Family Name ______________________ Given Name ______________________
ID. No. ______________________

DEPARTMENT OF MATHEMATICS AND STATISTICS
Theory of Interest 62-392 Test 1 M. Hlynka
Tuesday, February 10, 2009. Time allowed:75 minutes. Calculators encouraged.

Directions: You will be graded on your completeness as well as your correctness. You must show your work!

1. (MH) How long does it take for a fund to grow to two/three/four times its original value if growing at an annual rate of 7%?
(a) 7 (b) 10 (c) 16 (d) 20 (e) 28
SOLUTION:
$1.07^n = 2$ so $n\ln(1.07) = ln(2)$ so $n = \ln(2)/\ln(1.07) = 10.24477$ years.
$1.07^n = 3$ so $n\ln(1.07) = ln(3)$ so $n = \ln(3)/\ln(1.07) = 16.23757$ years.
$1.07^n = 4$ so $n\ln(1.07) = ln(4)$ so $n = \ln(4)/\ln(1.07) = 20.48954$ years.

2. (Exercise 1.7.6, 4th ed. Broverman, MH notes)
The nation of F has a unit of currency called the F. In the coming year in F, inflation is expected to be huge, 100%. Canada's expected inflation rate for the same year is 14%. An investor in Canada can make an interest rate of 18%. What must be the interest rate in F to be equivalent to the rate in Canada.
SOLUTION:
We want the real rates in both countries to be the same.
For Canada, $i = .18, r = .14$. Thus,

$$i_{real} = \frac{i-r}{1+r} = \frac{.18-.14}{1+.14}$$

For F, we want the same real interest rate and $r = 1.0$. We need to find i for F. Thus

$$\frac{.18-.14}{1+.14} = \frac{i-r}{1+r} = \frac{i-1}{1+1}$$

Solving for i gives $i = 1.070175 = 107.0175\%$.

3. (May, 2003 #8) Kathryn deposits 100 into an account at the beginning of each 4-year period for 40 years. The account credits interest at an annual effective interest rate of i. The accumulated amount in the account at the end of 40 years is X, which is 5 times the accumulated amount in the account at the end of 20 years. Calculate X.
(A) 4695 (B) 5070 (C) 5445 (D) 5820 (E) 6195
SOLUTION: SOLUTION:
$X = 100[(1+i)^{40} + (1+i)^{36} + \cdots + (1+i)^4] = \dfrac{100 * (1+i)^4 * (1-(1+i)^{40}))}{1-(1+i)^4}$.

$Y = A(20) = 100[(1+i)^{20}+(1+i)^{16}+\cdots+(1+i)^4] = \dfrac{100*(1+i)^4*(1-(1+i)^{20})}{1-(1+i)^4}$.
Using $X = 5Y$ and using a difference of squares on the left side gives
$1+(1+i)^{20} = 5$ so $(1+i)^{20} = 4$ so $(1+i)^4 = 4^{.2} = 1.319508$.
Hence $X = \dfrac{100*(1+i)^4*(1-(1+i)^{40}))}{1-(1+i)^4} = \frac{100*1.3195*(1-4^2)}{1-1.3195} = 6194.84$
Answer is E.

4. (May 2003, #50) Jeff deposits 10 into a fund today and 20 fifteen years later. Interest is credited at a nominal discount rate of d compounded quarterly for the first 10 years, and at a nominal interest rate of 6% compounded semiannually thereafter. The accumulated balance in the fund at the end of 30 years is 100. Calculate d.
(A) 4.33% (B) 4.43% (C) 4.53% (D) 4.63% (E) 4.73%
SOLUTION:

$$\frac{10}{(1-d/4)^{40}}(1+\frac{.06}{2})^{40} + 20(1+\frac{.06}{2})^{30} = 100$$

Solve for d to get $d = 0.04531789$ or 4.53%.

5. (Michigan 1.1) On July 1, 1999, a person invested 1000 in a fund for which the force of interest at time t is given by $\delta_t = \dfrac{3+2t}{50}$, where t is the number of years since January 1, 1999. Determine the accumulated value of the investment on January 1, 2000.
(A) 1036 (B) 1041 (C) 1045 (D) 1046 (F) 1051
SOLUTION:

$$AccAmt = 1000*exp(\int_{.5}^{1}\frac{3+2t}{50}) = 1000*exp(\frac{3t+t^2}{50}|_{.5}^{1})$$
$$= 1000*exp((4-7/4)/50) = 1046.028.$$

