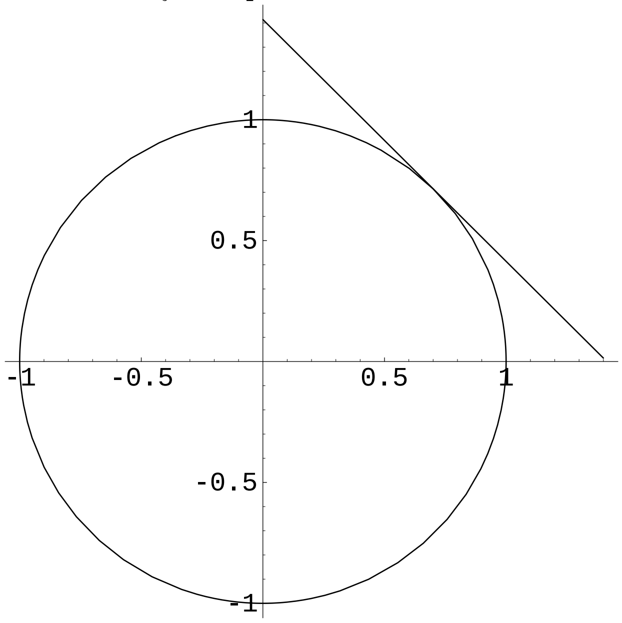
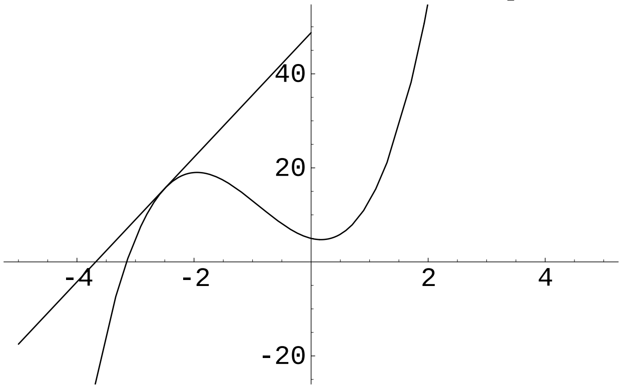
2 The tangent problem

We motivate the tangent problem not from real-world applications but rather from our previous mathematical understanding. Everyone knows that a tangent line to a circle is a line which touches the circle at only one point.



You will see as we go along that we will have a bit of trouble determining a precise definition of the tangent line to the graph of some function which is not part of a circle. But for the moment we can take our intuitive notion of a line touching the curve at only one point and picture what a tangent line to the function would be in the picture below.



Stated in its simplest form the tangent problem is to find the tangent line to the graph of a function $y = f(x)$ at some point $x = c$. If you aren't immediately fascinated by this problem you are proably asking yourself why you would care to do such a thing. Before we answer that question we remind you of something you already know about tangent lines and lines in general: a line is uniquely determined by one point and a slope. If we are given a point

(x_0, y_0) and a slope m recall that the point slope formula: $y = m(x - x_0) + y_0$ gives the equation of the line through this point (x_0, y_0) having slope m. (You may have seen this before in the form $y - y_0 = m(x - x_0)$, but of course the two are equivalent. Our form of the equation puts it in $y = f(x)$ form.) As we stated the tangent problem above, "find the tangent line to the graph of the function $f(x)$ at the point $x = c$", note that we know the point to be $(c, f(c))$, and what remains for us to do is to find the slope. So we can restate the tangent problem as follows: "Find the slope of the tangent line to the graph of the function $f(x)$ at the point $x = c$".

As we mentioned in the introduction there are real-world reasons why someone might care about the tangent problem – it is not merely a geometric curiosity. For you to fully appreciate these you will need to at least somewhat understand the method we are going to use to solve the tangent and area problems. With this understanding the reasons why you would be interested in the solution to the tangent problem will be evident, but for now you will have to take our word for it and excuse us if we delay discussion for the motivation for the tangent problem until later.