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Abstract

The collapse of housing prices in the aftermath of the U.S. subprime mortgage crisis of 2008 not only worsened the balance sheet positions of the banking sector but also led to a "bank run" in some cases such as the collapse of Lehman Brothers in September 2008. We develop a theoretical model featuring household debt (mortgages) and banking sector frictions. We show that mortgage risks can potentially lead to a bank run equilibrium. Such an equilibrium exists since mortgage risks reduce the liquidation prices of bank assets. We further show that mortgage market regulations such as loan-to-value requirements reduce the likelihood of bank runs.

JEL: classification: E32; E44; G01, G21, G33

Keywords: bank run, mortgage risk, loan-to-value ratio

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1 Introduction

One prominent feature of the 2008-09 financial crisis in the United States was the close connection between the housing market bust and banking panics. When housing prices started to collapse in 2007, the value of mortgage-backed securities (MBS) experienced a steep decline. The balance sheet positions of banks holding large amounts of MBS worsened considerably. This led to "bank runs" in some cases. The collapse of Bear Sterns and the bankruptcy of Lehman Brothers are but two well-known events in a string of bank runs that took place in 2007-2008 (e.g., Gorton 2010).

In this paper, we capture the link between mortgage loans and bank panics using a highly stylized theoretical model built upon Gertler and Kiyotaki (2015) (GK hereafter). GK provides a framework to study financial panics in an infinite horizon environment.[1] Banks in their model hold one type of asset, the capital claims issued by firms. Holding these assets is risky since the return to such claims is subject to shocks. When a large negative shock hits the economy, the return on the capital claims (and their price) fall, worsening banks' balance sheet conditions. The shock can give rise to a self-fulfilling bank-run equilibrium – a household stops rolling over its deposits since it believes that other households will not roll over theirs, and if the household keeps its deposits with the banks, it will lose them. If all households stop rolling over their deposits, then the banks would need to liquidate some assets on their balance sheets. A bank run happens if the liquidation value of the assets falls below the value of a bank's liabilities.

Our objective is to study bank runs caused directly by the risks related to mortgages. To do so, we extend GK by introducing mortgage claims. We allow banks to hold two types of assets: capital claims (as in GK) and mortgage claims. Mortgage claims are issued by households. Like capital claims issued by firms, mortgage claims are risky since their return is contingent. A negative shock to the mortgage claims' return leads to a decline of their price, worsening banks' balance sheets. The mortgage debt return being contingent is the key to generating bank runs. In the literature, household debt is typically modeled as a non-contingent contract. Without default, a non-contingent contract guarantees banks the return of their lending. In this environment, a shock hitting the household sector is unlikely to lead to a bank run. In our paper, on the other hand, although default is not explicitly modelled, since household debt is contingent, a decline in return can be thought of as a reduced-form way of capturing the rise in the default rate of mortgage debt.[2] In addition, the downfall in asset prices during a run in the model mirrors the collapse of the mortgage-backed securities during the financial crisis.[3]

We conduct numerical exercises using a perfect-foresight model for both the bank run and no-

[1] Similar to Diamond and Dybvig (1983), in GK the fundamental reason for bank runs to occur is liquidity mismatch – banks hold long-term assets which are financed by short-run liabilities. However, GK build an infinite horizon model while Diamond and Dybvig (1983) feature a three-period environment. In terms of technical approach, GK is more closely related to Cole and Kehoe (2000), which models self-fulfilling debt crises.

[2] When the contracts are non-contingent, the return is pre-determined and the loss of the banks is from the loan loss due to mortgage default. In our setup, the loss for banks is captured by a negative shock to the realized return instead of a shock to loans themselves.

[3] Modelling the long-term nature of mortgage debt and debt securitization is beyond the scope of our stylized model.

run cases. We find that an adverse mortgage risk shock can lead to a bank run. During panics, banks liquidate both capital and mortgage assets at extremely low prices. The probability of a bank run becomes positive due to the low liquidation prices. Different from the case of a technology shock, which hits the return on capital and affects the total endowment of the economy, the chance of having a bank run is lower in the case that the economy is hit by a mortgage shock (which only affects the return on household loans). However, once the economy is in the run equilibrium, the severity of the negative impact on the economy from a mortgage shock is similar to a technology shock. We also find that there is a significant spillover effect in both the run and no-run cases. For example, in both cases, a negative shock to the return of one asset increases the risk premium and depresses the price of the other asset.

Linking the mortgage debt risk with the possibility of a bank run has important policy implications, particularly for mortgage loan regulations. To explore this, we use our model to study the effect of changing the loan-to-value (LTV) ratio. For the no-run case, we find a conventional financial accelerator effect: a higher LTV ratio tends to amplify the shocks. For the run case, we show that a tightened LTV ratio reduces the probability of a bank run for both types of shock. This suggests that policy makers need to take into account the asymmetric nature of LTV regulation: a loose regulation might lead in adverse circumstances to a bank panic and consequently to a considerably larger decline in output.

Our paper is related to the following strands of literature. The first is the literature studying the role of frictions in models with financial intermediaries and business cycles (see, for example, Gertler and Kiyotaki 2010; Gertler and Karadi 2011). Our paper is particularly related to the recent development in the literature that links the banking sector financial frictions to financial panics (GK; Gertler, Kiyotaki, and Prestipino 2016, 2020a, 2020b, and 2020c). Gertler, Kiyotaki, and Prestipino (2016) extend GK to include a wholesale funding market to capture its role in the Great Recession; Gertler, Kiyotaki, and Prestipino (2020a) incorporate bank runs à la GK into a conventional macroeconomic framework to quantify the impact of bank panics on the aggregate economy; Gertler, Kiyotaki, and Prestipino (2020c) focus on the role of macroprudential tools in preventing bank panics. Our contribution is that we introduce household borrowing to the GK framework. With this, we are able to address the question of how a shock hitting the household sector can lead to a bank run.

The second direction in the literature examines the role of household debt in business cycle fluctuations (e.g., Iacoviello 2005; Iacoviello and Neri 2010; Justiniano, Primiceri and Tambalotti 2015; Forlati and Lambertini 2011). Our paper is particularly related to Ferrante (2019), which has a banking sector with financial frictions similar to those in GK; the banks in this model have both business assets and household debt on their balance sheets. However, bank panics are not addressed in Ferrante (2019).

The rest of the paper is organized as follows. In Section 2 we present the model. Section 3 discusses the bank run equilibrium. Section 4 presents numerical examples and describes how the model works. In Section 5 we discuss policy implications and in Section 6 we offer some concluding remarks.

2 The Model

2.1 Households

There is a continuum of infinitely lived households. They consist of two types, patient and impatient ones, which differ only by the rate at which they discount the future. Patient households are denoted by p, and impatient ones by m. For simplicity we assume that there is a measure one of each type. The discount factors for the patient and impatient households are denoted as β^p and β^m respectively, with $\beta^p > \beta^m$. In the model, the impatient households who discount the future at a higher rate become borrowers whereas the patient households become lenders. Impatient households can borrow either from the patient ones or from banks. We assume that the patient households are less efficient in making loans than the banks.

There is one non-durable commodity, consumption goods, and two durable goods, capital K and housing H. We assume away depreciation. The total stocks of capital and housing are normalized to 1. Capital K can be held by both patient households and banks; the respective quantities are denoted by K_t^p and K_t^b, with

$$K_t = K_t^b + K_t^p = 1. \tag{1}$$

Housing can be held by both patient and impatient households. The total housing stock is

$$H_t = H_t^m + H_t^p = 1. \tag{2}$$

The non-durable goods are produced by using capital. The production function is linear:

$$F(K) = Z_{t+1}^k K. \tag{3}$$

That is, one unit of capital produces Z_{t+1}^K units of non-durable goods. Both patient households and banks [4] have access to this technology. However, households are less efficient, and their production incurs additional costs $f(K_t^p)$, which are the consumption goods used in period t for producing $Z_{t+1}^K K_t^p$, where

$$f(K_t^p) = \frac{\gamma^k}{2}(K_t^p)^2. \tag{4}$$

Both patient and impatient households derive utility from consuming non-durable goods and housing.

[4] More precisely, it is entrepreneurs who produce using funds borrowed from banks.

2.1.1 Patient households' problem

The expected life-time utility for each patient household is

$$E_0 \sum_{t=0}^{\infty} \beta^p u(C_t^p, H_t^p), \tag{5}$$

where $u(C_t^p, H_t^p) = \log C_t^p + \eta \log H_t^p$. Here C_t^p and H_t^p denote the consumption of non-durable goods and housing consumption respectively, and the weight η indicates the relative importance of housing in the utility function. The household faces the following budget constraint:

$$\begin{aligned} & C_t^p + D_t + Q_t^K K_t^p + f(K_t^p) + Q_t^h H_t^p + Q_t^L L_t^p + g(L_t^p) \\ \leq \; & Z_t^K W_t^p + R_t D_{t-1} + (Z_t^K + Q_t^K) K_{t-1}^p + (Z_t^L + Q_t^L) L_{t-1}^p + Q_t^h H_{t-1}^p, \end{aligned} \tag{6}$$

where D_t is the patient household's deposits in the banking sector, R_t the risk-free rate, and K_t^p the capital stock held by the patient household. Q_t^h is the price of housing. The patient household also holds another type of asset, L_t^p, which can be thought of as claims issued by the impatient households (essentially, these are loans to impatient households). The patient household buys this type of assets at price Q_t^L at time t. At time $t+1$, the patient household receives the return on the claims, Z_{t+1}^L. When making loans, the patient household is less efficient than the banks, incurring an additional costs of $g(L_t^p)$ in terms of non-durable goods, where

$$g(L_t^p) = \frac{\gamma^l}{2} (L_t^p)^2. \tag{7}$$

The first-order conditions are:

C_t^p :

$$u_{c,t}^p = u_C(C_t^p, H_t^p) = \lambda_t^p, \tag{8}$$

D_t :

$$\lambda_t^p = \beta^p E_t R_{t+1} \lambda_{t+1}^p, \tag{9}$$

which can be rewritten as

$$E_t \Lambda_{t,t+1}^p R_{t+1} = 1, \tag{10}$$

where $\Lambda_{t,t+1}^p = \beta^p E_t \frac{\lambda_{1,t+1}^p}{\lambda_{1,t}^p} = \beta^p \frac{u_{c,t+1}^p}{u_{c,t}^p}$.

