INFOR, Vol. 46, No. 4, November 2008, pp. 29–46
ISSN 0315-5986 | EISSN 1916-0615

Examples of Applications of Queueing Theory in Canada

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Abstract—As part of the 50th anniversary of the Canadian Operational Research Society, we reviewed queueing applications by Canadian researchers and practitioners. We concentrated on finding real applications, but also considered theoretical contributions to applied areas that have been developed by the authors based on real applications. There were a surprising number of applications, many not well documented. Thus, this paper features examples of queueing theory applications over a spectrum of areas, years and types. One conclusion is that some of the successful queueing applications were achieved and ameliorated by using simple principles gained from studying queues and not by complex mathematical models.

Keywords Queueing theory, applications, history.

1. INTRODUCTION

This paper is a collection of examples of applications of queueing analysis in Canada or applications by Canadian researchers and practitioners, showing their contributions since the 1950's. This paper is dedicated to the 50th anniversary of CORS (Canadian Operational Research Society)[1].

We concentrated on finding applications to real queueing situations as much as possible, but we also considered theoretical contributions to applied areas that have been discovered by authors based on real applications. Hence, we were interested, to a lesser extent, in theoretical work directly motivated by real applications and done by someone closely involved, with the intent of helping the client organizations. Given that the intent was to obtain concrete and practical results, the work may not have resulted in a publication, but rather led to implementation or an internal report. Moreover, these applications often do not involve academic (or even consulting) queueing theorists. Thus, a survey of this nature can never be complete since many of the applications are silent; there may be no articles in the open literature and often there is no report or documentation, and memories of what happened long ago have faded. But our lack of completeness is also explained by the time constraints of this survey.

Queueing applications are abundant in Canada. The diverse areas where queueing analysis has been applied include: shipping and canals, grocery store checkout line count estimation, vehicle traffic flow, airline operations, airport terminal planning, forest fire management, medical wait times for surgery, patient scheduling, hospital service management, hospital emergency room management, ambulance management, banking, bus, truck, railroad and train operations, production and manufacturing, border crossings and customs, mail services, telecommunications and computer design, and call centre operations and staffing.

There is no good published description of the role of Canadians in queueing applications or theory. There are a number of papers on the birth of operational research in Canada, of which queueing was a fundamental part, and from them there is a hint at the role of Canadians in queueing research and application (Sandiford, 1963). Also, there are histories of queueing theory that include a little on Canadian researchers' contribution to theory. See the queueing history link on Hlynka's Queueing Theory Page (2008).

In order to prepare the current article and given the issues of incompleteness, lack of publications and fading memories, and knowing that we could not be all inclusive, our methodology was as follows. We first issued a call for information by sending an e-mail to all current CORS members through the

Received 00 March 2008; Revision 00 September 2008, Accepted 00 November 2008

[1] See the website http://www.cors.ca/

CORS list server and to a list of Canadian queueing researchers culled from Hlynka's Queueing Theory Page[2]. These attempts resulted in a very low response rate and minimal success. Using our knowledge and experience of people who worked in the field, we telephoned many practitioners directly, as well as contacted companies or organizations in areas where we thought there might be applications. Many of the people we contacted gave us information about others (living or dead, regardless of academic background) whose work included applied queueing. The interview approach with individuals, using the snowball technique, resulted in information on many of the applications discussed in this paper. We relied on the integrity of researchers and practitioners in telling us about these applications, because of the lack of documentation or references. We also had to make choices as to what was included and what we thought met the spirit of our intent. For example, what is meant by a "Canadian" application is somewhat vague. However everything referred to in this article has some Canadian connection and we used our discretion on an individual basis as to what was included. Hence, this is a very personal and unscientific study of queueing applications from our memory and the memory of those interviewed and is not meant to be exhaustive. Accordingly, we consider this paper as a collection of examples. We apologize in advance for any exclusions, omissions, errors, misrepresentations, or misinterpretations.

The many applications we consider in this paper can be categorized in different ways. A first category includes cases when knowledge of queueing methods was actually applied to a real problem and affected the choice of policies used. Traffic signal changes made by people with an understanding of queueing or using software with a queueing theoretical basis would be an example. Simulation techniques could be used in such situations as well.

A second category actually applies queueing techniques successfully to model a real system. This includes data collection and simulation techniques. The modelling does not necessarily result in changes to the system but rather increases one's understanding and has the potential to lead to a change in policy with improved performance. A study of ambulance services that results in no change would be an example.

A third category is the use of common sense and includes such areas as hospital emergency room triage nurses. These nurses certainly apply their expertise to assign priorities to patients and to provide the best service in the shortest time. But their applications of queueing could be labeled as applying sound intuition to a difficult situation.

A fourth category includes real situations that allow for a queueing model, but a model with an abstraction level that might be of limited value to the practitioner. An example is any theoretical queueing paper with a very real underlying model but which has not used data to select parameters and which is highly likely not to be used, and may be incapable of influencing policies. Such studies do increase understanding, in a way similar to the second category. An example is a traffic system in which we consider the effects of a behaviour change by vehicles users. If the behaviour change is not likely to happen and if the parameters of the model are theoretical rather than based on data, we have a category four example.

The presentation of the review below is ordered by application area.

2. TRANSPORTATION

In this section we discuss queueing applications in the area of transportation. This includes transportation on water, land and in the air. Transportation research played an important role in the early days of the application of operational research and this is reflected in the queueing applications we have selected.

2.1 The Welland Canal

One of the more successful projects that involved queues in Canada was a study[3] done on the Welland Canal for the St. Lawrence Seaway Authority by KCS, a firm established by a remarkable entrepreneur, Joe Kates, whose firm was involved in pivotal operational research work in the 1960's and 1970's. (See section 2.4.2 for more on Joe Kates.) The Seaway, made up of a succession of locks and canals, was built so that both large inland vessels and "Salties" (ocean going vessels) could go all the way up the St. Lawrence river and Lake Ontario to the Upper Great Lakes, including Chicago, Thunder Bay, Duluth and other important ports. For general descriptions of the seaway, see The St. Lawrence Seaway Management Corporation (2008a,b). This allowed a new influx of shipping traffic to go through the Welland Canal. Canada faced a major problem when this new traffic and a rise in grain exports clogged the Welland Canal in the early 1960's, not long after the Seaway, as we know it today, had been built. The number of ships wanting to get through the locks approached the system's ultimate capacity and when it crossed the limit due to fluctuations in demand, queues started to build up. It appeared that the system was unable to get rid of the queues without denying access to some of the ships. As well, the shipping companies were troubled by the waiting times and the great capital and operating expense of the ships caused by the delays. Canada's foreign trade was also seriously threatened by the delays and actual limitations of export capacity.

Given the importance to Canada's economy of shipping on the Great Lakes, government authorities wanted to expand the locks by *twinning* (creating parallel locks) but the cost amounted to billions of dollars. Moreover, the twinning project involved much work and would have taken a long time. In 1962–63,

[2] This website contains information on queueing theory collected by Myron Hlynka of the University of Windsor. http://web2.uwindsor.ca/math/hlynka/queue.html.

[3] Most of this story comes from interviews and correspondence with Frank Collins, Lee Sims and Andrew Elek, former KCS personnel.

ISSN 0315-5986 | EISSN 1916-0615
DOI 10.3138/infor.46.4.029

the company KCS was called in and given carte blanche to figure out what to do to get more ships through the canal, avoiding the huge expenditure. By this time much engineering of the twinning project had already been done and no one gave much hope to the success of the KCS contract. Joe Kates sold the government on the idea that by slightly increasing the capacity of the locks it would be possible to eliminate the queues. His reasoning was based on the nonlinearity of queueing performance to changes in capacity. A slight increase in capacity can make a drastic reduction in queue size and waiting times or delays.

