Dr. M. Hlynka. Math 62-392.01 Test 1. February 14, 2006. 75 minutes.
Calculators allowed. Theory of Interest
Show all work. Justify all answers (1-8). All questions are worth 5 points. For questions 9,10,11, only the correct letter answer is required. A guess is required if you cannot deduce the answer.

1. A fund earns interest at a rate of 5% per annum effective. A person makes three deposits of \$100 each to the fund, the first one three years from now, the second four years from now and the final one seven years form now. How much will be in the fund ten years from now?

2. Find the nominal rate of interest compounded monthly, equivalent to an effective rate of 42.95% per annum.

3. Find the present value of \$100 payable at the beginning of each three month period for the next five years, when interest is at the rate of 6% per annum, compounded quarterly.

4. A man deposited \$1000 into a savings account seven years ago. The account earns interest at 4% per annum effective. Today he withdraws half of the money in the accounts. In how many years from now will the remaining funds first exceed \$1000 again?

5. For a certain fund the force of interest δ_t is a linear function of t such that $\delta_0 = 0$ and $\delta_1 = ln(1.05)$. Three deposits of \$1000 were are made on Jan 1, 2001 (time 0), Jan 1, 2003 and Jan 1, 2005. What is the total accumulated value on Jan.1, 2006.

6. A business has been showing a net profit of \$10000 per annum (available at the end of the year) for many years. The owner feels that the business will continue to provide the same net profit forever. He offers the business for sale for the present value of its future earnings at an interest rate of 8% per annum effective. How much is the owner asking as a sale price for the business?

7. Kathryn deposits 100 into an account at the beginning of each 4-year period for 40 years. The account credits interest at an annual effective interest rate of i. The accumulated amount in the account at the end of 40 years is X, which is 5 times the accumulated amount in the account at the end of 20 years. Calculate X.

8. For a given positive integer n, a rate of interest i can be found such that $4s_{\overline{2n}|} = 9s_{\overline{n}|}$. Express in terms of n how long it will take for money to double at this rate of interest.

For the last three questions, 9, 10, 11, ONLY the answer is needed. You do not need to show any work for these questions (and any work shown will not be graded.)

9. A bank offers the following choices for certificates of deposit:

Term(in years)	Nominal annual interest rate convertible quarterly
1	4.00%
3	5.00%
5	5.65%

The certificates mature at the end of the term. The bank does NOT permit early withdrawals. During the next 6 years the bank will continue to offer certificates of deposit with the same terms and interest rates. An investor initially deposits 10,000 in the bank and withdraws both principal and interest at the end of 6 years. Calculate the maximum annual effective rate of interest the investor can earn over the 6-year period.
(A) 5.09% (B) 5.22% (C) 5.35% (D) 5.48% (E) 5.61%

10. The parents of three children, ages 1, 3, and 6, wish to set up a trust fund that will pay X to each child three years from now, and Y to each child upon attainment of age 21. They will establish the trust fund with a single investment of Z. Which of the following is the correct equation of value for Z?
(A) $\frac{X}{v^{17}+v^{15}+v^{12}}+\frac{Y}{v^{20}+v^{18}+v^{15}}$ (B) $3(Xv^{18}+Yv^{21})$ (C) $3Xv^3+Y(v^{20}+v^{18}+v^{15})$
(D) $(X+Y)\frac{v^{20}+v^{18}+v^{15}}{v^3}$ (E) $X[v^{17}+v^{15}+v^{12}]+Y[v^{20}+v^{18}+v^{15}]$

11. An investor accumulates a fund by making payments at the beginning of each month for 6 years. Her monthly payment is 50 for the first 2 years, 100 for the next 2 years, and 150 for the last 2 years. At the end of the 7th year the fund is worth 10000. The annual effective interest rate is i, and the monthly effective interest rate is j. Which of the following formulas represents the equation of value for this fund accumulation?
(A) $\ddot{s}_{\overline{24}|i}(1+i)[(1+i)^4+2(1+i)^2+3]=200$ (B) $\ddot{s}_{\overline{24}|i}(1+j)[(1+j)^4+2(1+j)^2+3]=200$
(C) $\ddot{s}_{\overline{24}|j}(1+i)[(1+i)^4+2(1+i)^2+3]=200$ (D) $s_{\overline{24}|j}(1+i)[(1+i)^4+2(1+i)^2+3]=200$
(E) $s_{\overline{24}|i}(1+j)[(1+j)^4+2(1+j)^2+3]=200$

SOLUTIONS:
1. $100(1+.05)^7+100(1+.05)^6+100(1.05)^3=115.76+134.01+140.71=390.48$

2. $(1+i)=(1+i^{(12)}/12)^{12}$ so $i^{(12)}=12[(1+i)^{1/12}-1]=12[(1.4295)^{1/12}-1]=.3627$

3. 5 years=20 quarters. Interest rate is .06/4=.15 per quater.
$PV=100\ddot{a}_{\overline{20}|.015}=100(1.015)a_{\overline{20}|.015}=100(1.015)\frac{1-1.015^{-20}}{.015}=1742.62$

4. $1000=(1/2)1000(1.04)^7(1.04)^n$ so $2=(1.04)^{7+n}$ so $(7+n)ln(1.04)=ln2$ so $n=-7+ln2/ln(1.04)=$ 10.673 years.