K_t^p :

$$\beta^p E_t \lambda_{t+1}^p (Z_{t+1}^K + Q_{t+1}^K) = (Q_t^K + f'(K_t^p)) \lambda_t^p. \tag{11}$$

Defining $R_{t+1}^{K,p} = \frac{Z_{t+1}^K + Q_{t+1}^K}{Q_t^K + f'(K_t^p)}$, the above equation can be rewritten as

$$E_t \Lambda_{t,t+1}^p R_{t+1}^{K,p} = 1. \tag{12}$$

L_t^p :

$$\beta^p E_t \lambda_{t+1}^p (Z_{t+1}^L + Q_{t+1}^L) = \lambda_t^p (Q_t^L + g'(L_t^p)). \tag{13}$$

Defining $R_{t+1}^{L,p} = \frac{Z_{t+1}^L + Q_{t+1}^L}{Q_t^L + g'(L_t^p)}$, the above equation can be rewritten as

$$E_t \Lambda_{t,t+1}^p R_{t+1}^{L,p} = 1. \tag{14}$$

H_t^p :

$$\frac{u_{H,t}^p}{u_{c,t}^p} = Q_t^h - E_t \Lambda_{t,t+1}^p Q_{t+1}^h, \tag{15}$$

where

$$u_{H,t}^p = \frac{\eta}{H_t^p}. \tag{16}$$

2.1.2 Impatient households' problem

The impatient household maximizes the following expected lifetime utility:

$$E_0 \sum_{t=0}^{\infty} \beta^m u(C_t^m, H_t^m), \tag{17}$$

where

$$u(C_t^m, H_t^m) = \log C_t^m + \eta \log H_t^m. \tag{18}$$

The impatient households do not hold capital; however, they can issue claims to either patient households or banks (let us denote these amounts by L_t^p and L_t^b respectively); and the total amount of claims is

$$L_t = L_t^p + L_t^b. \tag{19}$$

The price of the claims is denoted as Q_t^L, and the return on the claims as Z_t^L. The budget constraint of each impatient household is

$$C_t^m + Q_t^h H_t^m + (Z_t^L + Q_t^L) L_{t-1} \leq Z_t W_{m,t} + Q_t^L L_t + Q_t^h H_{t-1}^m. \tag{20}$$

We assume that each patient household is subject to a collateral constraint:

$$Q_t^L L_t \leq \theta E_t Q_{t+1}^h H_t^m, \tag{21}$$

where θ is the loan-to-value (LTV) ratio. We assume that $0 \leq \theta \leq 1$.

The first-order conditions are:

C_t^m :

$$\lambda_{1,t}^m = u_{C,t}^m = \frac{1}{C_t^m}, \tag{22}$$

L_t :

$$\lambda_{1,t}^m Q_t^L = \beta^m E_t \lambda_{1,t+1}^m (Z_{t+1}^L + Q_{t+1}^L) + \lambda_{2,t}^m Q_t^L, \tag{23}$$

where $\lambda_{2,t}^m$ is the Lagrangian multiplier for the collateral constraint.

H_t^m :

$$u_{H,t}^m + \beta^m E_t \lambda_{1,t+1}^m Q_{t+1}^h + \theta \lambda_{2,t}^m E_t Q_{t+1}^h = Q_t^h \lambda_{1,t}^m, \tag{24}$$

where

$$u_{H,t}^m = \frac{\eta}{H_t^m}. \tag{25}$$

Using the above equations, we obtain

$$u_{H,t}^m + \beta^m E_t u_{C,t+1}^m Q_{t+1}^h + \left(u_{C,t}^m - \beta^m E_t u_{C,t+1}^m \frac{Z_{t+1}^L + Q_{t+1}^L}{Q_t^L} \right) \theta E_t Q_{t+1}^h = Q_t^h u_{C,t}^m. \tag{26}$$

2.2 Financial intermediaries

There is a continuum of banks of measure 1. Note that we use lowercase letters for individual bank variables and capital letters for their aggregate counterparts. Each bank funds capital investment of the business borrowers and makes loans to household borrowers by issuing deposits to patient households and using their own net worth. More precisely, in each period, bank j's assets a_{jt} are financed by deposits d_{jt} and bank's own net worth n_{jt}:

$$a_{jt} = d_{jt} + n_{jt}. \tag{27}$$

The assets consist of capital assets and loans to impatient households, l_{jt}^b:

$$a_{jt} = Q_t^K k_{jt}^b + Q_t^L l_{jt}^b. \tag{28}$$

Bank j maximizes

$$V_{jt} = \max E_t \{ \Lambda_{t,t+1}^p ((1-\sigma) n_{jt+1} + \sigma V_{jt+1}) \} \tag{29}$$

subject to the flow of funds condition

$$a_{jt} = Q_t^K k_{jt}^b + Q_t^L l_{jt}^b = d_{jt} + n_{jt}. \tag{30}$$

Note that we assume that banks use the patient households' discount factor. Define

$$R_{jt}^{K,b} = \frac{Z_t^K + Q_t^K}{Q_{t-1}^K}, \tag{31}$$

and

$$R_{jt}^{L,b} = \frac{Z_t^L + Q_t^L}{Q_{t-1}^L}. \tag{32}$$

The law of motion for net worth is

$$n_{jt+1} = R^{K,b}_{t+1} Q^K_t k^b_{jt} + R^{L,b}_t Q^L_t l^b_{jt} - R_t d_{jt}, \tag{33}$$

and the incentive constraint is

$$V_{jt} \geq \kappa^k Q^K_t k^b_{jt} + \kappa^l_t Q^L_t l^b_{jt}. \tag{34}$$

Substituting the flow of funds condition to the net worth equation, we have

$$n_{jt+1} = (R^{K,b}_{t+1} - R_t) Q^K_t k^b_{jt} + (R^{L,b}_{t+1} - R_t) Q^K_t l^b_{jt} + R_t n_{jt}. \tag{35}$$

Define R^B_{t+1}, the weighted average return on total bank assets, as

$$R^B_{t+1} = R^{K,b}_{t+1} \frac{Q^K_t k^k_{jt}}{a_{jt}} + R^{L,b}_{t+1} \frac{Q^L_t l^b_{jt}}{a_{jt}}.$$

Define the bank's leverage as

$$\phi_{jt} = \frac{a_{jt}}{n_{jt+1}}.$$

The net worth equation can be further written as

$$n_{jt+1} = E_t[(R^B_{t+1} - R_t)\phi_{jt} + R_t] n_{jt}.$$

Using the undetermined coefficients method, we guess that the bank's value function is a linear function of net worth:

$$V_{jt} = \mu_{n,t} n_{jt}, \tag{36}$$

where $\mu_{n,t}$ can be thought of as the expected discounted marginal gain of having one more unit of net worth. We also guess that

$$\phi_t = \phi_{jt} = \frac{a_{jt}}{n_{jt+1}}.$$

Appendix A shows that

$$\begin{aligned} & \frac{E_t \Lambda^p_{t,t+1}[((1-\sigma) + \sigma\mu_{n,t+1})(R^{K,b}_{t+1} - R_t)]}{\kappa^k} \\ = \quad & \frac{E_t \Lambda^p_{t,t+1}[((1-\sigma) + \sigma\mu_{n,t+1})(R^{L,b}_{t+1} - R_t)]}{\kappa^l_t}. \end{aligned} \tag{37}$$

Equation (37) implies that the expected discounted return from holding capital assets must equal the expected discounted return from holding mortgage assets, after adjusting for the diversion rates difference. This gives us a spillover effect – a rise in risk premium of one of the assets, for example, $R^{K,b}_{t+1} - R_t$, will lead to a rise in the other, $R^{L,b}_{t+1} - R_t$.

When the incentive constraint binds, the following equation holds:

$$\mu_{n,t} n_{jt} = \kappa^k Q_t^K k_{jt}^b + \kappa_t^l Q_t^L l_{jt}^b. \tag{38}$$

Define $\kappa_t^B = \kappa^k \frac{Q_t^K k_{jt}^b}{a_{jt}} + \kappa_t^l \frac{Q_t^L l_{jt}^b}{a_{jt}}$; we then can obtain this:

$$\mu_{n,t} = \kappa^B \frac{a_{jt}}{n_{jt}} = \phi_t \kappa_t^B. \tag{39}$$

Thus,

$$\phi_t = \frac{\mu_{n,t}}{\kappa_t^B}. \tag{40}$$

Equation (40) suggests that (i) the bank leverage is endogenously determined; and (ii) this leverage decreases as κ_t^B increases.

2.3 Aggregation and resource constraints

Since all banks face the same problem, their behavior will be identical. Therefore we will omit the subscript j in our exposition below. At the aggregate level, we have

$$Q_t^K K_t^b + Q_t^L L_t^b = \phi_t N_t, \tag{41}$$

with

$$N_t = N_t^e + N_t^n, \tag{42}$$

where N_t^e and N_t^n denote the aggregate net worth of the existing and newly entering banks respectively.

Using

$$N_t^e = \int_0^1 N_{jt}\, dj$$

and

$$N_t^n = \omega^b, \tag{43}$$

we have the law of motion of the aggregate net worth:

$$\begin{aligned} N_t &= N_t^e + N_t^n && (44)\\ &= \sigma\left[(Z_t^K + Q_t^K)K_{t-1}^b + (Z_t^L + Q_t^L)L_{t-1}^L - R_t D_{t-1}\right] + (1-\sigma)\omega^b && (45)\\ &= \sigma N_t^e + W^b, && (46)\end{aligned}$$

with

$$W^b = (1-\sigma)\omega^b. \tag{47}$$

The exiting banks simply consume their net worth:

$$\begin{aligned} C_t^b &= (1-\sigma)N_t^e && (48)\\ &= (1-\sigma)\left[(Z_t^K+Q_t^K)K_{t-1}^b+(Z_t^L+Q_t^L)L_{t-1}^b-R_tD_{t-1}\right] && (49)\\ &= \frac{1-\sigma}{\sigma}(N_t-W^b). && (50) \end{aligned}$$

Define the aggregate output as

$$Y_t = Z_t^K + Z_t^K(W^p+W^m)+W^b. \tag{51}$$

The output is demanded by households and banks, i.e.

$$Y_t = f(K_t^p)+g(L_t^p)+C_t^p+C_t^m+C_t^b. \tag{52}$$

The amount of total loans is

$$L_t^p + L_t^b = L_t. \tag{53}$$

Similarly, the housing supply and capital supply equations are

$$H_t^p + H_t^m = 1, \tag{54}$$

and

$$K_t^p + K_t^b = 1. \tag{55}$$

3 Bank Runs

In this section we consider an unanticipated bank run. As in GK, we assume that when patient households make deposits at $t-1$ that mature in t, they assign a probability of zero to a run at time t. However, a run can happen ex post as follows. When deposits mature at time t, the patient households must decide whether to roll the deposits over for another period. If an individual household believes that the other households will not roll over the deposits and thus itself decides not to roll over its own deposits, then this becomes a self-fulfilling prophecy. (A certain condition should hold for existence of such an equilibrium; it will be discussed later in the paper.) Then the banks will be forced into liquidation. They will need to liquidate both capital and mortgage claims and turn the proceeds over to patient households.

Impatient households take loans at $t-1$, which mature in period t. Similarly, they assign a probability of zero to a run at time t. When a run happens ex post, it is assumed that the impatient households still repay their debts to the banks but the banks have to liquidate their assets related to mortgage debt, which will be acquired by the patient households.