We can see Joe's idea by looking at the waiting time for a simple M/M/1 queue:

$$E[W_s] = \frac{1}{\mu - \lambda}.$$

If the arrival rate approaches the service rate (i.e. the system is near capacity), then the wait is very large. But if we increase the capacity by increasing the service rate by a small percentage, the average wait falls substantially more in percentage terms. For example, if the service rate is 1.1 and the arrival rate is one, then the expected wait is 10; however, if we raise the service rate to 1.2 (which is less than a 10% increase), the expected waiting time falls to five, a 50% drop.

Actually, the locks would operate more like an M/D/1 queue. As a simple hypothetical but telling illustration, Joe explained it in the following way: If the canal could handle 15 down bound ships a day and 16 wanted to pass, in 270 days (by December) there would be 270 ships in the queue, with a waiting time of 17 days. If the capacity were to be increased by just 10 percent, there would be little or no queue. This is a case in which the rate of arrival exceeds the service capacity. Joe promised that he could increase capacity enough to lower wait times to acceptable levels.

Around 1965, he put a team of consultants on the job. Some of those participating in the work, besides Joe Kates, were Frank Collins, Lee Sims, Arthur Mittermaier and Peter Sandor, all professionals[4] with industrial engineering and operational research backgrounds, and Peter was also an economist. The study cost about 10 million dollars, including all the experiments, and actually saved the Seaway Authority several billion dollars. Joe was a very enthusiastic thinker and came up with many ideas on how to carry out the study and to make the system better. Several operational improvements were suggested, concerning vessel tie-ups and manoeuvres, meant to make the service more efficient, some of which were implemented rather promptly.

Some operational measures to increase the capacity suggested by KCS were:

- *Empty (turn back) lockages*. When the traffic was imbalanced (primarily at the beginning of the season) KCS suggested that some of the lockages should be "empty". This meant that after a ship had been dropped via a down-bound lockage, the lock was immediately filled with water again, without waiting for the immediate availability of an up-bound vessel (i.e. from a small up-bound queue). This speeded up service since an empty lockage could actually be accomplished in a few minutes instead of the long time it would have taken to move a ship up as part of a complete cycle. Previously, this was only done occasionally in obvious circumstances. Now, with a simple calculation, it could be determined how many empty lockages the system could afford, given the asymmetry of the demand. New traffic control strategies led to much more use of this procedure to maximize overall traffic flow.
- *Multiple small ships in one lockage.* Doubling and tripling up (or even more) of small ships in one lockage was suggested. When small ships showed up requesting passage they were told to wait until enough showed up to fill a lock. Since this was contrary to past practice, this policy at first angered many users. Later a booking system was introduced so that ships could sign up days in advance for specific passage times, which could then be coordinated.
- *Flying lockage*. The usual procedure was to let a ship into the lock, immediately tie it to the lock sides, close the lock and then raise or lower the water in the lock. At Lock 8 (Port Colborne), a long lock with a low lift, the flying lockage procedure was not to tie the ship up but to allow the ship to keep on going in the lock and by the time the boat reached the other end the water level had changed and the locks were opened and the boat sailed out. This speeded up the service operation of the lock enormously.

Some of the ideas to advance the study were:

- *Air time-lapse photos:* Time-lapse photographs were taken from a helicopter to understand the interconnection of the Seaway's system of locks and canals.
- *Consultants imbedded with the locals:* Local staff resisted the ideas of a newcomer, hired by the Board of Directors, who knew nothing about shipping. The local Seaway staff often pleaded that a new idea was unsafe or not operationally implementable. KCS had an engineer consultant live near the canal (at Lock 7) and become close with the operational supervisor and his staff. By developing an informal friendly relationship with the consultant, the supervisor agreed to try out new procedures.
- The local consultant spent months watching the operations at the locks, especially Lock Seven, which had the tightest space available for operations and was the major bottleneck.
- A simulation model of the operation of the lock system in terms of a series of queues was built to determine strategies to improve the throughput of the system. One heuristic, based on a necessary condition for optimal throughput, was to be sure that the locks where always full when there was balanced traffic. When this occurred there were queues in every reach, the waterway between locks of the system.

[4] Most of these people later joined Kates, Peat, Marwick when KCS merged with Peat, Marwick to form KPM around 1967. Some later went on to work at IBI, an off shoot of KPM, like Neal Irwin, Lee Sims and Arthur Mittermaier. Peter Sandor has since passed away.

And if the locks were always full then at each move of a lock a boat went up and down the lock so that queues between locks always remained the same and the idea was to determine the optimal number of ships in each of the queues.

Improvements were made to traffic control procedures, hardware and software, controller training, etc. As part of this, an extensive traffic control centre was built with vastly improved communications, telemonitoring of operations and a dynamic physical model of the system showing the current status of all locks and ships.

As a result of all the work there were dramatic improvements in a relatively short time. Frank Collins indicated that through a multiplicity of improvements, the Canal's service time was reduced and resulted in a 40% increase in the ship handling capacity. People ceased talking about twinning the canal, and Joe became a hero. KCS established its reputation with Transport Canada. This reputation was inherited by KPMG and has lasted to this day.

The improvements described above were significant but not unlimited. What about the long term anticipated growth? A larger "Seaway-class ship" was born: a ship that exactly matched the dimensions of the Seaway locks, with not an inch to waste. These larger ships had great economies of scale compared to the older smaller ships. Gradually, as older ships were retired, fewer ships were needed to carry the same cargo.

In the long run, growth of grain exports was negligible, as farmlands have not grown and productivity improvements were limited. But more importantly: much of the export market shifted from the Atlantic to the Pacific. So, today, Seaway capacity is no longer a topic of concern.

2.2 Airline Planning and Operations

Another very successful series of applications of queueing were carried out by Alec Lee. In the late 1950's, Air Canada brought Alec Lee from England and made him their Director of Operational Research. He already had experience from British European Airways (BEA), having been the head of their operational research group. During these two tenures, he was involved in some of the early work in operational research and particularly queueing applications in regards to airline planning and operations. He was one of the first Canadians to apply operational research and queueing theory. Much of the work involved the collection of applicable data and its formal analysis and presentation, and the use of queueing models. Fortunately, Alex Lee published a good deal of his work, much of it preserved in his book "Applied Queueing Theory" (Lee, 1966). In it, he gives sources and references to earlier published papers. Around 1970, he left Air Canada to go to Rolls Royce and has since passed away. Below we refer to the various studies he participated in as recounted in the book.

Air Freight Reservation: Chapter 9 is a recounting of a study of the air freight reservation process. The volume of calls requesting service sometimes involved considerable waiting, suggesting that it might be beneficial to increase the number of servers. To substantiate this hypothesis, it was necessary to collect data on inter-arrival times, waiting times, and service times. In his book, Lee presents some of the real data which allowed a prediction of how many extra servers were needed and the resulting improvement in service. As part of the analysis, an $M/M/c$ model was used.

Passenger Check-in Process: Alec Lee also began using queueing theory to determine airline check-in counter capacity and policies in an airline terminal. Chapter 10 discusses this passenger check-in process. Passengers often arrived in groups and data were collected on the group size. One of the main points that emerged here was that it is worthwhile to have an operational research team in place that can propose and study alternative policies with the aim of improving quality of service. The studies involved data collection, simulation, and validation aspects. Theoretical models were used to estimate the effect of policy change before it was implemented, often using simulation. Some of this work appeared in Lee and Longton (1959).

Passenger Air Terminal Sizing: Chapter 13 addresses the issue of passenger terminal sizing where one of the major issues is space. Statistics were collected on previous terminals to help make decisions. Short term parking is a type of queueing problem. This led to collecting real data on loss and probability of rejection. Pricing can be used as a control on the arrival and service rates of parking. Loading times for taxis were recorded. Loading and unloading times for buses were recorded. Terminal design must address not only the current state of affairs but also predictions about the future. The resulting study and preparation by Lee's team resulted in a successful air terminal design in London, England, resulting in a publication on queueing problems in air terminal design (Lee, 1962).