6. (May, 2005 #13) At a nominal interest rate of i convertible semi-annually, an investment of 1000 immediately and 1500 at the end of the first year will accumulate to 2600 at the end of the second year. Calculate i.
(A) 2.75% (B) 2.77% (C) 2.79% (D) 2.81% (E) 2.83%
SOLUTION: Let j =semi annual interest.
$2600 = 1000(1+j)^4 + 1500(1+j)^2$ This is a quadratic in $x = (1+j)^2$, which simplifies to $10x^2+15x-26 = 0$ so $x = \dfrac{-15 \pm \sqrt{15^2-4(10)(-26)}}{2(10)} = 1.028342$
Thus $(1+j)^2 = 1.028342$ so $j = 1.028342^{.5} - 1 = 0.01407199 = \dfrac{i^{(2)}}{2}$.
Finally $i^{(2)} = 2(.01407199) = .02814 = 2.81\%$.

7. (Michigan 1.3) Deposits are made to a fund each January 1 and July 1 for the years 1995 through 2005. The deposit made each July 1 will be 10.25% greater than the one made on the immediately preceding January 1. The deposit made on each January 1 (except for January 1, 1995) will be the same amount as the deposit made on the immediately preceding July 1. The fund will be credited with interest at a nominal rate of 10%, compounded semiannually. On December 31, 2005, the fund will have a balance of 11000. Determine the initial deposit to the fund.
(A) 160 (b) 165 (C) 175 (D) (E)195 (F) 200
SOLUTION:
We work with rate $i = .05$ for 6 months. Let X be the initial deposit.
The Jan 1, 2005 deposit is $X(1.1025)^{10}$. Note that $1.05^2 = 1.1025$.
The Jan 1 deposits accumulate, measuring from 2005 down, to

$$A1 = X(1.1025)^{10}(1.05)^2 + \cdots + X(1.1025)^0(1.05)^{22} = 11X(1.1025)^{11}$$

The July 1 deposits accumulate, measuring from 2005 down, to

$$A2 = X(1.1025)^{11}(1.05)^1 + X(1.1025)^{10}(1.05)^3 + \cdots + X(1.1025)^1(1.05)^{21} = 11X(1.1025)^{11.5}$$

Thus $11000 = 11X(1.1025^{11} + 1.1025^{11.5})$ so $X = 1000/(1.1025^{11} + 1.1025^{11.5}) = 166.7560$.

8. (May,2003, #22) A perpetuity costs 77.1 and makes annual payments at the end of the year. The perpetuity pays 1 at the end of year 2, 2 at the end of year 3,..., n at the end of year (n+1). After year (n+1), the payments remain constant at n. The annual effective interest rate is 10.5%. Calculate n.
(A) 17 (B) 18 (C) 19 (D) 20 (E) 21
SOLUTION:

$$\begin{aligned}
PV = 77.1 &= 1v^2 + 2v^3 + \ldots nv^{n+1} + nv^{n+2} + \ldots \\
&= v(v + 2v^2 + \cdots + nv^n) + nv^{n+2}(1 + v + v^2 + \ldots) \\
&= v(Ia)_{\overline{n}|i} + nv^{n+2}\frac{1}{1-v} = v\frac{\ddot{a}_{\overline{n}|} - nv^n}{i} + nv^{n+2}\frac{1+i}{i} = \frac{a_{\overline{n}|} - nv^{n+1}}{i} + \frac{nv^{n+1}}{i} \\
&= a_{\overline{n}|}/i = \frac{1-v^n}{i^2}
\end{aligned}$$

Thus $v^n = 1 - 77.1i^2$ so $n = ln(1 - 77.1i^2)/ln(v) = 19.002$

9. (May, 2003, #45) A perpetuity-immediate pays 100 per year. Immediately after the fifth payment, the perpetuity is exchanged for a 25-year annuity-immediate that will pay X at the end of the first year. Each subsequent annual payment will be 8% greater than the preceding payment. Immediately after the 10th payment of the 25-year annuity, the annuity will be exchanged for a perpetuity-immediate paying Y per year. The annual effective rate of interest is 8%. Calculate Y.

(A) 110 (B) 120 (C) 130 (D) 140 (E) 150

SOLUTION: At time 5, the value of the perpetuity and annuity must be equal so

$$100a_{\overline{\infty}|} = Xv + X(1.08)v^2 + \cdots + X(1.08)^{24}v^{25} \text{ or}$$

$$100/i = 25Xv.$$

At time 15, the remaining value of the annuity must equal the value of the perpetuity of Y. Thus

$$X(1.08)^{10}v + X(1.08)^{11}v^2 + \cdots + X(1.08)^{24}v^{15} = Ya_{\overline{\infty}|}$$

$$15Xv(1.08)^{10} = Y/i.$$

Thus $Y = \dfrac{15}{25}(1.08)^{10} = 129.5355.$