Thus, as in GK, there are two equilibria: a "normal" one where patient households roll over their deposits; and a "run" equilibrium where patient households stop rolling over their deposits. In the event of a run, banks' net worth goes to zero, and the patient households hold the entire capital stock and mortgage claims. In what follows we describe the conditions under which a bank run can exist and the liquidation prices for the two assets in the event of a run.

3.1 Conditions for a bank run equilibrium

We consider a run on the entire banking system. After the realization of a negative shock to Z_t^K or Z_t^L, if depositors decide not to roll over their deposits, banks have to liquidate their assets. To distinguish the run case from the no-run one, we use $*$ to indicate the case where depositors stop rolling over deposits. For example, Q_t^{K*} and Q_t^{L*} are the asset prices of capital and mortgage claims when the banks have to liquidate assets. Quantities Q_t^{K*} and Q_t^{L*} have to be low enough to support a run equilibrium in which the liquidation value of bank assets $(Z_t^K + Q_t^{K*})K_{t-1}^b + (Z_t^L + Q_t^{L*})L_{t-1}^b$ is lower than the value of its liabilities $R_t D_{t-1}$. (And this is the condition we referred to earlier.) When a run happens, the banks return the proceeds from liquidation back to patient households (depositors). This means that the banks are left with zero equity. We define a recovery rate x_t as

$$x_t = \frac{(Z_t^K + Q_t^{K*})K_{t-1}^b + (Z_t^L + Q_t^{L*})L_{t-1}^b}{R_t D_{t-1}}. \tag{56}$$

For convenience, define a variable run_t as follows

$$run_t = 1 - x_t. \tag{57}$$

The value of run indicates a possibility of having a bank run. A run equilibrium exists when $run_t > 0$.

Let us describe the conditions that will hold when a run occurs. First, the patient households stop rolling over deposits, $D_t^* = 0$. Banks liquidate both types of assets, $K_t^{b*} = 0$ and $L_t^{b*} = 0$, and hold zero equity, $N_t^* = 0$. Patient households hold the entire stock of capital, $K_t^{p*} = 1$, and become the only source from which impatient households can borrow, $L_t^{p*} = L_t^*$.

The aggregate return for banks at the time of a run is

$$\begin{aligned} R_t^{B*} &= \frac{(Z_t^K + Q_t^{K*})K_{t-1}^b + (Z_t^L + Q_t^{L*})L_{t-1}^b}{Q_{t-1}^K K_{t-1}^b + Q_{t-1}^L L_{t-1}^b} & (58) \\ &= \frac{Q_{t-1}^K K_{t-1}^b}{Q_{t-1}^K K_{t-1}^b + Q_{t-1}^L L_{t-1}^b} R_t^{K,b*} + \frac{Q_{t-1}^L L_{jt-1}^b}{Q_{t-1}^K K_{jt-1}^b + Q_{t-1}^L L_{t-1}^b} R_t^{L,b*}, & (59) \end{aligned}$$

where $R_t^{K,b*} = \frac{Z_t^K + Q_t^{K*}}{Q_{t-1}^K}$ and $R_t^{L,b*} = \frac{Z_t^L + Q_t^{L*}}{Q_{t-1}^L}$. Using equations $Q_{t-1}^K K_{t-1}^b + Q_{t-1}^L L_{t-1}^b = \phi_{t-1} N_{t-1} = A_{t-1}$, and $A_{t-1} = D_{t-1} + N_{t-1}$, equation (56) can be rewritten as

$$x_t = \frac{R_t^{B*}}{R_t} \frac{1}{1 - \frac{1}{\phi_{t-1}}}. \tag{60}$$

Equation (60) implies that a bank run becomes possible when R_t^{B*} is low or leverage ϕ_{t-1} is high. There are two potential reasons for a low value of R_t^{B*} and high value of ϕ_{t-1}: one stems from low capital returns and the other from low returns on mortgage claims.

After the run, new banks enter the industry and the banking system starts to rebuild itself. As in GK, we assume that new banks cannot immediately start their operations – there is a short delay. Although the new banks start to enter at $t+1$, they only start to operate from period $t+2$. Banks' net worth is

$$N_{t+1} = W^b + \sigma W^b, \tag{61}$$

and

$$N_{t+i} = \sigma \left[(Z_t^K + Q_t^K) K_{t-1}^b + (Z_t^L + Q_t^L) L_{t-1}^b - R_t D_{jt-1} \right] + W^b, \quad i \geq 2. \tag{62}$$

At the time of the run t, the output produced equals

$$Y_t = Z_t^K + Z_t^K (W^p + W^m), \tag{63}$$

whereas the output demanded by households (banks' consumption is zero) is

$$\begin{aligned} Y_t &= f(K_t^{p*}) + g(L_t^{p*}) + C_t^{p*} + C_t^{m*} \\ &= f(1) + g(L_t^*) + C_t^{p*} + C_t^{m*}. \end{aligned} \tag{64}$$

3.2 The liquidation price

In this section we discuss how the liquidation prices Q_t^{K*} and Q_t^{L*} are determined. When a run happens at time t, before going out of business banks need to fully liquidate their assets carried from time $t-1$. They sell all their assets to patient households at time t. The liquidation prices Q_t^{K*} and Q_t^{L*} are determined by the first-order conditions for patient households by imposing the run conditions $K_t^{p*} = 1$ and $L_t^{p*} = L_t^*$. Using the first-order conditions for the patient households, we can show that [5]

$$\begin{aligned} Q_t^{K*} &= E \sum_{i=1}^{\infty} \Lambda_{t,t+i}^p \Big[Z_{t+i}^K - f'(K_{t+i}^p) \Big] - f'(K_t^{p*}) \\ &= E \sum_{i=1}^{\infty} (\beta^p)^i \frac{u_{c,t+i}^p}{u_{c,t}^{p*}} \Big[Z_{t+i}^K - \gamma^k K_{t+i}^p \Big] - \gamma^k. \end{aligned} \tag{65}$$

It is more likely to have a large reduction in Q_t^{K*} when there is a large negative shock to Z_t^K and the adjustment cost parameter γ^k is large. How low the liquidation price Q_t^{K*} could go is also related to the value of $u_{c,t}^{p*}$. A large decline in C_t^{p*} at the time of the run tends to lead to a low liquidation price.

[5] See Appendix B for derivation details.

Similarly, we can show that

$$\begin{aligned} Q_t^{L*} &= E\sum_{i=1}^{\infty}\Lambda_{t,t+i}^{p}[Z_{t+i}^{L}-g'(L_{t+i}^{p})]-g'(L_t^{p*}) \\ &= E\sum_{i=1}^{\infty}(\beta^p)^i\frac{u_{c,t+i}^{p}}{u_{c,t}^{p*}}[Z_{t+i}^{L}-\gamma^l L_{t+i}^{p}]-\gamma^l L_t^{*}. \end{aligned} \tag{66}$$

Equation (66) suggests that a large decline in Z_{t+i}^{L} will lead to a large decline in Q_t^{L*}. A high debt level L_{t+i}^{p} and high adjustment costs γ^l will also contribute to the low liquidation price. As in the case with Z_t^K, a large decline in C_t^{p*} at the time of the run tends to lead to a low liquidation price.

4 Numerical Examples and Workings of the Model

In this section, we present some numerical examples to illustrate the nuts and bolts of the model. We first present the cases where a bank run does not happen; and then we present the bank run cases.

4.1 Parameters and steady state values

Table 1 displays the parameter values used in our numerical example. For most of the parameter values, we follow GK. We choose the discount rates for the patient and impatient households, β_p and β_m, to be 0.99 and 0.95, respectively. The weight of the housing in the utility function, η, is set to 0.5. The adjustment cost parameter for managing capital, γ^k, is 0.007, while that for mortgage claims, γ^l, is set to 0.08. The loan-to-value ratio, θ, is set to 0.85. We set the endowments for the patient and impatient households, ω_p and ω_m, to be 0.045 and 0.45, respectively.

For the banking sector, we set the steady-state level of leverage ϕ to 10 and annual risk premium for both capital claims and mortgage claims to 100 basis points. These numbers are similar to the ones in GK, and they render the diversion rates of 0.19 for both assets. We assume that the shock to the return on capital and shock to the return on mortgage claims are both quite persistent, with $\rho_{z^K} = \rho_{z^L} = 0.95$.

Table 2 reports the steady state values. Consumption of non-durables goods by the patient households is more than two times that by the impatient households. The housing consumption of patient households is about five times that of impatient households. Banks' capital asset holding is about eight times their holdings of mortgage claims. Due to the high adjustment costs, the amounts of both assets (capital and mortgage) held by the patient households are smaller than those held by the banks. The ratio of total consumption over output is about 80 percent.

4.2 Baseline case: no runs

Figure 1 shows the baseline no-run case where the economy is subject to the technology shock Z_t^K.The size of the shock is five percent. As in GK, the immediate response to the shock is a decline in net worth N_t. The 30 per cent decline in net worth tightens the banks' incentive constraint, leading to a rise in the risk premium for capital assets $ER^K_{t+1} - R_t$. With the higher costs of acquiring capital, the demand for capital assets declines, which in turn leads to a decline in the asset price Q^K_t. Different from GK, there is a spillover effect – the risk premium of the mortgage assets $ER^L_{t+1} - R_t$ rises due to the tightened incentive constraint of the banking sector. Together, the aggregate risk premium for the banks $ER^B_t - R_t$ rises by about 40 basis points. The higher borrowing costs lower the demand for bank loans. The asset price of the mortgage claims Q^L_t declines, which further deteriorates the balance sheet position of the banking sector. Since banks hold less capital and mortgage claims, patient households have to hold more of the two assets. Due to the high management costs when households hold these assets, output declines by about 6 percent. On the other hand, the demand for housing declines due to the weaker bank balance sheet positions, and Q^h_t falls. The run variable is negative, suggesting that banks are solvent when the assets are sold at regular prices.

Figure 2 shows the case where the economy is subject to a mortgage shock Z^L_t. The size of the shock is also five percent. Compared to Z^K_t, the overall impact on output is much smaller, only about 0.4%. This is because Z^K_t directly affects the endowment in the economy while Z^L_t does not. Z^L_t is rather a redistributional shock – a negative shock results in a loss to lenders (patient households and banks), and a gain to borrowers (impatient households). Similar to the Z^K_t shock, the immediate impact of the mortgage shock is a decline in net worth N_t, and a rise in the risk premium for mortgage claims $ER^L_{t+1} - R_t$. There is also a spillover effect, which leads to a rise in $ER^K_{t+1} - R_t$. The prices for both capital and mortgage claims Q^L_t and Q^K_t fall. The banks are more leveraged and households need to manage more assets; this leads to a decline in output due to the high management costs. Again, the run variable is negative, suggesting that banks are solvent when the assets are sold at regular prices.