Telephone Seat Reservation (TSR): Chapter 15 discusses standards of service. "In 1961, the manager of Sales Performance of Air Canada, approached H.J.G. Whitton of the Operational Research Department with a request that he study the relationship between the number of trunk lines [in the TSR office] and agents available, and the call reject rate and queueing time distribution." This request had the aim of determining the best organization of TSR offices. Whitton collected data, fit the data to a log normal distribution, and constructed a simulation study. The second phase of the study was carried out by Lee and Linder (1965). The end result was a heuristic technique for planning and scheduling of agents in TSR offices.

Overall, Lee's book is a very useful guide for real implementation of examples of applied queueing models.

2.3 Airport Master Planning

In the 1960's, there was considerable growth in air traffic, and Transport Canada, the government body responsible for all

INFOR, Vol. 46, No. 4, November 2008, pp. 29–46
ISSN 0315-5986 | EISSN 1916-0615
DOI 10.3138/infor.46.4.029

major airports in Canada at the time, including Montreal, Toronto, Vancouver and Calgary, needed to upgrade and to construct facilities at each of these airports. Thus, in addition to the work by Alec Lee, which focused on airline operations and planning, there was considerable airport master planning and analysis work done by Transport Canada. This led to work by Canadians on master planning of airports in other parts of the world, as well. In the initial stages, much of this work was done by a team of consultants from KPM.

2.3.1 Lod Airport Planning

In the middle 1960's a team of consultants from the Israel Institute of Productivity (IIP), including B. Avi-Itzhak, J. Blecher, and S. Saltzman did work on the master planning of the Ben Gurion (formerly Lod) International Airport Terminal[5]. The team included a young Canadian engineer, Marvin Mandelbaum, who would later work on similar projects in Canada.

The team studied both arrival and departure processes, and used flow forecasts and various mathematical models developed by the team to determine the facilities and operational requirements of the terminal. In studying departures Avi-Itzhak and Mandelbaum (1968) used filtered Poisson stochastic processes that took into account passenger ground arrival patterns to the airport relative to their flight departure times to determine passenger arrival patterns to the airport check-in counters. These filtered Poisson processes were also used to determine aircraft apron requirements as well as some terminal sizing. $M/M/c$ models were used to determine facility sizing. In the arrival process, a computer simulation model was used for the queues at immigration and baggage carousels, which included a matching process of bags with passengers at the carousels, based on the hypergeometric distribution (Avi-Itzhak *et al.*, 1966). This work introduced for the first time "split and match" queueing (Mandelbaum, 1968, Mandelbaum and Avi-Itzhak, 1968), since the passengers were split from their baggage and then matched at the carousels.

2.3.2 Prototype Air Passenger Terminal Simulation

In the late 1960's, Van Ginkel Associates, a Montreal architectural firm, was contracted to design a generic prototype modular air passenger terminal, that could be pieced together to form any size or type (Van Ginkel, 1970). KPM consultants Andrew Elek, Marvin Mandelbaum and Peter Hichelheim were to assist the architects with performance evaluation of the prototype[6] to determine the capacity and viability of the units. Joe Kates had the idea of developing a dynamic visual simulation tool. The simulation model was built as a general tool that included facility and service nodes and links so that any queueing configuration could be constructed. It was the forerunner of general simulation models like Arena. However, the model turned out to be very cumbersome for planning since it required a large amount of input to define any terminal configuration. Besides, the computer technology at the time limited the graphical interface for defining models and for visualizing dynamic results. Therefore, a simpler simulation model based on multiple server queueing was built and, along with analytical work, proved more useful for this project. It became apparent, however, that the generic airport terminal concept was not viable, since there was no opportunity for local issues to be addressed and the prototype air terminal did not make it to the operational stages. However, similar analytical and simulation work was later used on an individual basis in the master planning of Mirabel Airport. This simulation model or the idea also was used in Vancouver.

2.3.3 Airport Terminal Master Planning

Analytical work[7] was performed by KPM and later by a spinoff firm (IBI) in airport planning for: Toronto-Malton (now Lester B. Pearson), Montreal-Mirabel (the New Montreal International Airport Project NMIAP), and the Toronto-Pickering project, in the late 1960's and early to middle 1970's. Similar methods were later used for other airports throughout the world. Facility requirements needed to be determined in settings that included many queueing situations. However, the uncertainties inherent in traffic forecasts for 20 or 30 years later were felt to be so significant that the accuracies provided by the application of detailed models were not consistent with the uncertainties of the demand itself. In essence, in the real conceptual airport planning, calculations were based on infinite server models with some adjustment factors. Queueing models were used to get ideas about the relationships between free flow models and constrained ones. See NMIAP (1970/71). There was a large set of projects carried out over several years under this approach.

2.3.4 Surveys Study

In the 1960's, Transport Canada began developing and carrying out a survey protocol based on time stamped punched cards, as well as on other observations, called the CASE survey. The survey was implemented at many of the larger airports and was especially adapted for use at Toronto. The cards were given to passengers and greeters/well-wishers as they entered the airport and time stamped at various locations. The survey was used to obtain arrival patterns of people relative to flights times, and to measure queues, flows and accumulations

[5] There were four internal reports produced in 1967 by the Israel Institute of Productivity, which was commissioned to do the study.

[6] The source is the memory of the author as well as discussions with Andrew Elek. The Van Ginkel prototype terminal report, provided to Transport Canada, would be hard to find and nevertheless would not have included the analytical or simulation work.

[7] The main source was the author, as well as discussions with Andrew Elek, members of the KPM team who did many of the studies. Other participants in the studies included Peter Sandor and Harvey Kriss both of whom have passed away.

ISSN 0315-5986 | EISSN 1916-0615
DOI 10.3138/infor.46.4.029

at various points in the terminal, as well as other characteristics such as well-wishers to passenger ratios. Because of the complexity of terminal facilities, these surveys were often very large and difficult to analyze but did provide, after much work, useful results for both short and long term planning (Transport Canada, 1982).

One interesting result from this survey occurred in the evaluation of the elevator system at Toronto International Airport Terminal. Using the data, a very simple model was developed to show how perceived elevator (as server) capacity changed as the load or arrival rates on the elevators grew, carried out by Marvin Mandelbaum and Gordon Albright (Transport Canada, 1980).

2.4 Vehicle Transportation

In this section, we consider issues of vehicular transportation, particularly in an urban setting, as well as on an international bridge and customs setting

2.4.1 Trucks and Buses

In 1990, a queueing problem arose related to the Ambassador Bridge connecting Detroit and Windsor. Car travelers entering Canada from the United States with no purchases were to be allowed to choose a proposed express booth for customs and duty declarations. By examining the interarrival and service distributions, and the proportions of cars and trucks, Hlynka and Lucier (1990) used simulation to predict that the proposed express booth would not be successful, because other booth lineups would block the express booth and cause it to be starved of traffic for much of the time. When the express booth concept was actually implemented, the predictions proved to be correct and the concept was soon abandoned.

2.4.2 City Traffic

Vehicular traffic flow in a cosmopolitan city is a large queueing network, where the vehicles are the customers or packets, the roads are the links as well as the queue buffers, and the traffic intersections and the lights are the servers controlling the packet flow. Delays are caused by the traffic lights, the traffic density or congestion, capacity and configuration of the network. The entity responsible for setting the traffic light sequences and timing is the controller of the network. One of the first firms to use sophisticated computer controllers for setting traffic lights was the firm KCS run by Josef Kates, (Vardalas, 2001).

