Dr. M. Hlynka. Math 62-392.01 Test 1. February 19, 2007. 75 minutes.
Calculators allowed. Theory of Interest
Show all work. Justify all answers for questions 1-7. All questions are worth 5 points. For questions 8,9, only the correct letter answer is required. On these last 2 questions, no work need be shown and no work will be graded. A guess is REQUIRED if you cannot deduce the answer.

1. A loan of $30,000 is to be repaid by a level annuity payable monthly at the end of each month for 25 years, and calculated on the basis of an nominal interest rate of 12% per year, compounded monthly. Calculate the monthly repayments.

2. Interest rate is i per annum. Begin with $100 in a bank account. The money accumulates for two years and then $50 is withdrawn. How many more years are needed for the amount in the account to accumulate to $200? (The answer should be a function of i.)

3. What is the nominal discount rate convertible 3 times per year which is equivalent to a nominal interest rate of 6% convertible 6 times per year?

4. Suppose you deposit $100 in a bank account at nominal annual rate of interest 6%, convertible quarterly, and six months later deposit $200. How big will your account be two years after your second deposit?

5. A perpetuity paying 1 at the end of each 6 month period has a present value of 20. A second perpetuity pays X at the end of every 2 years. Assuming the same effective interest rate, the present value of the two perpetuties are equal. Find X.

6. An investment of 1000 accumulates to 1200 at the end of 4 years. If the force of interest is 1.25δ during the first two years and δ during the next two years, find the equivalent effective annual interest rate i for the first year.

7. If $s_{\overline{n}|} = 60$, $s_{\overline{2n}|} = 240$, find $s_{\overline{3n}|}$.

8. What is the value of

$$(1 + \frac{i^{(m)}}{m})(1 - d)^{1/m}?$$

(A) $(1+i)^{2/m}$ (B) $\frac{i^{(m)}d^{(m)}}{m}$ (C) $1 - \frac{i^{(m)}d}{m}$ (D) $(1 - \frac{d^{(m)}}{m})^2$ (E) 1 (F) 0

9. Given $a_{\overline{n+1}|} = 6.25$, find $a_{\overline{n}|}$, assuming an effective annual rate of interest 12.36%.
(A)6.02 (B) 6.07 (C) 6.12 (D) 6.65 (E) 5.63 (F) 5.90

1. SOLUTION: (question modified from Oxford web site) Effective rate per month is $.12/12 = .01$. There are 25x12=300 months. Then

$$30000 = Ka_{\overline{300}|.01} = K\frac{1-v^{300}}{.01} = K\frac{1-1.01^{-300}}{.01} = K(94.94655).$$

Thus $K = \frac{30000}{94.94655} = 315.97$.

2. SOLUTION: (question modified from old actuarial practice problems)
We want $(100(1+i)^2 - 50)(1+i)^n = 200$. Thus $(1+i)^n = \frac{200}{(100(1+i)^2-50)}$ so
$n = \dfrac{ln(200) - ln(100(1+i)^2 - 50)}{ln(1+i)}$.

3. SOLUTION: (question from old actuarial practice problems)
$(1+.06/6)^6 = (1-d^{(3)}/3)^{-3}$ so $d^{(3)} = 3(1-1.01^{-2}) = .0591$

4. SOLUTION: (question modified from Queen's website)
We are given $i^{(4)} = .06$, so $1+i = (1+\frac{i^{(4)}}{4})^4 = 1.06136$.
The first deposit occurs at time 0, the second at time 0.5. We need to calculate the amount accumulated at time 2.5. The first deposit accumulates to $100(1+i)^{2.5} = 116.054$, the second deposit to $200(1+i)^2 = 225.2985$. The sum is 341.35.

5. SOLUTION:(question modified from 2.2.3)
Let i be the interest rate for a 6 month period. Then

$$\begin{aligned} 20 = a_{\overline{\infty}|} &= 1/i \\ &= Xv^4 + Xv^8 + \cdots = \frac{Xv^4}{1-v^4}. \end{aligned}$$

Thus $i = 1/20 = .05$ so $v^4 = 1/(1.05)^4$.
Thus $X = 20(\dfrac{1-v^4}{v^4}) = 20(\dfrac{1}{v^4} - 1) = 20(1.05^4 - 1) = 4.31$.

6. SOLUTION: (question modified from 1.5.5)

$$1200 = 1000exp(\int_0^4 \delta_r dr) = 1000exp(\int_0^2 1.25\delta dt + \int_2^4 \delta dt) = 1000exp(2.5\delta + 2\delta) = 1000exp(4.5\delta)$$

For the first year $1+i = e^{\int_0^1 1.25\delta} = (1200/1000)^{1.25/4.5}$ so $i = 1.2^{1.25/4.5} - 1 = 0.05195$.

7. SOLUTION: (question from Exercise 2.1.8, modified)
$s_{\overline{2n}|} = s_{\overline{n}|} + (1+i)^n s_{\overline{n}|}$ so $240 = 60 + 60(1+i)^n$.
Thus $(1+i)^n = 3$. Hence

$$s_{\overline{3n}|} = s_{\overline{n}|} + (1+i)^n s_{\overline{2n}|} = 60 + 3(240) = 780.$$

8. SOLUTION: (from old actuarial practice problems) Let A be the answer. Then

$$A^m = (1+\frac{i^{(m)}}{m})^m(1-d) = (1+i)(1-d) = (1+i)v = 1.$$

So A=1. (E)

9. SOLUTION:(from old actuarial practice problems)
$a_{\overline{n+1}|} = v + v^2 + \cdots + v^{n+1} = v + v(1+\cdots+v^n) = v + va_{\overline{n}|}$
Now $v = 1/(1+i) = 1/1.1236$, so $a_{\overline{n}|} = \frac{a_{\overline{n+1}|}-v}{v} = \frac{a_{\overline{n+1}|}}{v} - 1 = (1+i)a_{\overline{n+1}|} - 1 = 1.1236(6.25) - 1 = 6.0225$.
SOL2: $6.25 = \frac{1-(1+.1236)^{-(n+1)}}{.1236}$ so $-(n+1) = \frac{ln(1-6.25(.1236))}{ln(1.1236)} = -12.705$. Then
$a_{\overline{n}|} = \frac{1-1.1236^{-n}}{.1236} = \frac{1-1.1236^{-11.705}}{.1236} = 6.0225$.