4.3 Case of a bank run

In this section, we study whether a bank run equilibrium can occur in our model. We first study the Z^K_t shock. We assume that a five percent Z^K_t shock hits the economy at the beginning of the first period $t = 1$ with persistence of 0.95, which we also assumed in the no-run case. To see whether a run equilibrium exists, we conduct the following exercise. For each period $t = 1, 2, ..., 40$, we compute the liquidation prices Q^{K*}_t and Q^{L*}_t by assuming that a bank run happens beginning at time t. We substitute Q^{K*}_t and Q^{L*}_t to equation (56) to check if $x_t < 1$. If that is the case (meaning $run_t > 0$), we conclude that a run equilibrium indeed exists in that period. If the run variable turns out to be negative, a run equilibrium cannot exist in that period. We plot out the run variable for the first 40 period in Figure 3. It shows that a run is possible for all of the first 40 quarters after the shock, with the chance of having a run being the highest right after the shock.

We conduct a similar exercise to a five percent Z^L_t shock. Figure 4 shows the result. Similar to the Z^K_t shock, the run variable is positive upon impact and remains positive for the entire 40 quarters. The magnitude of the run variable, however, is much smaller than that under the Z^L_t

shock. Another difference is that the likelihood of a bank run becomes larger and peaks later in the case of a Z_t^L shock. This is due to different patterns of behavior of asset prices under the two different shocks.

An inspection of equations (64), (65), and (66) can explain this difference. Equation (64) shows that a Z_t^K shock reduces the endowment in the economy directly, and that a larger decline in Z_t^K will lead to a larger decline in C_t^{p*} at the time of the run. Since C_t^{p*} appears in both (65), and (66), both Q_t^{K*} and Q_t^{L*} are at their lowest when the run happens, and it is when the decline in Z_t^K is the largest. For a Z_t^L shock, Q_t^{L*} appears to reach the lowest point when Z_t^L experiences the largest decline. This is easy to see from (66), where Z_t^L enters directly to the equation. However, a Z_t^L shock does not enter equation (64) and thus does not affect the endowment in the economy. This leads us to conclude that the decline in C_t^{p*} is not necessarily related to the size of the decline in Z_t^L. As a result, the time when Q_t^{K*} reaches its lowest point does not coincide with that whenZ_t^L has the largest drop.

We use Figures 5 and 6 to further illustrate the responses of the key variables to the shocks when a run happens. In Figure 5, we assume that a bank run happens in the second period after the Z_t^K shock. The red dash line displays the run case. For comparative purposes, the no-run case responses are also displayed as the solid blue line. When a run happens, depositors stop rolling over the deposits, leading to liquidation of both capital and mortgage assets by banks. As a result, K_t^b and L_t^b drop to zero. The asset prices of both capital and mortgage claims Q_t^K and Q_t^L drop by about 15 percent. The value of $ER_{t+1}^K - R_t$ rises by about 300 basis points and $ER_{t+1}^l - R$ by about 400 basis points. The extremely high costs of obtaining funds also cause a decline in housing price Q_t^h by about 15%. Output declines by almost 30% in the run economy, partly due to the fact that the Z_t^K shock decreases the endowment directly and partly due to the sharp rise in management costs since now households have to hold large amounts of assets. Consumption of the patient households declines by almost 15% and consumption of the impatient households declines by about 10%.

We next assume that a run happens in the second period after the Z_t^L shock and display the responses of the key variables in Figure 6. The responses are similar to those in the case of the Z_t^K shock but the damage to the economy is less severe. When a run takes place, the banks liquidate both types of assets, and quantities K_t^b and L_t^b drop to zero. This leads a decline in prices Q_t^K and Q_t^L, but the magnitude of the decline is smaller than in the case of the Z_t^K shock (a 10 percent price drop as a response to Z_t^L vs. a 15 percent price drop as a response to Z_t^K). The premium $ER_{t+1}^K - R_t$ rises by about 250 basis points, and $ER_{t+1}^l - R_t$ by about 300 basis points. Overall, output declines by about 20 percent, less than in the Z_t^K shock case. The decline in consumption of both patient and impatient households is also smaller compared to the Z_t^K shock case.

5 Policy Implications

The high leverage for both household and banking sectors was a key factor leading to the 2008-09 financial crisis. A related important policy question is whether tighter regulations on household borrowing and bank lending can reduce the likelihood of bank runs. In this section, we address this issue by lowering the leverage of households and banks in an alternative economy and by comparing the results for the alternative economies with those for the baseline economy in both the no-run and run scenarios.

5.1 Household Leverage – Loan-to-value ratio

Figures 7 and 8 illustrate what would happen in the no-run case when we reduce the value of the LTV ratio θ faced by households from 0.85 to 0.75. We compare the impulse response functions of the key variables. We find that for both types of shocks, a fall of the LTV ratio dampens the impact of a negative shock, a typical financial accelerator effect. This is particularly true when the economy is hit by a Z_t^L shock. In the alternative economy where LTV=0.75, the output only declines by 0.15 percent, much less than in the baseline model. The decline of the banks' net worth is much milder, two percent compared to six percent in the baseline model. This leads to a dampening responses of the risk premium $ER_{t+1}^K - R_t$ and $ER_{t+1}^L - R_t$ and a more moderate rise in the bank leverage ϕ_t.

Another key question is whether a lower LTV ratio helps reduce the likelihood of a bank run equilibrium. We compute the likelihood of having a run equilibrium for both shocks when LTV=0.75. Figures 9 and 10 show that for this lower level of the LTV ratio, the run_t variable is lower than in the baseline case. When the economy is hit by a Z_t^K shock, the likelihood of a run on impact is half of that in the baseline case, and become negative after about 20 quarters. In the case of the Z_t^L shock, the run_t becomes negative, suggesting that a run equilibrium does not exist when LTV=0.75.

5.2 Bank Leverage

In this exercise, the steady state level of bank leverage in the alternative economy is set to $\phi = 8.8$, slightly lower than the value $\phi = 10$ in the baseline model, which gives us about 12 percent reduction in the value of ϕ. We compare the alternative economy with the baseline economy for both the run and no-run cases. The results are similar. We observe a typical financial accelerator effect for the no-run case (Figures 11-12). In the run case, we find that the likelihood of a run is smaller when $\phi = 8.8$ (Figures 13-14).

6 Conclusions

GK have shown that a shock to a productive asset can potentially lead to a bank run. We have extended the GK model to incorporate household debt and show that a shock to the return on a non-productive asset can also lead to a bank run. The extension is important since it allows us to model and analyze a salient issue at the center of the 2008-09 financial crisis: a large shock to and a subsequent collapse of the mortgage-backed securities market can lead to a bank panic and large decline in output. Having two assets on the banks' balance sheets also allows us to explicitly address the spillover effect: a shock to the mortgage assets weakens the banks' balance sheet position, causing a rise of the borrowing costs of all assets held by the banks. The spillover effect leads to a fall in the price of capital increasing the likelihood of a bank run. Our policy experiments suggest that tightening households' borrowing condition and banks' lending condition can reduce the likelihood of a bank run.

Although our model has shed light on the role of household mortgages, which was prominent during the recent financial crisis, the simplicity of our model forced us to abstract from many interesting issues observed in the data. In the future research, our model could be extended to incorporate both a commercial banking and an investment banking sectors. In such an environment, a bank run would start in the investment banking sector that holds mortgage-backed securities and then impact the commercial banking sector (a spillover effect) that sells mortgage loans to households.[6] Another avenue of extending our model is to embed it in a conventional macroeconomic framework to be able to conduct a quantitative analysis.

[6]This is similar to the setup in Gertler, Kiyotaki and Prestipino (2016), which has an unregulated wholesale banking sector and a traditional retail banking sector. However, they do not consider household debt.

Appendix A Banks' problem

The Lagrangian for bank j's problem is

$$L_t = E_t\{\Lambda^p_{t,t+1}[(1-\sigma)n_{jt+1} + \sigma V_{jt+1}(n_{jt+1}) - \lambda_{bank}(\kappa^k Q^K_t k^b_{jt} + \kappa^l_t Q^L_t l^b_{jt} - V_{jt})\}, \tag{67}$$

where λ_{bank} is the multiplier on the incentive constraint.

The first-order condition with respect to k^b_{jt} is

$$\frac{dL_t}{dk^b_{jt}} = E_t\left\{\Lambda^p_{t,t+1}[(1-\sigma)\frac{dn_{jt+1}}{dk^b_{jt}} + \sigma\frac{dV_{jt+1}}{dn_{jt+1}}\frac{dn_{jt+1}}{dk^b_{jt}} - \lambda_{bank}\kappa^k Q^K_t]\right\} = 0, \tag{68}$$

which, together with equation (35), gives us

$$E_t\Lambda^p_{t,t+1}[((1-\sigma)+\sigma\mu_{n,t+1})(R^{k,b}_{t+1} - R_t)] = \lambda_{bank}\kappa^k. \tag{69}$$

Similarly, the first-order condition with respect to l^b_{jt} is

$$\frac{dL}{dl^b_{jt}} = E_t\left\{\Lambda^p_{t,t+1}[(1-\sigma)\frac{dn_{t+1}}{dl^b_{jt}} + \sigma\frac{dV_{jt+1}}{dn_{jt+1}}\frac{dn_{jt+1}}{dl^b_{jt}} - \lambda_{bank}\kappa^l_t]\right\} = 0, \tag{70}$$

or

$$E_t\Lambda^p_{t,t+1}[((1-\sigma)+\sigma\mu_{n,t+1})(R^{L,b}_{t+1} - R_t)] = \lambda_{bank}\kappa^l_t. \tag{71}$$

Note that equations (69) and (71) give us

$$\begin{aligned} & \frac{E_t\Lambda^p_{t,t+1}[((1-\sigma)+\sigma\mu_{n,t+1})(R^{K,b}_{t+1} - R_t)]}{\kappa^k} \\ = \ & \frac{E_t\Lambda^p_{t,t+1}[((1-\sigma)+\sigma\mu_{n,t+1})(R^{L,b}_{t+1} - R_t)]}{\kappa^l_t}. \end{aligned} \tag{72}$$

Below, we verify whether our guess is correct and that the bank's value is indeed linear in net worth with

$$\mu_{n,t} = E_t\{\Lambda^p_{t,t+1}((1-\sigma)+\sigma\mu_{n,t+1})(R^B_{t+1} - R_t)\phi_t + R_t\}.$$

To verify it, we substitute $V_{jt} = \mu_{n,t}n_{jt}$ and equation (35) into the value function equation (29), to obtain

$$\begin{aligned} V_{jt} &= E_t\{\Lambda^p_{t,t+1}((1-\sigma)n_{jt+1} + \sigma V_{jt+1})\} \\ &= E_t\{\Lambda^p_{t,t+1}((1-\sigma)+\sigma\mu_{n,t+1})\{[(R^B_{t+1} - R_t)\phi_t + R_t]n_{jt}\}\}. \end{aligned}$$