Dr. Josef Kates, a fascinating individual, joined the University of Toronto's new computer centre as a graduate student in 1948, obtaining his Ph.D. in physics in 1951. Joe worked out the theory of the cathode ray tube (CRT) parallel memory in his thesis. He was part of a small group of engineers and physicists that began designing and building a pilot model of the first Canadian electronic computer (UTEC) in 1951 under C.C. Gotlieb. Unfortunately, it was never fully implemented. Joe also developed the first electronic game-playing machine, Bertie the Brain, which became a hit at the 1951 Canadian National Exhibition (CNE). Kates cofounded KCS which distinguished itself in the early 1960's when it created the first computerized urban traffic control system for the city of Toronto (Vardalas 2001, p.41). KCS joined Peat Marwick around 1966 to form Kates Peat Marwick (KPM), the consulting wing of the large and famous accounting firm Peat Marwick. Kates left KPM shortly thereafter to serve as Chair of the Science Council of Canada. KPM later after a few mergers became Klynveld Peat Marwick Goerdeler (KPMG). Joe Kates also was Chancellor of the University of Waterloo. See University of Waterloo (2005), Griffith (1994) and Nangini (1998).

Neal Irwin[8], the director of TRC (Traffic Research Corporation) a division of KCS in the 1960's, described in a conversation the use of the principles of queueing in traffic work, for example, the setting of left/right turn lane lengths and numbers, and their impact on the configuration and traffic congestion. He also recalled one traffic control issue where they could only afford one detector at each traffic stop and needed to determine its optimal location. The detector location was influenced by the wave of motion of the cars and how the quickly the wave moved when the light changed to green. In the end, the optimal position to get the best information from the detectors was 200 feet from the intersection. See Irwin *et al.* (1980.)

Lee Sims[9], currently a director of the IBI group in Toronto, an engineer and transportation analyst at KCS at the time (1960's), remembers that they had an optimization program (Sigrid) for signal timing in a network for the traffic control system. They tried to use queueing theory but it did not work well especially if the buffers were not large. However they did use M/M/c models to help them set the optimal traffic light splits, and determined the time for each direction which would minimize the queues while taking into account the delays.

Lee Sims also recounted that he did some work on pedestrian flows at Union Station in Toronto, an important Canadian rail/subway hub. Measurements of queues were used to assess the capacity of stairwells to disperse arriving train passengers.

We interviewed Raj Bissessar[10], currently Senior Engineer, Traffic Control Systems, City of Toronto. He stated that there are many issues in the study and control of traffic that involve queueing insight. For their traffic network problems, especially to coordinate signals, they now use software called SYNCHRO (2008), designed for grid or arterial networks. Raj said that the queueing models tend to be hidden in the software. This same software is used in Windsor, and in other

[8] This story came from a discussion with Neal Irwin.

[9] We interviewed Lee Sims

[10] Telephone interview with Raj Bissessar and John Wolff on traffic operations by Myron Hlynka

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Canadian cities. Raj mentioned a SCOOT (2008) system developed in England, which has detection loops to measure traffic volume in real time. It controls about 300 traffic signals in Toronto. The same system is used in Halifax and Red Deer. An interesting issue he related concerns intersection crossing timings and pedestrian traffic. Raj said that it is not good to have pedestrians waiting too long because if this happens, the pedestrians will jaywalk. (We believe that this suggests interesting queueing problems, which model aberrant behaviour by customers when their wait is too long.)

In another interview, John Wolff at Traffic Operations in Windsor, Ontario, said his team uses queueing terminology in dealing with traffic situations. He has taken specialized courses for his job at Georgia Tech. One typical short course studied was "Signal Timing Analysis in Oversaturated Conditions."

3. TELECOMMUNICATIONS, NETWORKS AND COMPUTER PERFORMANCE

The field of telecommunications plays an important role in the Canadian economy and there have been major contributions in the area by Canadian researchers. In this section, we discuss examples and issues in telecommunications.

3.1 WAN/LAN Internet Performance Tool

Bob Lieberman, while working at Network Design Corp in the 1970's, developed a WAN/LAN network Internet performance tool[11]. It could be used to predict the performance of Internet networks connected together by bridges, routers, and gateways. The system supported the Ethernet, Token Ring, Token Bus, and FDDI LANs as well as various routing protocols. The routing of applications through the network was performed automatically using an algorithm based on shortest number of hops and available bandwidth, but could also be manually specified by the user. The purpose of the tool was to predict application and file transfer times, line and equipment utilizations, and application throughput in WAN/LAN Internet networks. The outputs consisted of throughput, mean message network transit times for each application or traffic stream, and line and internet device utilizations.

The main components in the traffic model were:

- The *Flow Generator* – calculated the traffic inputs required by the Flow Solver
- The *Flow Solver* – calculated the traffic flows to each queueing model.
- The *Queueing Model Solver* – invoked the appropriate component model and solved for the packet queueing delays. Its outputs were packet delays through each network component, and the component utilization.
- The *Response Time Calculator* – took as input traffic stream routes and calculated packet network transit time for the route using results of the packet delay calculations.

Inputs entered through the GUI were: (1) *LAN Parameters* (such as LAN type and speed, and packet length); (2) *Internet Device Specification* (such as maximum packet processing rate, buffer size, and WAN protocols); (3) *Topology* (such as the line connections); (4) *Traffic Specification* (such as for each traffic stream its ID, source, destination, arrival type (bursty or non bursty) and application mix; for each application its ID, message size, message generation rate, network protocol, and priority; (5) *Model Directives* such as report generation.

Queueing theory was applied to this problem. The model took the instance of a communication network specified by the user and mapped it to a network of queues, which was solved using Ward Whitt's Queueing Network Analyzer (Whitt, 1983). Once the load or flows on each device and link was calculated the mean packet delays through devices and links were found by applying the appropriate queueing model. The Kleinrock (1976) independence assumption allowed each queue in the network to be treated independently. There were various queueing models for calculating packet delays through the network devices such as M/G/1 for Internet device delay and single link delay, and queueing models from Schwartz (1977) for CSMA/CD LAN, Token Ring/Bus LAN and FDDI LAN. The application was packaged and sold as one of Network Design Corp's products and a few copies were sold. Unfortunately, Bob left the company shortly after it was developed.

3.2 Data Networks

Adrian Conway and Nicolas Georganas[12,13] developed some network queueing algorithms. Conway and Georganas (1989) present exact computational algorithms for queueing networks. In Conway and Georganas (1986a) they developed an algorithm called RECAL (REcursion by Chain ALgorithm), which computed performance measures for closed multi-chain product form queueing networks. RECAL involves a recursion that is different in form from the recursions used in other convolution and MVA (Mean Value Analysis) algorithms and was a breakthrough, for its time, for analyzing networks with many routing paths called chains. They also developed the (parallel) MVAC algorithm, for chain analysis in networks that had few service centres but many chains, which had lower computational costs than MVA. See Conway and Georganas (1986b) and Conway *et al.* (1989). Both algorithms, described above

[11] The description comes from a discussion with Bob Lieberman and notes from this study.

[12] We corresponded with both Conway and Georganas, but much of our material came from the references to their work.

[13] Nicolas D. Georganas, Distinguished University Professor, Associate Vice-President, Research (External), University of Ottawa. Nicolas Georganas is a prolific author in the field of telecommunication and the application of analytical techniques, including queueing, to performance analysis.

ISSN 0315-5986 | EISSN 1916-0615
DOI 10.3138/infor.46.4.029

for analyzing queueing networks, were developed in Canada by Conway and Georganas. Subsequently, Conway applied them in the telecommunications industry in the United States at both IBM and at GTE (now Verizon). The algorithms now appear in a number of well-known applied textbooks in data networks and computer systems, and these queueing results are presumed[14] to be used by other practitioners.

3.3 Telephone and Hydro Systems

At laboratories owned by large telephone companies such as AT&T, much research into telephony service occurred especially in the United States. Bell Northern Research (now a part of Nortel, but in 1995 partially owned by Bell Canada Enterprises), also supported such studies for application to their telephone systems.

3.3.1 Northern Electric, NORTEL, BNR

Digital switching and buffering are required for various types of data conversion. Digital switches can be viewed as service nodes of a queueing network, where queues develop as input arrives. So queueing theory plays a significant role in telecommunication networks. The position of Nortel and BNR in advancing this field were (and are) immense and thus it is worth presenting a brief history of these companies.