5. Since δ_t passes throught the origin, $\delta_t=kt$ where k is the slope. But $ln(1.05)=\delta_1=k1=k$ so $\delta_t=ln(1.05)t$.

$$\begin{aligned}Accum.Value &= 1000e^{\int_0^5\delta_t dt}+1000e^{\int_2^5\delta_t dt}+1000e^{\int_4^5\delta_t dt}\\ &= 1000e^{(ln(1.05))(25/2-0)}+1000e^{(ln(1.05))(25/2-4/2)}+1000e^{(ln(1.05))(25/2-16/2)}\\ &= 1000(1.05^{12.5}+1.05^{10.5}+1.05^{4.5})=1000(1.840205+1.669120+1.245523)=4754.85\end{aligned}$$

6. $10000a_{\overline{\infty}|.08}=10000\frac{1}{i}=10000\frac{1}{.08}=10000(12.5)=125000.$

7. There are 10 payments at times 0,4,8,...,36. Let j be the interest rate for a 4 year period and let i be the annual rate. Then $(1+j)=(1+i)^4$. Thus $100\ddot{s}_{\overline{10}|j}=5(100)\ddot{s}_{\overline{5}|j}$. So
$100\frac{(1+j)^{10}-1}{j}(1+j)=5(100)\frac{(1+j)^5-1}{j}(1+j)$. Simplifying, using
$(1+j)^{10}-1=((1+j)^5-1)((1+j)^5+1)$ gives $((1+j)^5+1)=5$ so $(1+j)^5=4$ so
$X=100\frac{(1+j)^{10}-1}{j}(1+j)=100\frac{4^2-1}{4^{1/5}-1}4^{1/5}=6194.72.$

8. We need t such that $(1+i)^t=2$ so $t=\ln 2/(\ln(1+i))$. But $4s_{\overline{2n}|}=9s_{\overline{n}|}$ so $4\frac{(1+i)^{2n}-1}{i}=9\frac{(1+i)^n-1}{i}$ so $4\frac{((1+i)^n-1)((1+i)^n+1)}{i}=9\frac{(1+i)^n-1}{i}$ so $(1+i)^n+1=9/4$ so $(1+i)^n=5/4=1.25$.
Thus $n\ln(1+i)=\ln 1.25$.
Finally $t=\ln 2/(\ln(1+i))=n\ln 2/\ln 1.25=3.106n$.

9. There are essentially four possible ways to invest for 6 years. (6=1+1+1+1+1+1=3+1+1+1=3+3=5+1). Just looking at the interest rates tells us that the best investment is 5+1 years. Then
$(1+i)^6 = (1+0565/4)^{20}(1+.04/4)^4$ so $i = .0548$

10. Child 1 needs $Xv^3 + Yv^{21-1}$.
Child 2 needs $Xv^3 + Yv^{21-3}$.
Child 3 needs $Xv^3 + Yv^{21-6}$ So total amount needed is $3Xv^3 + Y(v^{20} + v^{18} + v^{15})$.

11. Note that 6 years=6×12 months = 72 months; 4 years = 48 months; 2 years = 24 months.
After 6 years, the first two years of payments have grown to $50\ddot{s}_{\overline{24}|j}(1+i)^4$.
After 6 years, the next two years of payments have grown to $100\ddot{s}_{\overline{24}|j}(1+i)^2$.
After 6 years, the last two years of payments have grown to $150\ddot{s}_{\overline{24}|j}(1+i)^0$.
Let T be the total after 6 years. Then the total after 7 years is $T(1+i)$. Thus

$$10000 = (1+i)[50\ddot{s}_{\overline{24}|j}(1+i)^4 + 100\ddot{s}_{\overline{24}|j}(1+i)^2 + 150\ddot{s}_{\overline{24}|j}(1+i)^0].$$

Divide by 50 and factor to get

$$200 = \ddot{s}_{\overline{24}|j}(1+i)[(1+i)^4 + 2(1+i)^2 + 3].$$