Equation (36) gives us

$$\mu_{n,t} = E_t\{\Lambda^p_{t,t+1}[(1-\sigma)+\sigma\mu_{n,t+1}][(R^B_{t+1} - R_t)\phi_t + R_t]\}\},$$

and thus we have verified that our conjecture is correct and that

$$V_{jt} = \mu_{n,t} n_{jt}.$$

Moreover, the bank's incentive constraint can be rewritten as

$$\mu_{n,t} n_{jt} \geq \kappa^k Q_t^K k_{jt}^b + \kappa_t^l Q_t^L l_{jt}^b. \tag{73}$$

When the incentive constraint binds, we have

$$\mu_{n,t} n_{jt} = \kappa^k Q_t^K k_{jt}^b + \kappa_t^l Q_t^L l_{jt}^b. \tag{74}$$

Define $\kappa_t^B = \kappa^k \frac{Q_t^K k_{jt}^b}{a_{jt}} + \kappa_t^l \frac{Q_t^L l_{jt}^b}{a_{jt}}$; we can then have

$$\mu_{n,t} = \kappa^B \frac{a_{jt}}{n_{jt}} = \phi_t \kappa_t^B. \tag{75}$$

Thus

$$\phi_t = \frac{\mu_{n,t}}{\kappa_t^B}, \tag{76}$$

$$\phi_t \kappa_t^B = \mu_{n,t} = E_t\{\Lambda_{t,t+1}^p [(1-\sigma) + \sigma \mu_{n,t+1}][(R_{t+1}^B - R_t)\phi_t + R_t]\}.$$

Thus the leverage ϕ_t can be also expressed as

$$\phi_t = \frac{E_t\{\Lambda_{t,t+1}^p [(1-\sigma) + \sigma \phi_{t+1} \kappa_{t+1}^B] R_t\}}{\kappa_t^B - E_t\{\Lambda_{t,t+1}^p [(1-\sigma) + \sigma \phi_{t+1} \kappa_{t+1}^B][(R_{t+1}^B - R_t))\}}. \tag{77}$$

Appendix B Expressions for Q_t^{K*} and Q_t^{L*}

B.1 Deriving Q_t^{K*}

Combining the first-order condition for the patient households

$$E_t \Lambda_{t,t+1}^p R_{t+1}^{K,p} = 1, \tag{78}$$

with $R_{t+1}^{K,p} = \frac{(Z_{t+1}^K + Q_{t+1}^K)}{Q_t^K + f'(K_t^p)}$, we obtain

$$E_t \Lambda_{t,t+1}^p \frac{Z_{t+1}^K + Q_{t+1}^K}{Q_t^K + f'(K_t^p)} = 1. \tag{79}$$

This gives us

$$Q_t^K + f'(K_t^p) = E_t \Lambda_{t,t+1}^p (Z_{t+1}^K + Q_{t+1}^K), \tag{80}$$

or

$$Q_t^K = E_t \Lambda_{t,t+1}^p (Z_{t+1}^K + Q_{t+1}^K) - f'(K_t^p). \tag{81}$$

Similarly, we can obtain

$$Q_{t+1}^{K} = E_t \Lambda_{t+1,t+2}^{p} (Z_{t+2}^{K} + Q_{t+2}^{K}) - f'(K_{t+1}^{p}), \tag{82}$$

$$Q_{t+2}^{K} = E_t \Lambda_{t+2,t+3} (Z_{t+3}^{K} + Q_{t+3}^{K}) - f'(K_{t+2}^{p}). \tag{83}$$

Substituting equations (82) and (83) to the equation (81), we have

$$\begin{aligned}
Q_t^K &= E_t \Lambda_{t,t+1}^{p} (Z_{t+1}^{K} + \Lambda_{t+1,t+2}^{p} (Z_{t+2}^{K} + Q_{t+2}^{K}) - f'(K_{t+1}^{p})) - f'(K_t^p) && (84)\\
&= E_t \Lambda_{t,t+1}^{p} (Z_{t+1}^{K} + \Lambda_{t+1,t+2}^{p} (Z_{t+2}^{K} + \left(\Lambda_{t+2,t+3}^{p} (Z_{t+3}^{K} + Q_{t+3}^{K}) - f'(K_{t+2}^{p})\right)) && (85)\\
&\quad - f'(K_{t+1}^{p})) - f'(K_t^p) && (86)\\
&= E_t \left(\Lambda_{t,t+1}^{p} [Z_{t+1}^{K} - f'(K_{t+1}^{p})] + \Lambda_{t,t+2}^{p} [Z_{t+2}^{K} - f'(K_{t+2}^{p})] + ... - f'(K_t^p)\right) && (87)\\
&= E_t \sum_{i=1}^{\infty} \Lambda_{t,t+i}^{p} [Z_{t+i}^{K} - f'(K_{t+i}^{p})] - f'(K_t^p), && (88)
\end{aligned}$$

where

$$\Lambda_{t,t+i}^{p} = (\beta^p)^i \frac{u_{c,t+i}^{p}}{u_{c,t}^{p}}. \tag{89}$$

During a run, $K_t^{p*} = 1$, and thus

$$f^{'}(K_t^{p*}) = \gamma^k K_t^{p*} = \gamma^k, \tag{90}$$

and the liquidation price during the run can be written as

$$\begin{aligned}
Q_t^{K*} &= E \sum_{i=1}^{\infty} \Lambda_{t,t+i}^{p} [Z_{t+i}^{K} - f'(K_{t+i}^{p})] - f'(K_t^{p*}) && (91)\\
&= E \sum_{i=1}^{\infty} (\beta^p)^i \frac{u_{c,t+i}^{p}}{u_{c,t}^{p*}} [Z_{t+i}^{K} - \gamma^k K_{t+i}^{p}] - \gamma^k.
\end{aligned}$$

B.2 Deriving Q_t^{L*}

From the first-order condition

$$E_t \Lambda_{t,t+1}^{p} R_{t+1}^{L,p} = 1, \tag{92}$$

and with $R_{t+1}^{L,p} = \frac{Z_{t+1}^{L} + Q_{t+1}^{L}}{Q_t^L + g'(L_t^p)}$,we have

$$Q_t^L + g'(L_t^p) = E_t \Lambda_{t,t+1}^{p} (Z_{t+1}^{L} + Q_{t+1}^{L}), \tag{93}$$

or

$$Q_t^L = E_t \Lambda_{t,t+1}^{p} (Z_{t+1}^{L} + Q_{t+1}^{L}) - g'(L_t^p). \tag{94}$$

Similarly, we can obtain

$$Q^L_{t+1} = E_t \Lambda^p_{t+1,t+2}(Z^L_{t+2} + Q^L_{t+2}) - g'(L^p_{t+1}), \tag{95}$$

$$Q^L_{t+2} = E_t \Lambda^p_{t+2,t+3}(Z^L_{t+3} + Q^L_{t+3}) - g'(L^p_{t+2}). \tag{96}$$

Substituting equations (95) and (96) to the equation for Q^L_t yields

$$Q^L_t = E_t \sum_{i=1}^{\infty} \Lambda^p_{t,t+i}[Z^L_{t+i} - g'(L^L_{t+i})] - g'(L^p_t). \tag{97}$$

That is

$$Q^L_t = E \sum_{i=1}^{\infty} (\beta^p)^i \frac{u^p_{c,t+i}}{u^p_c}[Z^L_{t+i} - g'(L^p_{t+i})] - g'(L^p_t).$$

During a run, $L^{p*}_t = L^*_t$, and thus

$$g^{'}(L^{p*}_t) = \gamma^l L^*_t, \tag{98}$$

and using

$$\Lambda^p_{t,t+i} = (\beta^p)^i \frac{u^p_{c,t+i}}{u^p_{c,t}}, \tag{99}$$

the liquidation price can be expressed as

$$\begin{aligned} Q^{L*}_t &= E \sum_{i=1}^{\infty} \Lambda^p_{t,t+i}[Z^L_{t+i} - g'(L^p_{t+i})] - g'(L^{p*}_t) \\ &= E \sum_{i=1}^{\infty} (\beta^p)^i \frac{u^p_{c,t+i}}{u^{p*}_{c,t}}[Z^L_{t+i} - \gamma^l L^p_{t+i}] - \gamma^l L^*_t. \end{aligned} \tag{100}$$

Appendix C Numerical procedure when there is a run

We follow the procedure described in GK to compute the impulse responses to shocks during a run. The details are as follows. Suppose the economy starts in a no-run equilibrium in the steady-state and then it is hit with a negative shock to either capital or mortgage claims at $t = 1$. It stays at a no-run equilibrium until t^* when a bank run occurs. The condition for a bank run is

$$x_t = \frac{(Z^K_t + Q^{K*}_t)K^b_{t-1} + (Z^L_t + Q^{L*}_t)L^b_{t-1}}{R_t D_{t-1}} < 1. \tag{101}$$

After the run, the economy returns to the no-run equilibrium and converges back to the steady state after T periods from the initial shock.

As in GK, we compute the path using the following steps. Step 1: we calculate the path of the variables $\{X_t\}^{t^*+T}_{t=t^*+1}$ from t^*+1, the first period after the run, to the time the economy is back to the steady state. Step 2: using $\{X_t\}^{t^*+T}_{t=t^*+1}$ and the fact that a run occurs at t^*, we compute X_{t^*}, the value of the endogenous variables at the time of the run. Step 3: we compute the path of the variables from the initial shock at time $t = 1$ to the steady state and then select the first $t^* - 1$ elements of the path $\{X_t\}_{t=1}^{t^*-1}$. Among the three steps, the second one is crucial. In what follows, we describe Step 2 in details.

When a run happens, we have $K_{p,t^*} = 1, K_{b,t^*} = 0$, $L^p_{t^*} = L_{t^*}$, $L^b_{t^*} = 0$, $D^b_{t^*} = 0$, $N_{t^*} = 0, C^b_{t^*} = 0$, and $(1-\sigma)\omega^{b*} = 0$. We also have

$$f(K^p_{t^*}) = \frac{\gamma^f}{2} K^{*2}_{p,t} = \frac{\gamma^f}{2}, \tag{102}$$

$$f^{'}(K^p_{t^*}) = \gamma^f, \tag{103}$$

$$g(L^p_{t^*}) = \frac{\gamma^g}{2} L^2_{t^*}, \tag{104}$$

and

$$g^{'}(L^p_{t^*}) = \gamma^g L_{t^*}. \tag{105}$$

Given that $(1-\sigma)\omega^{b*} = 0$ and $C^b_{t^*} = 0$, the resources are used for consumption of patient and impatient households and for adjustment costs:

$$Y_{t^*} = f(K^p_{t^*}) + g(L^p_{t^*}) + C^p_{t^*} + C^m_{t^*}. \tag{106}$$

This means

$$C^p_{t^*} = Z^K_{t^*} + Z_{t^*}\left[W^p + W^m\right] - f(K^p_{t^*}) - g(L^p_{t^*}) - C^m_{t^*}. \tag{107}$$

In our model, since we have two types of households, pinning down consumption of the patient households is not as straightforward as in GK.[7] The equation that we use for solving for $C^p_{t^*}$ is equation (107), which has $C^m_{t^*}$ and $L^p_{t^*}$ and their values are unknown. To find $C^p_{t^*}$ $C^m_{t^*}$ and $L^p_{t^*}$, we use the following numerical procedure.