The following account of the Canadian communication giant comes from Wikipedia (2008a,b) and from Telephone World (2008a). The company Northern Electric was originally owned by Bell Canada. In 1914, after a merger, Northern Electric was co-owned by Bell Canada and Western Electric in the United States, but by 1964, Bell Canada acquired all of Northern Electric. Subsequently, in 1969, Northern Electric Research Labs worked on optical cables and digital communications. In another reorganization in 1971, Bell Canada and Northern Electric combined research offices and formed Bell-Northern Research (BNR). Around 1975, Northern Electric began selling one of the earliest digital switching systems and competed in the North American market with other large companies like Western Electric. In 1976 Northern Electric became Northern Telecom Ltd (Nortel), which introduced its DMS telephone switches one year later and became a world-leading supplier.

In 1983, Bell Canada Enterprises (BCE) was formed holding both Bell Canada and Nortel and these later companies jointly owned BNR. By the early 1980's, BNR operated research facilities in the United States, the United Kingdom, and Australia, but ceased to exist as a separate entity in the 1990's when Nortel acquired all of BNR. By 1995, Nortel was trying to dominate the communications network market and in the late 1990's stock speculators pushed up the price of Nortel stock. However, in a short two-year period from the fall of 2000, the share price went from $124 Canadian to 50 cents. After huge layoffs and more reorganization, Nortel continues to exist and to make telecommunications equipment, and still has some strong researchers and developers.

The following quotes show how important the Canadian contribution was to the building of telecommunication queueing networks.

In an interview (Hochfelder, 1999), Mischa Schwartz, a networking expert and pioneer related that:

> "*People knew about layers: physical layer; data link layer and a third layer, normally a networking layer. There was a lot of work published. One of the first administrations to latch onto this was that of Canada, with Bell Northern Research developing products. In fact, I attended a communications meeting in 1976 and they presented some of their work at that time too. So they developed some of their products. The Canadians were one of the first in this area.*"

The following statement appears in an article, at the website Telephone World (2008b) (in the United States), talking about the race to digital switching and confirming Northern Telecom's pivotal role:

> "*Northern Electric (later Northern Telecom) and engineers with Bell Northern Research (BNR) of Canada had a raging debate on whether to make an electronic #5XB or go fully digital. The digital team was successful in getting their project direction approved, and the rest is history ... Northern Telecom started the public "digital awareness/fever" ball rolling with their "Digital World" announcement in February 1976 at Walt Disney World near Orlando, Florida.*"

3.3.2 Queues, Switches and Schedulers

Attahiru Alfa[15] is a prolific researcher at the University of Manitoba in the field of queueing theory as applied to communication and telephone services and has many papers in the field. He founded the annual CanQueue conference that brings together Canadian queueing researchers and still is a driving force. Alfa's work and the CanQueue conferences will undoubtedly foster many more application of queueing.

Alfa[16] did work with BNR (1994–95) where he used a hybrid queue discipline, combining FIFO and LIFO, for the analysis and efficient management of a dial tone telephone system. In such systems there are a finite number of spaces in the primary FIFO queue, however when the system becomes overloaded customers that come when the FIFO queue is full are placed in a LIFO queue that has infinite capacity. The

[14] Conway in an interview indicated that this was his understanding.

[15] Much of his applied work cannot be disclosed due to confidentiality agreements with those companies he works for.

[16] Came from private conversations and communications with Attahiru Alfa, as well as the papers that he indicated were connected with the applications described here.

ISSN 0315-5986 | EISSN 1916-0615
DOI 10.3138/infor.46.4.029

assumption is that these customers will have to wait a long time anyway and are likely to misbehave as a function of their wait time, in that they will consume server resources, yet are unlikely to contribute to throughput. For example, such customers could be those that abandon the queue. Alfa and Fitzpatrick (1999) report on a particular analysis of such a system. They found that this type of system provides more useful throughput and was more effective than either the FIFO or LIFO system alone when the server is fast. That is, it has a high capacity under normal conditions and yet robust properties on overload. Gordon Fitzpatrick at the time was the manager of Nortel's Networks Advanced Modelling Technology group.

In the development of a telephone switch, it must be decided how to execute the various call processing functions required to establish and disconnect a call. The functions involved in establishing a call would be: detecting an incoming call, receiving the digits, determining the call destination, finding the best route for the call, setting up the speech path, apply ringing, and detecting an answer. The processing of one call is called a job, yet it is not run as a single uninterrupted entity but broken up into tasks each with its own priority. How these tasks are formed and what priorities they arc given, greatly influences their sojourn times. Martens and Alfa (1998) discuss and analyze a model of such a system with forking to obtain sojourn times. Forking, where several tasks of different types enter the priority queues upon completion of a predecessor task, is a common occurrence in call processing.

Frigui and Alfa (1999) is another paper motivated by Alfa's work for BNR, this time on schedulers that attempt to provide jobs a fair share of the processors. The authors consider a polling system with a polling table, which indicates which job types should be done next. The system tried to avoid situations in which the heavy load jobs hogged the system by limiting the service time given to jobs. This type of polling scheme is used very extensively in scheduling processes in an operating system of the central module of a telecommunication switch. The paper presents a simple method to approximate the waiting time.

3.3.3 Hydro Meter Reading

Frigui, Stone, and Alfa (1997) discuss work done with Manitoba Hydro. They used polling system analysis to study data acquisition of meter readings when Manitoba Hydro wanted to move to automatic meter readings. Meter reading is important since the messages are used for billing, for discovering customer consumption patterns, for projection of demand and for protecting against meter tampering. The automatic meter reading can be done more often and be more economical than manual data collection, and more timely and accurate. A module in the meter does the reading, and then sends the messages on to repeaters, which collect data from many meters. There are many repeaters that need to be polled by the central server. To save resources and reduce complexity no communication is allowed between stations and the central server, but stations must hold messages and wait to be polled and requested before transmitting. The performance of different configurations of an acquisition system needed to be studied, which is often done by costly simulations. They developed a simple and quick analytical approximation method based on a polling queueing model with cyclic order and arbitrary switchover times that allowed for multiple priority messages and nonpremeptive service to estimate the mean waiting time under exhaustive and time limited service policy, with more than one type of message at each station.

3.4 Computer and Component Performance

Kenneth Sevcik (1944–2005)[17] was a member (since 1971) of the University of Toronto Computer Science department and one of the pioneers in the field of computer system performance analysis. In an interview in Ubiquity (2005) with Sevcik on the origins of this field, Ken stated "the origins of performance evaluation for computer systems come from queueing theory." He also recalled that in the early 1980's "mainframe computer systems cost millions of dollars, so making a purchasing decision often meant betting the ranch." Ken and others developed the queueing network modelling software MAP distributed by Amdahl, which benefited "because the models could quantify the price/performance advantage that Amdahl systems held at that time over their competitors." Ken supervised a huge number of Ph.D. and M.Sc. students and their influence will be felt for years to come. See Lazowska and Zahorjan (2005).

Alberto Leon-Garcia is a professor in Electrical and Computer Engineering at the University of Toronto. Much of his work involves queueing as applied to the internet. He helped to develop ATM (Asynchronous Transfer Mode) switches and he is the holder of several patents on switches and schedulers. At the 2003 CanQueue conference, he talked about a queueing issue that arose in the company AcceLight, which he co-founded (Leon-Garcia, 2003). The company was based in Pittsburgh with research and development (R&D) work done in Ottawa. The company developed an optical transparent switch for SONET and packet traffic. One of the issues was to intelligently waste bandwidth to meet delay requirements. The solution involved replacing weighted fair queueing with strict priority queueing. The company AcceLite was in existence from 1999 to 2003. As Leon Garcia put it in the abstract of his talk:

> "*We will discuss performance issues in large-scale multi-service data switches, switches with aggregate throughputs in excess of one terabit per second. We will focus on the "skinny flow" problem that arises when small flows must be provided with delay guarantees, and contrast the circuit-switched and scheduled packet switched approaches to the problem.*"

[17] Our main sources were from his colleagues' recollections when he passed away.

