LECTURE 21-23

Date: Nov 23, 2004

1. Maximum and Minimum Values

Let $f(x)$ be a function.

Definition:

A function f has an *absolute maximum* (or global maximum) at c if $f(c) \geq f(x)$ for all x in D, where D is the domain of f, and the number $f(c)$ is called the maximum value of f.

Similarly, f has an *absolute minimum* (global minimum) at c if $f(c) \leq f(x)$ for all x in D, and the number $f(c)$ is called the minimum value of f.

The maximum and the minimum value of f are called the <u>extreme values</u> of f.

Definition:

A function f has a <u>*local maximum*</u> (or *relative maximum*) at c if $f(c) \geq f(x)$ when x is near c (for x sufficiently close to c on both sides of c, or for all x in some open interval containing c).

Similarly, f has an <u>*local minimum*</u> (or *relative minimum*) at c if $f(c) \leq f(x)$ when x is near c (for x sufficiently close to d on both sides of c, or for all x in some open interval containing c).

The Extreme Value Theorem:

If f is continuous on a closed interval $[a, b]$, then f attains an absolute maximum value $f(c)$ and an absolute minimum value $f(d)$ at some numbers c and d in $[a, b]$.

Fermat's Theorem:

If f has a local maximum or minimum at c, and if $f'(c)$ exists, then $f'(c) = 0$.

Definition:

A *critical number* (or critical point) of a function f is a number c in the domain of f such that either $f'(c) = 0$ or $f'(c)$ does not exist.

2. Find Extreme Values of a continuous function on a closed interval $[a, b]$

If f is a continuous function on a closed interval $[a, b]$, then f attains an absolute maximum value $f(c)$ and an absolute minimum value $f(d)$ at some numbers c and d in $[a, b]$.

To find the absolute maximum and minimum values of a continuous function f on a closed interval $[a, b]$:

(1) **Find the values of f at the critical numbers of f in (a, b).**

(2) **Find the value of f at the endpoints of the interval, i.e., evaluate $f(a)$ and $f(b)$.**

(3) **The largest of the values from step 1 and 2 is the absolute maximum value, the smallest of these values is the absolute minimum value.**

Rolle's Theorem:

Let f be a function that satisfies the following three hypothesis:

(1.) f **is continuous on the closed interval** $[a, b]$**;**

(2.) f **is differentiable on the open interval** (a, b)**;**

(3.) $f(a) = f(b)$

Then there is a number c in (a, b) such that $f'(c) = 0$.

The intermediate Value Theorem:

Suppose that f **is continuous on the closed interval** $[a,]$ **and let** N **be any number between** $f(a)$ **and** $f(b)$**, where** $f(a) \neq f(b)$**. Then there exists a number** c **in** (a, b) **such that** $f(c) = N$**.**

The Mean Value Theorem:

Let f be a function that satisfies the following three hypothesis:

(1.) f **is continuous on the closed interval** $[a, b]$**;**

(2.) f **is differentiable on the open interval** (a, b)**;**

Then there is a number c in (a, b) such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

or equivalently,

$$f(b) - f(a) = f'(c)(b - a)$$

Theorem:

If $f'(x) = 0$ **for all** x **in an interval** (a, b)**, then** f **is constant on** (a, b)**.**

Corollary:

If $f'(x) = g'(x)$ **for all** x **in an interval** (a, b)**, then** $f - g$ **is constant on** (a, b)**; that is,** $f(x) = g(x) + c$ **where** c **is a constant.**

Definition:

Let f be a function defined on an interval I, and x_1, x_2 in I.

($\bullet$) **f is said to be increasing on an interval I if $f(x_1) \leq f(x_2)$ whenever $x_1 < x_2$;**

($*$) **f is said to be decreasing on an interval I if $f(x_1) \geq f(x_2)$ whenever $x_1 < x_2$;**

Increasing and Decreasing Test:

(a.) If $f'(x) > 0$ on an interval, then f is increasing on that interval.

(b.) If $f'(x) < 0$ on an interval, then f is decreasing on that interval.

The First Derivative Test

Let c be a critical number of a continuous function f.

(1) If $f'(x)$ changes from positive to negative at c, i.e.,

$$\textbf{If} \quad \begin{cases} f'(x) > 0 \textbf{ for } x \in (a, c) \\ f'(x) < 0 \textbf{ for } x \in (c, b) \end{cases}$$

then $f(x)$ has a local maximum value at c.

(2) If $f'(x)$ changes from negative to positive at c, i.e.,

$$\textbf{If} \quad \begin{cases} f'(x) < 0 \textbf{ for } x \in (a, c) \\ f'(x) > 0 \textbf{ for } x \in (c, b) \end{cases}$$

then $f(x)$ has a local minimum value at c.

(3) If $f'(x)$ does not chenge sign at c, then $f(x)$ has neither local maximum or local minimum value at c.

Definition:

IF the graph of f lies above all of its tangents on an interval I, then it is called concave upward on I. If the graph of f lies below all of its tangents on I, it is called concave downward on I.

Definition:

A point P on a curve is called an *inflection point* if the curve changes from concave up to concave down, or vice versa, from concave down to concave up at P.

To locate possible points of inflection, we need only determine the values of x for which $f''(x) = 0$, or for which $f''(x)$ is undefined.

Second Derivative Test For Concavity

Let $f(x)$ **be a function that has second derivative in an interval** I

(1) If $f''(x) > 0$ **on** I**, then** $f(x)$ **is *concave up* on** I**.**

(2) If $f''(x) < 0$ **on** I**, then** $f(x)$ **is *concave down* on** I**.**

Second Derivative Test For Local Extrema

Suppose that $f''(x)$ is continuous near c.

(1) If $f'(c) = 0$ and $f''(c) > 0$, then f has a *local minimum* at c;

(2) If $f'(c) = 0$ and $f''(c) < 0$, then f has a *local maximum* at c;

(3) If $f'(c) = 0$ and $f''(c) = 0$, then test does not work.