We start with initial guesses for $C^m_{t^*}$ and $L^p_{t^*}$, and we use equation (107) to obtain $C^p_{t^*}$. We then pin down R_{t^*} using

$$R_{t^*} E_t \left[\beta^p \frac{u^p_{C,t^*+1}}{u^p_{C,t^*}}\right] = 1. \tag{109}$$

Since the values of $C^p_{t^*}$ and $C^p_{t^*+1}$ are already found (Step 1), we know the values of u^p_{C,t^*} and u^p_{C,t^*+1}. We then use

$$\frac{(Z^K_{t^*+1} + Q^K_{t^*+1})}{Q^K_{t^*} + \gamma^f} E_t \left[\beta^p \frac{u^p_{C,t^*+1}}{u^p_{C,t^*}}\right] = 1 \tag{110}$$

[7] Note that in GK the consumption for households C_{t^*} is pined down by using

$$C_{t^*} = Z_{t^*} + Z_{t^*} W - f(K_{t^*}). \tag{108}$$

Since Z_{t^*} is an exogenous variable, $K_{t^*} = 1$ at the time of a run (and W is a parameter), obtaining C_{t^*} in GK is straightforward. Once C_{t^*} is solved for, GK use the Euler equation for the capital price to pin down $Q^K_{t^*}$ and the value for the other endogenous variables at the time of the run.

to pin down $Q^K_{t^*}$. Again, we are able to obtain $Q^K_{t^*}$ since we already know u^p_{C,t^*} and u^p_{C,t^*+1}. We also know $Q^K_{t^*+1}$ from Step 1. We then use

$$\frac{(Z^L_{t^*+1} + Q^L_{t^*+1})}{Q^L_{t^*} + \gamma^g L_{t^*}} E_t \left[\beta^p \frac{u^p_{C,t^*+1}}{u^p_{C,t^*}} \right] = 1 \tag{111}$$

to obtain $Q^L_{t^*}$. Note that at the time of a run, $L_{t^*} = L^p_{t^*}$. Once we know $Q^L_{t^*}$, we can use the collateral constraint

$$Q^L_{t^*} L^*_t \leq \theta E_t Q^h_{t^*+1} H^m_{t^*} \tag{112}$$

to pin down $H^m_{t^*}$ (assuming the constraint is binding). Note that we know $Q^h_{t^*+1}$ from Step 1. The housing owned by patient households $H^p_{t^*}$ is easy to pin down given the housing constraint $H^m_{t^*} + H^p_{t^*} = 1$. Using the Euler equation for housing for the patient households,

$$\frac{u^p_{H,t^*}}{u^p_{c,t^*}} = Q^h_{t^*} - E_t \Lambda^p_{t^*,t^*+1} Q^h_{t^*+1}, \tag{113}$$

we can pin down $Q^h_{t^*}$ since we know u^p_{H,t^*} $\left(u^p_{H,t^*} = \frac{1}{H^p_{t^*}}\right)$ and $u^p_{c,t}$ $\left(u^p_{c,t^*} = \frac{1}{C^p_{t^*}}\right)$, and we know $Q^h_{t^*+1}$ from Step 1.

We then use the first-order condition for the impatient households to update $Q^h_{t^*}$ ($Q^{h,update}_{t^*}$):

$$u^m_{H,t} + \beta^m E_t u^m_{C,t+1} Q^h_{t+1} + \left(u^m_{C,t} - \beta^m E_t u^m_{C,t+1} \frac{(Z^L_{t+1} + Q^L_{t+1})}{Q^L_t} \right) \theta E_t Q^h_{t+1} = Q^h_t u^m_{C,t}. \tag{114}$$

Furthermore, we substitute $Q^{h,update}_{t^*}$ back to equation (113) to get $u^{p,update}_{H,t^*}$. We now solve for $C^m_{t^*}$ and $L^p_{t^*}$ so that they satisfy the following two equations:

$$Q^{h,update}_{t^*} - Q^h_{t^*} = 0,$$

and

$$u^{p,update}_{H,t^*} - u^p_{H,t^*} = 0.$$

Appendix D System of equations

$$E_t[\Lambda^p_{t,t+1}R_{t+1}] = 1, \tag{115}$$

$$E_t\left[\Lambda^p_{t,t+1}\frac{Z^K_{t+1}+Q^K_{t+1}}{Q^K_t+f'(K^p_t)}\right] = 1, \tag{116}$$

$$E_t\left[\Lambda^p_{t,t+1}\frac{Z^L_{t+1}+Q^L_{t+1}}{Q^L_t+g'(L^p_t)}\right] = 1, \tag{117}$$

$$\frac{u^p_{H,t}}{u^p_{C,t}} = Q^h_t - E_t\left[\Lambda^p_{t,t+1}Q^h_{t+1}\right], \tag{118}$$

$$Q^L_t L_t \leq \theta E_t Q^h_{t+1} H^m_t, \tag{119}$$

$$u^m_{H,t} + \beta^m E_t u^m_{C,t+1} Q^h_{t+1} + \left(u^m_{C,t} - \beta^m E_t u^m_{C,t+1}\frac{Z^L_{t+1}+Q^L_{t+1}}{Q^L_t}\right)\theta E_t Q^h_{t+1} = Q^h_t u^m_{C,t}. \tag{120}$$

$$C^m_t + Q^h_t H^m_t + (Z^L_t + Q^L_t)L_{t-1} \leq Z^K_t W_{m,t} + Q^L_t L_t + Q^h_t H^m_{t-1}. \tag{121}$$

$$Q^K_t K^b_t + Q^L L^b_t = D_t + N_t, \tag{122}$$

$$Q^K_t K^b_{jt} + Q^L L^b_t = \phi_t N_t, \tag{123}$$

$$N_t = \sigma\left[(Z^K_t + Q^K_t)K^b_{t-1} + (Z^L_t + Q^L_t)L^b_{t-1} - R_t D_{t-1}\right] + (1-\sigma)\omega^b, \tag{124}$$

$$\phi_t = \frac{E_t\{\Lambda_{t,t+1}[(1-\sigma)+\sigma\phi_{t+1}\kappa^A_{t+1}]R_t\}}{\kappa^B - E_t\{\Lambda_{t,t+1}[(1-\sigma)+\sigma\phi_{t+1}\kappa^A_{t+1}][R^B_{t+1}-R_t]\}}, \tag{125}$$

$$\phi_t \kappa^B_t = \mu_{n,t},$$

$$\begin{aligned} & \frac{E_t\Lambda_{t,t+1}[((1-\sigma)+\sigma\mu_{n,t+1})(R^{k,b}_{t+1}-R_t)]}{\kappa^k} \\ = \; & \frac{E_t\Lambda_{t,t+1}[((1-\sigma)+\sigma\mu_{n,t+1})(R^{L,b}_t-R_t)]}{\kappa^l_t}, \end{aligned} \tag{126}$$

$$Y_t = Z^K_t + Z^K_t\left[W^p + W^m\right] + (1-\sigma)\omega^b, \tag{127}$$

$$Y_t = f(K^p_t) + g(L^p_t) + C^p_t + C^m_t + C^b, \tag{128}$$

$$C^b_{t+1} = (1-\sigma)\left[(Z^K_t + Q^K_t)K^b_{t-1} + (Z^L_t + Q^L_t)L^b_{t-1} - R_t D_{t-1}\right], \tag{129}$$

$$L^p_t + L^b_t = L_t, \tag{130}$$

$$K^p_t + K^b_t = 1, \tag{131}$$

$$H^p_t + H^m_t = 1, \tag{132}$$

Tables and Figures

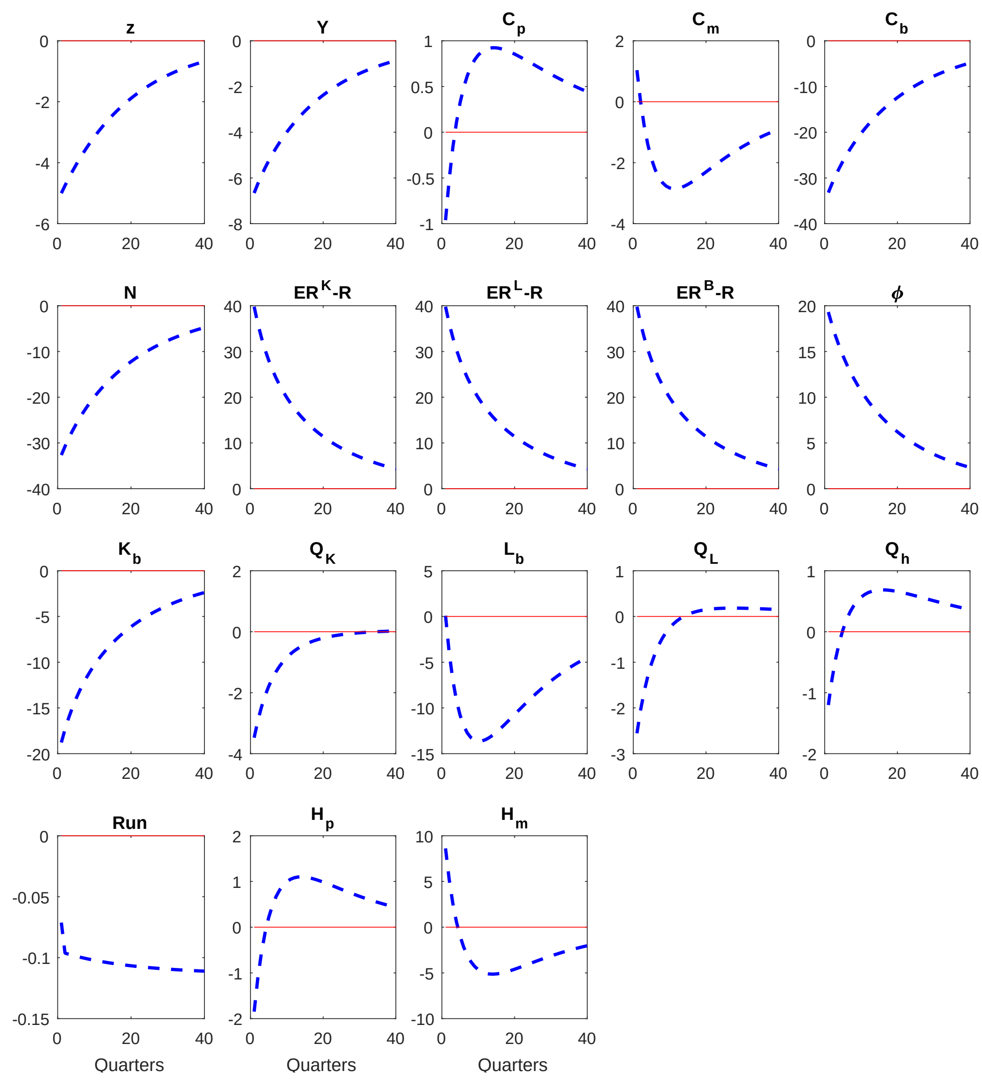
Table 1: Parameter values

Symbol	Value	Description
		Households
β_p	0.990	Discount factor for patient households
β_m	0.950	Discount factor for impatient households
η	0.500	Relative utility weight of housing
γ^k	0.007	Cost parameter for households to manage capital
γ^l	0.080	Cost parameter for households to manage loans
θ	0.850	LTV ratio
w_p	0.045	Patient households endowment
w_m	0.450	Impatient households endowment
		Financial Intermediaries
κ_k	0.193	Fraction of capital that can be diverted
κ_l	0.193	Fraction of mortgage claims that can be diverted
σ	0.950	Survival rate of the bankers
ω^b	0.024	Endowment for new bankers
		Shocks
ρ^k	0.950	Technology shock persistence
ρ^l	0.950	Mortgage shock persistence

