Lecture 20

Trigonometric Functions

1. Radian Measure of Angles.

We consider a circle of radius 1 and measure angles in term of distances around the circumference. The central angle determined by an arc of length along the circumference is said to have a measure of *one radian*. See page ♯431:

Illustrations of Angles in Radians: See page ♯250 and ♯251.

$$2\pi \text{ radian} = 360^o$$

$$1 \text{ radian} = \frac{180^o}{\pi} \approx 57^o$$

2. Definitions.

Given a number θ, we consider an angle of θ radians placed in standard position.

Let $P(x, y)$ be a point on the terminal side of this angle. Let r be the length of the segment OP, i.e., $r = \sqrt{x^2 + y^2}$. Now, we define four functions:

$$\sin\theta = \frac{y}{r}, \ \cos\theta = \frac{x}{r}$$

$$\tan\theta = \frac{y}{x}, \ \cot\theta = \frac{x}{y}$$

θ	radians	$\sin\theta$	$\cos\theta$	$\tan\theta$
0^o	0	0	1	0
30^o	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
45^o	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
60^o	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
90^o	$\frac{\pi}{2}$	1	0	∞

$$\begin{array}{lll} \cos(\frac{7\pi}{6}) & = & -\cos(\frac{\pi}{6}) = -\frac{\sqrt{3}}{2} \\ \sin(\frac{3\pi}{4}) & = & \sin(\frac{\pi}{4}) = \frac{\sqrt{2}}{2} \\ \sin(\frac{5\pi}{4}) & = - & \sin(\frac{\pi}{4}) = -\frac{\sqrt{2}}{2} \end{array}$$

3. Trigonometric Identities:

$$\sec x = \frac{1}{\cos x}, \quad \csc x = \frac{1}{\sin x}$$
$$\tan x = \frac{\sin x}{\cos x}, \quad \cot x = \frac{\cos x}{\sin x}$$
$$\sin^2 x + \cos^2 x = 1$$
$$\sin(2x) = 2\sin x \cos x$$
$$\cos(2x) = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$$
$$\sec^2 x - \tan^2 x = 1$$
$$\csc^2 x - \cot^2 x = 1$$
$$\cos^2 x = \frac{1 + \cos(2x)}{2}$$
$$\sin^2 x = \frac{1 - \cos(2x)}{2}$$

All previous angles are in radians.

4. Extra Properties of Trigo Functions:

$$\sin(x \pm 2\pi) = \sin x$$
$$\cos(x \pm 2\pi) = \cos x$$
$$\sin(-x) = -\sin x$$
$$\cos(-x) = \cos x$$
$$\sin(\frac{\pi}{2} - x) = \cos x$$
$$\cos(\frac{\pi}{2} - x) = \sin x$$
$$\sin(\pi - x) = \sin x$$
$$\cos(\pi - x) = -\cos x$$
$$\sin(\pi + x) = -\sin x$$
$$\cos(\pi + x) = -\cos x$$

$$\sin(A+B) = \sin A \cdot \cos B + \cos A \cdot \sin B$$

$$\cos(A+B) = \cos A \cdot \cos B - \sin A \cdot \sin B$$

$$\sin(A-B) = \sin A \cdot \cos B - \cos A \cdot \sin B$$

$$\cos(A-B) = \cos A \cdot \cos B + \sin A \cdot \sin B$$

5. Limits of Trigo Functions:

$$\lim_{\theta \to 0} \frac{\sin \theta}{\theta} = 1$$

$$\lim_{\theta \to 0} \frac{\cos \theta - 1}{\theta} = 0$$

6. Derivatives of Trigo Functions:

$$(\sin x)' = \cos x$$

$$(\cos x)' = -\sin x$$