490 (D) 500 (E) 510
10. SOLUTION:
Let K be annual payment. $v = (1+i)^{-1} = 8/9$. Then $153.60 = K(1-v) = K/9$ so $K = 153.86(9)$.
$6009.12 = K(v^n + \cdots + v^2) = Kv^2(1-v^{n-1})/(1-v) = (64K/9)(1-v^{n-1}) = 153.86(64)(1-v^{n-1})$
Thus $v^{n-1} = 1 - 6009.12/(153.86(64)) = .3897659$
$Y = K(v^n) = K(vv^{n-1}) = K.3897659v = 153.86(9).3897659(8)/9 = 153.86 * .389659 * 8 = 479.6235$. B