Table 2: Steady State Values

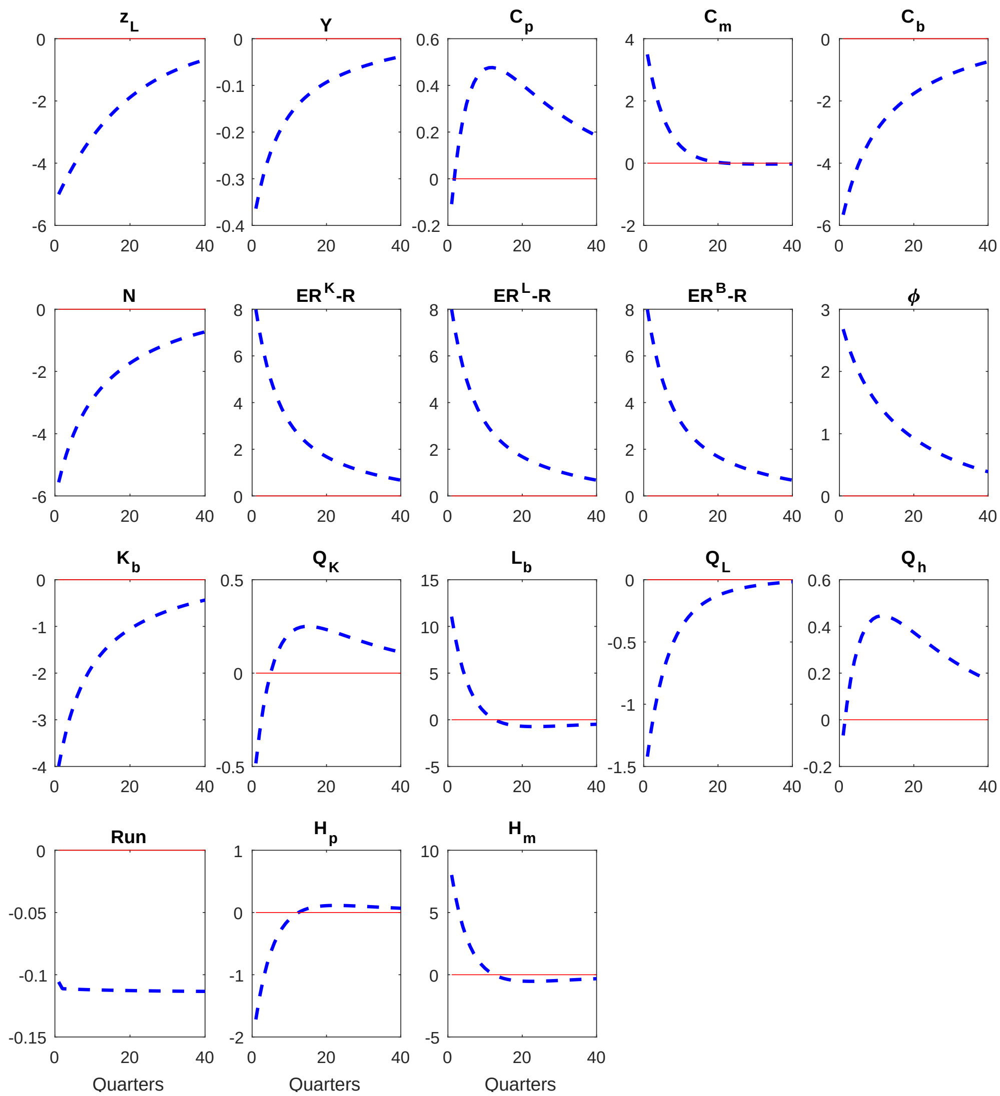
Symbol	Value	Description
K	1.000	Total capital stock
K^b	0.646	Capital stock held by banks
K^p	0.354	Capital stock held by patient households
L	0.105	Total loans
L^b	0.074	Loans held by banks
L^p	0.031	Loans held by patient households
Q^K	1.000	Capital price
Q^L	1.000	Mortgage-backed security price
Q^H	0.697	Housing price
C^p	0.011	Patient households consumption
C^m	0.004	Impatient households consumption
H^p	0.823	Patient households housing
H^m	0.177	Impatient households housing
S	100.000	Annual risk premium for both assets (basis points)
R	1.010	Quarterly risk free rate
C^b	0.004	Banks consumption
D	0.648	Deposits held by banks
N	0.072	Net worth
ϕ	10.000	Banks leverage
X	1.114	Recovery rate
Y	0.020	Output
		Some steady state ratios
C^p/C^m	2.639	Consumption: Patient/Impatient
H^p/H^m	4.664	Housing: Patient/Impatient
K^b/L^b	8.771	Bank capital holding/Bank mortgage holding
L^b/L^p	2.382	Mortgage Loans: Bank/Patient households
K^b/K^p	1.828	Capital: Bank/Patient households
$(C^p+C^m)/Y$	0.809	Total household consumption/Output

FIGURE 1 – Impulse Responses after a Technology Shock – Baseline No Run Case



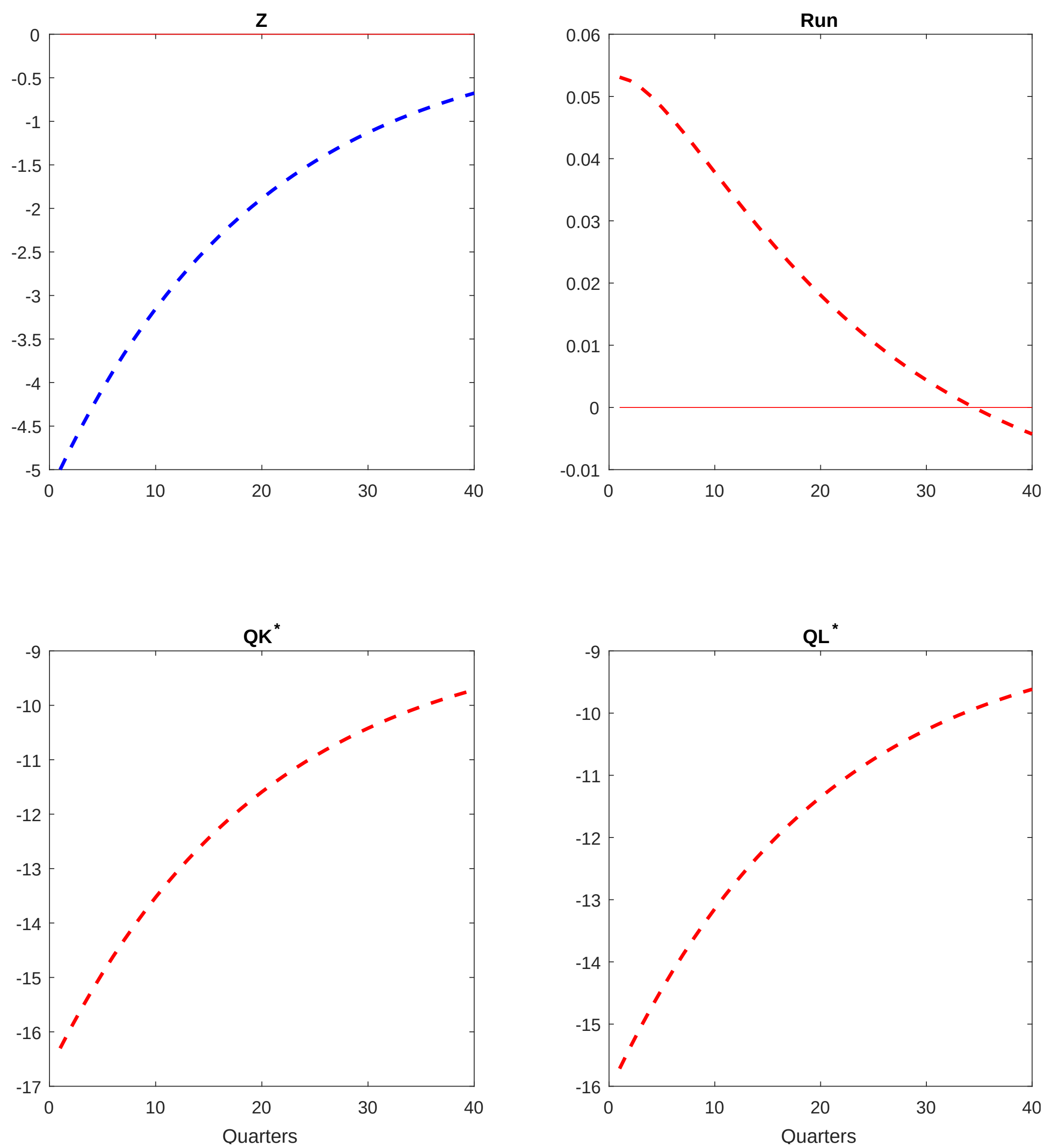
Spreads are in annual basis points. Run is expressed in absolute values. All of the remaining variables are in percentage deviations from their steady-state values.