ISSN 0315-5986 | EISSN 1916-0615
DOI 10.3138/infor.46.4.029

4. CALL CENTRES

Call centres[18] represent examples of applications that regularly use queueing results and analysis. The most common type of call centre provides service to customers or agents, while other call centres are used to sell items over the phone.

4.1 Introduction

Telephone calls represent arriving customers, which form a queue to multiple servers who answer the calls on a priority or FIFO basis. Most call centres have performance targets (percentage of calls handled within a certain number of minutes) and are often managed by software that incorporates queueing concepts. The number of servers needed can vary considerably with time of day (day of week, season), and queueing formulae (Erlang C) are sometimes used to estimate the number of servers required to meet quality targets. Automated systems attempt to make the system more efficient by reducing the number of servers or lowering the service time and can involve networks to transfer the calls to other centres. The systems also keep statistics on the queueing characteristics to evaluate effectiveness. Certainly an understanding of queueing theory is helpful to call centre managers.

Call centres are prevalent in Canada employing around 370,000 workers in the year 2000 according to The Centre for Spatial Economics (2004), and Call Center Week (2001) estimates the number of Canadian call centres to be 6,500 in 2001. For example, Grand and Toy has 120 service representatives in Toronto, Ottawa, and Edmonton, handling over a million contacts (phone, e-mail, fax, etc.) per year. The Shopping Channel (owned by Rogers Broadcasting) has, in addition to the regular staff, "120 home based agents to assist with the unique call spikes associated with the on-air broadcast." FedEx Canada Customer Service handles 1.3 million calls per year. See ICCM (International Contact Centre Management) (2008). Ford Motor company had a large phone call centre in Toronto area handling customer queries and complaints, and they have an other centre in Dearborn that handles calls from dealerships about incentive programs.

In the next section we give details on a call centre in the Canadian government, which further shows that Canadian companies are indeed involved in technology development relative to an important queueing application.

4.2 Service Canada Call Centres

Service Canada[19] call centres and Canada Customs and Revenue Agency (CCRA) call centres handle 80% of all government call centre traffic. The Service Canada call centres alone handle a total volume of 11.5 million agent answered calls per year (2006). There are 1200 employees including managers (some part time) in call centres located across Canada in Vancouver, Edmonton, Regina, Winnipeg, Sudbury, Hamilton, Toronto, Scarborough, Ottawa, Montreal, Shawinigan, Bathurst, Glace Bay, and St. Johns.

The variation and the peak times of the call volume (i.e. the arrival rate), or calls offered (number of calls arriving), depend on various factors such as seasonal issues (e.g. Christmas and winter weather), customer behaviour (e.g. Mondays give 20% of weekly calls), as well as policy (e.g. client reporting requirements set out by employment insurance). Given the variable demand, agents are scheduled based on expected call volume and thus the number of employees depends on the time of day, time of month, and time of year. If there are not enough agents locally, the calls will be routed elsewhere, based on various routing algorithms, because the system is nationally networked.

Call centre managers are familiar with queueing terminology. The goal of Service Canada is to answer 95% of the calls within three minutes. In mathematical terms, if we use M/M/s as the model, then we might write this as $P[W_q \leq 3] \geq 0.95$ (or $P[W_q > 3] \leq 0.05$), where W_q is the waiting time of a call before it is answered. From Winston (2004) we have the following expression for a M/M/s queueing system:

$$P[W_q > t] = \frac{(s\rho)^s \pi_0}{s!(1-\rho)e^{-s\mu(1-\rho)t}},$$

where

$$\pi_0 = \left(\sum_{i=0}^{s-1} \frac{(s\rho)^i}{i!} + \frac{(s\rho)^s}{s!(1-\rho)} \right)^{-1}$$

and $\rho = \pi/s\mu$.

For given parameters we set $t = 3$ and compute $P[W_q > t]$ for increasing values of s until the probability drops below 0.05. The value of s for which this occurs will determine the number of servers (agents) needed.

The language skills of Service Canada are English and French. In Ottawa, all agents are fluently bilingual, but most other call centres have unilingual agents. Calls are routed based on skills using the software Symposium produced by Canadian company, Nortel(2008a)[20]. A queueing paper relevant to the bilingual issue would be Stanford and Grassmann (1993). Another product made by Nortel (2008b), CallPilot, plays recorded announcements while clients wait in queue and also reads prompts and voice mailboxes. There are also some training modules that have been developed within Service Canada.

A "wait time in queue" message was used in the past but it did not work well. Currently, when the volume of calls is high, the message says that there is a "high level of calls" and invites

[18] The main source for this information is our own experience in visiting a call centre in Dearborn, Michigan run by Percepta, a subsidiary of Ford, where we gave a lecture on queueing to their staff and got insight into the industry from Doug Tanoury.

[19] We interviewed Ginette Éthier, Director for Operational Support, Service Canada Call Centres, of Ottawa.

[20] We also referred to the websites of the software and hardware manufactures who contributed to Service Canada's operations. It is clear that NORTEL, a Canadian telecommunication giant, plays a part in providing solutions of call centres and management of queues.

ISSN 0315-5986 | EISSN 1916-0615
DOI 10.3138/infor.46.4.029

callers to "call back later." or to self-serve on the IVR (Interactive Voice Response system). Overall 23% of calls are dropped although a number of these calls become retrials, or in call centre terminology, "redials." Generally, the average length of a call to Service Canada is six minutes. Their software keeps track of statistics such as the number of calls that come in, the number of phone calls that are dropped, the average speed of answer (the time to pick up the phone), service levels, abandoned calls, and the number of agent interactions (the number of calls handled by agents). Since each call centre works differently, there still seem to be opportunities for much study using queueing analysis.

5. HEALTH CARE PERFORMANCE

Public health care expenditures are a significant proportion of the budgets of governments in Canada. Thus, making the provision of health care more efficient and effective is a major issue. Furthermore, long patient wait times and wait lists, which arise because of large and increasing demand and limited resources, have become highly contentious and political issues. Canadian queueing researchers in the health care sector have made contributions to the understand and control of wait times in areas such as, ambulance service, patient scheduling and emergency room operations, some of which are described below. Efforts to reduce wait times for medical procedures in Canada are discussed in Saunders (2008).

5.1 Hospital Ambulance Service

Duncan Taylor[21], a Ph.D. student at the University of Toronto in the early1970's, studied the Ontario ambulance system with supervisor Jim Templeton and modelled it using queueing theory. See Taylor (1976) and Taylor and Templeton (1978). The goal was to predict the effect of changing the ambulance reserves (cut-off points) and the number of ambulances on the quality of service performance measures. Separate measures are required for emergency victims and for the non-urgent patients on routine hospital transfers. The latter constitute over half of the day time calls served. Duncan interviewed managers and dispatchers and determined that they keep reserves for emergencies, that is, they should use a cut-off dispatch rule so that routine calls are left to wait when free ambulances go below the reserve cutoff point. The quality of service measures used were the average time non-urgent calls wait and the probability that an emergency is delayed. Of the non-urgent routine demand, long distance transfers have very long service times relative to other services and may lead to the need for extra ambulances and for overtime crews to staff them. Thus, a third measure of performance is the probability of a call-in for extra staff.

[21] Unfortunately Duncan passed away on a boating accident not long after completing his thesis. Also Jim Templeton passed away in 2003, so our discussion here is based on just the two publications on the topic.

Actual statistical data were collected on arrivals and service patterns. The analysis showed that arrival pattern of emergencies was Poisson and the service time for emergency and short routine calls (when long-distance transfers were removed) was Erlang distributed.

Using priority models from the literature, Duncan developed analytical results to give service utilization, the waiting time distribution for priority customers, and complete solutions under queue length dependent cutoff rules. He calibrated the model using numerical and simulation outcomes to get a good fit to reality. A model of ambulance service with long distance transfers and an overtime call-in system was then incorporated into the cutoff priority model and validated using a system with a small number of ambulances. The model was meant to be used as a policy making tool.

Armann Ingolfsson[22] at the University of Alberta, and his collaborators, including Erhan Erkut, Susan Budge, and Dan Haight did sponsored research work for ambulance services in Calgary, Edmonton, and St. Albert, Alberta, using an approximate hypercube model (Larson, 2001). Ingolfsson's characterization of the value of queueing in this application is that "it is part of a set of tools that help to understand, confirm, and sometimes change" the models that were currently used. Budge, Ingolfsson and Erkut (2008) extended the approximate hypercube model to allow for multiple units per location and location-specific service times, features that are important for emergency medical systems. The authors developed an algorithm to estimate station specific busy fractions and vehicle dispatch probabilities. The algorithm has been used as part of a decision support system for the Calgary Emergency Medical Services system.

Ingolfsson is also well known for his queueing package Queueing ToolPak (Ingolfsson and Gallop, 2008), written in Microsoft Excel, and available on his website. Besides using it for ambulance work, this package has been used by many, including by a consulting company that used it to plan the number of checkout lines in a grocery store, and by planners to determine staffing for the check in area at the airport in Reykjavik, Iceland.

5.2 Emergency Rooms

In 1994, John Blake, Michael Carter and Susan Richardson won the CORS Practice Prize Competition for work on queueing and hospital emergency rooms (Blake *et al.*,1996).

Their paper studied the emergency room at the Children's Hospital of Eastern Ontario in Ottawa. The authors note that "an emergency room can be thought of as a network of queues." Because of the recommendations in the study, the hospital implemented a fast track facility for treating patients with minor problems and increased the number of physician hours in the emergency room.

[22] We interviewed Armann Ingolfsson.

DOI 10.3138/infor.46.4.029
ISSN 0315-5986 | EISSN 1916-0615

Michael Carter is a Professor in Mechanical and Industrial Engineering at University of Toronto, and Director of the Centre for Research in Healthcare Engineering. He works in such areas as Health Care modelling, scheduling operating rooms and determining the cause and relationship between overcrowding and waiting in emergency departments. Some applications of operational research to health care (including queueing) are discussed in Carter (2006a,b) and Carter *et al.* (2006). Carter has been involved in issues of wait times for medical treatment. In Carter (2002), he refers to health care waiting lists with the following perceptive comment: "As the queueing rate increases, the reneging increases. People on the list either look elsewhere for service, become more seriously ill and go to a hospital or perhaps die waiting."

5.3 Patient Scheduling

Jonathan Patrick[23], currently at the University of Ottawa, worked in a health care setting as a graduate student under the supervision of Marty Puterman. In 2003, he and others conducted a project with Vancouver Coastal Health Authority through the Centre for Operations Excellence at the University of British Columbia. They addressed the scheduling of patients for a diagnostic resource. The Health Authority had no system in place for scheduling computed tomography (CT) scanning slots, and it had multiple priority classes and a single resource. The project measured the current wait times by looking at electronic and paper records. The wait time targets for the highest priority class was one week, the next priority class was two weeks and the third priority class target was one month. Most patients were waiting longer than the wait time targets before the suggested improvements. Rather than maximizing revenue, the challenge facing the resource manager was to dynamically allocate available capacity to incoming demand so as to achieve wait time targets in a cost effective manner.

Patrick and Puterman developed a multiple priority class scheduling policy for a single resource. The model used a Markov decision policy under stochastic demand. The aim was to schedule patients for minimal cost yet maintaining reasonable waiting times. They present a model that gives the optimal reservation policy. One of the recommendations that would apply to many systems is that good data be collected on wait times. The work was documented in Jonathan Patrick's Ph.D. thesis (Patrick 2006) and in Patrick *et al.* (2008).

6. FORESTRY

Forests cover a large part of Canada, thus forestry management and timber export are major activities. Although queueing and forestry may not seem to be related, below we discuss interesting work relating the two in the control of forest fires.

6.1 Forest Fires

Dave Martell[24], of the Faculty of Forestry at the University of Toronto, was a member of a small group that pioneered the application of operations research and queueing to forest fire management in Canada. Those involved in the operations research work included Glen Doan of the Ontario Department of Lands and Forests and Peter Kourtz and Albert Simard who worked in the Canadian Forestry Service's Forest Fire Research Institute in Ottawa. While those most connected to the queuing applications included Jerry Drysdale, Glen Doan and Dennis Boychuk, all from the Ontario Ministry of Natural Resources (OMNR). They developed a simulation model of a forest fire initial attack system comprised of fire fighters, airtankers, transport aircraft and trucks in 1984. It was designed to capture the complex queueing system behaviour of Ontario's forest fire initial attack system. It was used to help evaluate airtankers for a Cabinet submission and was considered by Treasury Board analysts to be a welcome departure from traditional "seat of the pants" decision-making. The above work, described in an article in Interfaces by Martell *et al.* (1984), is believed to be the first documented implementation of operational research in forest fire management. The initial attack model was later enhanced by the OMNR and re-named LEOPARDS, as noted by McAlpine and Hirsch (1999), and which has been used for strategic planning purposes to guide, for example, airtanker fleet modernization strategies and how the OMNR might respond to the challenges that climate change will pose to fire managers. Dennis Boychuk is updating the model to a decision support system to be called LEOPARDS II.

In 2007, he was editor of a special issue of INFOR on this topic (Martell, 2007). In Fraumeni (2006), Martell gives the following description: "The fire becomes the "customer." The "servers" are the resources used to control the fire – air tankers, helicopters and fire fighters. The longer fires "wait," the more "annoyed" (and damaging) they become. Air tankers cost roughly $30 million each. So the government has to figure out how it can balance air tanker costs with response time. I develop queueing models that can be used to help determine an optimal number of air tankers."

The earliest paper to explicitly mention queueing in a forest fire setting may have been by Bookbinder and Martell (1979). Dave Martell believes[25] that his work on queueing, along with the work of others, has influenced forest management thinking, mainly through education and demonstration.

It is interesting to note that Martell's early interest in queueing theory motivated him and many others to develop daily fire occurrence models to characterize the fire arrival process, and two of these models are described by Cunningham and

[23] Jonathan Patrick shared his insight in correspondence and an interview.

[24] Much of the historical material and its relevance to our paper came from correspondence and discussions with Dave Martell.

[25] We got this from correspondence and discussion with Dave Martell.

ISSN 0315-5986 | EISSN 1916-0615
DOI 10.3138/infor.46.4.029

Martell (1973) and Wotton and Martell (2005). Besides the extensive work of Martell, and others already mentioned, other Canadian contributions to forest management and queueing include work done by Stanford, Woolford, Kulperger, and Hunchak. Special queueing models called *fluid queues* were used to analyze the evolution of the fire perimeter. Knowledge about the perimeter movement helps forest fire planners determines how to allocate resources to fight the fire. At recent CanQueue conferences, Woolford *et al.* (2004) and Woolford (2005) presented theoretical results in fluid queues, which illustrated the potential application of this methodology to the forest fire perimeter evolution. A two-dimensional process with five underlying states as a preliminary model was presented. See also Woolford *et al.* (2008). David Stanford (2005) at CanQueue2005 described the characteristic of forest fires giving practical information and data collected about them. He also addressed myths about forest fire control, and presented issues related to model development and application of queueing to forest fire spread.

7. MANUFACTURING, PRODUCTION AND SERVICE

Manufacturing has contributed significantly to the Canadians economy providing equipment, machinery, durable and personal goods in all economic sectors. According to Thibault (2008), between 1945 and the 1990s, manufacturing had accounted for 22–24% of Canada's total real output of goods and services, and employment of roughly one to two million workers. He also indicated that the value of goods shipped by Canada's 47,000 manufacturing establishments was almost $250 billion in 1986. Thus, manufacturing provides many opportunities for the practice of queueing and operational research.

John Buzacott[26] is one who contributed significantly over many years to the application of analytical methods of simulation and queueing to manufacturing while a professor of industrial engineering and management science at the University of Toronto, the University of Waterloo, and York University. The book Buzacott and Shanthikumar (1993) is a standard reference classic on the application of queueing theory to manufacturing and production. John Buzacott, with Ken McKay and Jim Bookbinder and others were the principle researcher at The University of Waterloo Management of Integrated Manufacturing Systems (WATMIMS) research group. This research group was formed in 1984 to promote and coordinate applied studies involving industry and academic researchers in the areas of manufacturing and logistics. There were other researchers who applied queueing to production such as Attahiru Alfa from the University of Manitoba. Below we provide four examples from steel, electronics, and bus and automotive manufacturing.

[26] Much of the insights and leads to other work in this manufacturing section came from a long interview with John Buzacott.

7.1 Steel Production

In his Ph.D. thesis, Callahan (1971), at the University of Toronto studied a problem in the production of steel at the Hilton Works of Stelco in Hamilton, Ontario. The problem was introduced to him by his supervisor, John Buzacott. See Buzacott and Callahan (1971). The necessity of having uniformly hot steel ingots, that is, at a uniform temperature throughout the ingots, provided by a battery of soaking pits to prepare the steel for rolling, limits the production capacity of a rolling mill and its efficiency. The lack of uniformly hot steel ingots delays the mill's production. These situations are called "nothing hot" delays. Callahan determined the amount of nothing hot delay to be expected from the parameters of the soaking pit rolling mill complex given the characteristics of the ingots being put into the soaking pits. The obvious approach is to consider the soaking pits as parallel servers feeding the mill in a tandem manner. However, a more fruitful approach is to consider the pits as units circulating in a cyclic queue. Closed form solutions are found for the probability and the capacity of the pit-mill complex which take into account the individual heating rates of each pit, the potential rolling rate of the mill, as well as the possible necessity of repairs. Often when ingots arrive at the soaking pits, there is no space available and the ingots must wait. As they wait, they cool and hence must stay longer in the soaking pits when space becomes available. The system is a queue with service times dependent on waiting time and this application led to theoretical work on such queues by Callahan (1973) and others such as Brill and Posner (1977). Substantial assistance on Callahan's work was provided by Stelco and by individual employees Arie Verduijn and Paul Wendling of the Hilton works.

7.2 Vehicle Production

According to Dennis DesRosiers (2006) from 1995 to 2005 Canadian vehicle production had been above 2.5 million cars per year, new capital expenditures had been in the 2.7 to 4 billion dollar range each year and one in seven jobs in Canada were tied directly or indirectly to the automotive sector. Canadian researcher applied operational research and queueing within this industry. In fact, John Buzacott related that, in 1985, General Motors (GM) was redesigning their production plant at Oshawa, at a cost of about two billion dollars, switching from the traditional flow assembly lines to the use of automatic guided vehicles (AGV's) to move car bodies through the plant. The workers now had four to five minutes rather than one minute to complete particular assembly operations. However, to implement the new approach required parallel workstations to achieve the same throughput capacity, and required buffers between stations to smooth out the workflow. A main objective of the company was to improve car quality. Also, in the prepainting stage, the company began using

INFOR, Vol. 46, No. 4, November 2008, pp. 29–46
ISSN 0315-5986 | EISSN 1916-0615
DOI 10.3138/infor.46.4.029

robots for welding rather than personnel resulting in six workers running a plant section instead of 100 people as before.

Bookbinder and Kotwa (1987) reported doing simulations on the welding, where one of the objectives was to determine the number of automated guided vehicle needed to obtain the desired throughput. The plant included queues stations in the line as buffers between operations to account for station breakdowns. According to John Buzacott, the WATMIMS group did 10 or 15 such simulations of various operations, while General Motors personnel, like Terrance Kotwa, did the actual simulations.

At certain points in the final assembly the plant needed to maintain sequence to match up cars with the correct characteristic such as: trim, seat or colour. For example, the plant had to sequence the seats to match the order of the car bodies from the paint shop. Thus, waiting banks were introduced, where cars bodies waited to be sure they left in the correct sequence and part supply was organized to meet the needed sequence. The company could not re-sequence on the fly.

Various WATMIMS group staff members and students did analytical work related to queueing applications motivated by the new automobile assembly plant. For example Kostelski *et al.* (1987) describes work done by the group on a macro simulation of the whole automotive assembly plant that incorporated the individual simulations. The plant was modelled by nodes that were separated by inventory banks of finite capacity. The objective was to balance production rates for plant areas and to determine buffer area capacities, which took in to account breakdowns. Liu and Buzacott (1990) describe analytical research of the development of approximate analytical models, which used Markov chain analysis, to analyze the production line and to help validate the simulation models.

7.3 Trucks and Buses

Attahiru Alfa[27], at the University of Manitoba, worked for Motor Coach Industries in Winnipeg on problems in the manufacturing and assembly process of a class of coaches produced by the company. He and his team developed a tandem queue model and implemented the results on a spreadsheet that was developed using the simple and quick approximation algorithms from Yannopoulos and Alfa (1994). Motor Coach Industries used this to analyze and manage their E-line coach production (1997–98).

7.4 Electronics and Computers

IBM has had an important presence in Canada and has fostered industrial collaboration with academics researchers in the country, and no less so with the WATMIMS group. At the IBM Toronto plant the company was manufacturing circuit boards and were attempting to change from pin to surface mount circuit board manufacturing technologies. Many of the assembled printed circuit boards (PCB) failed during the electrical testing, and needed reworking, affecting throughput and work-in-process inventory. A circuit boards may be tested and reworked several times at a single testing station before passing the test and then proceeding on to the next operation. Elisabeth Jewkes[28] and Steve Shevell spent much time at the Toronto plant observing the testing and repair operation and the associated queues, Resulting in a confidential report. In addition, several published articles appeared. Shevell *et al.* (1986) reported on simulation work to determine the affect on production performance. The authors reported that a variety of printed circuit boards were being manufactured at the plant with 80 assembly operations and test stations producing over 300 different parts, and the total production exceeded 1.5 million units per annum. Jewkes and Buzacott (1991) modelled a particular feedback queues in which customers may feed back to obtain several phases of service before they finally depart the system. In the inspection and repair feedback systems, Jewkes (1995) looked at the tradeoff between inspection standards and queueing congestion, in which lowering inspection standards reduced queueing but at the same time decreased quality.

8. SUMMARY AND CONCLUSIONS

This paper gives a wide range of examples of the application of queueing as done by Canadian researchers and practitioners. There has been a deliberate attempt to include applications from wide variety of areas, including transportation, airport planning, health care, call centres, telecommunications, forest fires, computer networks, and manufacturing. We are certain that what we have presented is only a small portion of the applications of queueing in Canada. We do not claim to be exhaustive in our coverage and this article is only a sample. We hope that the article motivates others to write about their queueing applications and experiences, and to let others know about them. This article is dedicated to the fiftieth anniversary of CORS and to those applied queueing practitioners who are no longer with us.

ACKNOWLEDGMENT

The support of NSERC (Natural Sciences and Engineering Research Council of Canada is gratefully acknowledged. The suggestions of the referee were very helpful in improving this paper.

NOTES ON SOURCES

We give the sources and notes for each section of this paper including formal references as well as reports that the author

[27] We took this from a discussion and correspondence with Attahiru Alfa.

[28] Beth Jewkes shared here experience with us and led us to several references we use here.

or any of the interviewees remember most of the reports are no longer in the procession of the authors or interviewees.

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ISSN 0315-5986 | EISSN 1916-0615
DOI 10.3138/infor.46.4.029

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