FIGURE 2 – Impulse Responses after a Mortgage Shock – Baseline No Run Case



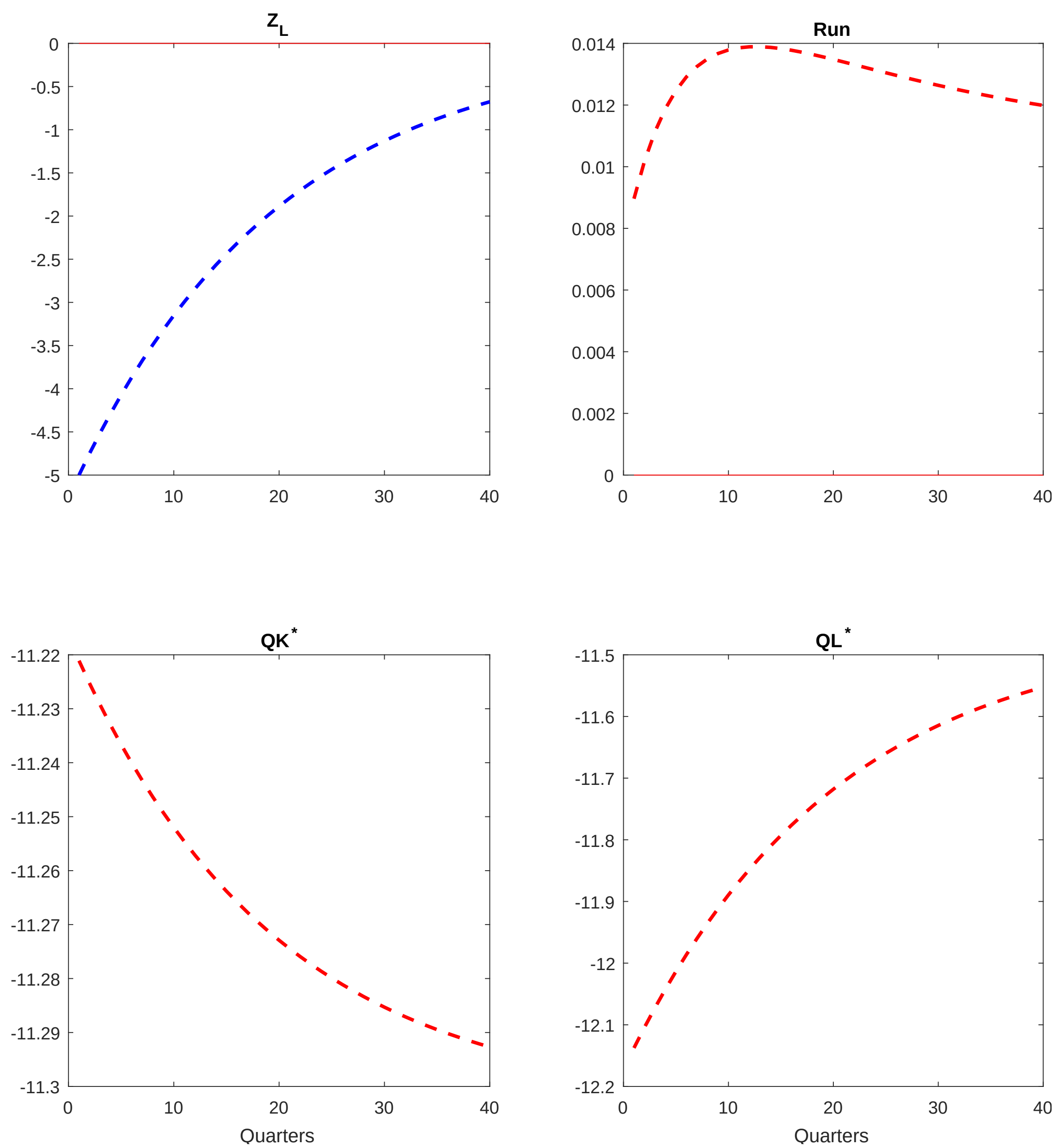
Spreads are in annual basis points. Run is expressed in absolute values. All of the remaining variables are in percentage deviations from their steady-state values.

FIGURE 3 Technology Shock – Run Variable and Liquidition Prices



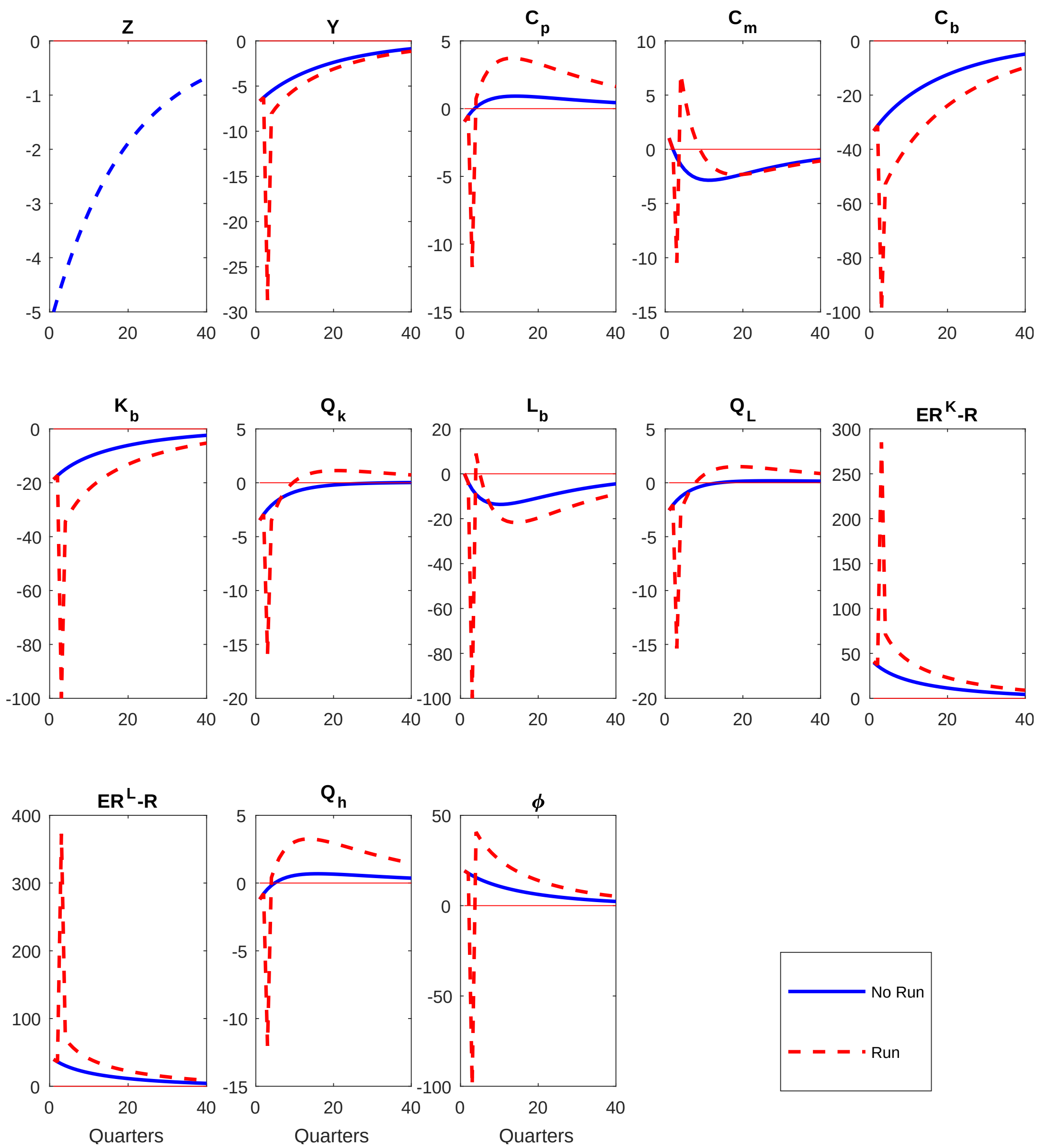
Run is expressed in absolute values. All of the remaining variables are in percentage deviations from their steady-state values.

FIGURE 4 – Mortgage Shock – Run Variable and Liquidition Prices



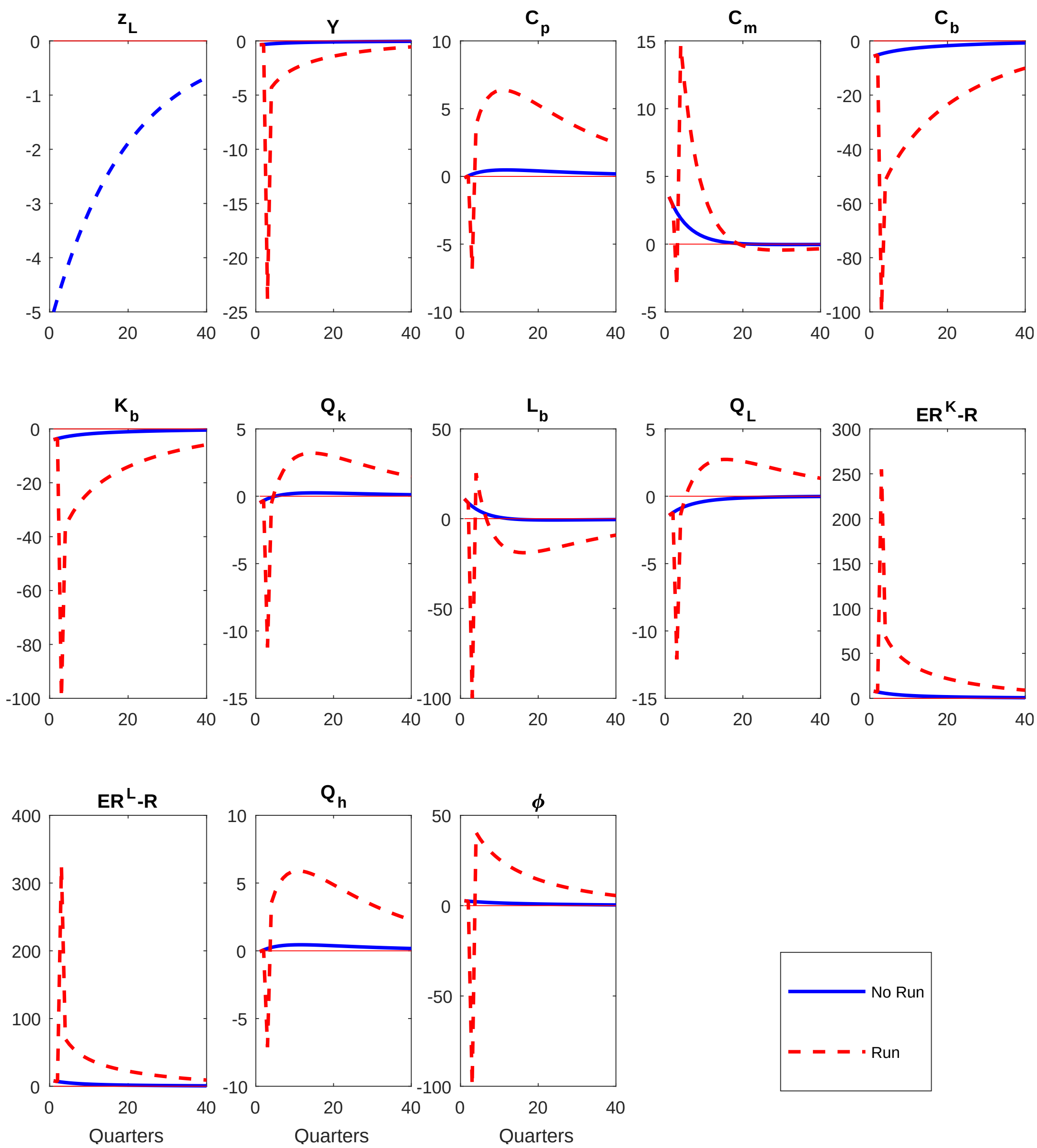
Run is expressed in absolute values. All of the remaining variables are in percentage deviations from their steady-state values.

FIGURE 5 – After a Technology Shock – Run vs. No Run



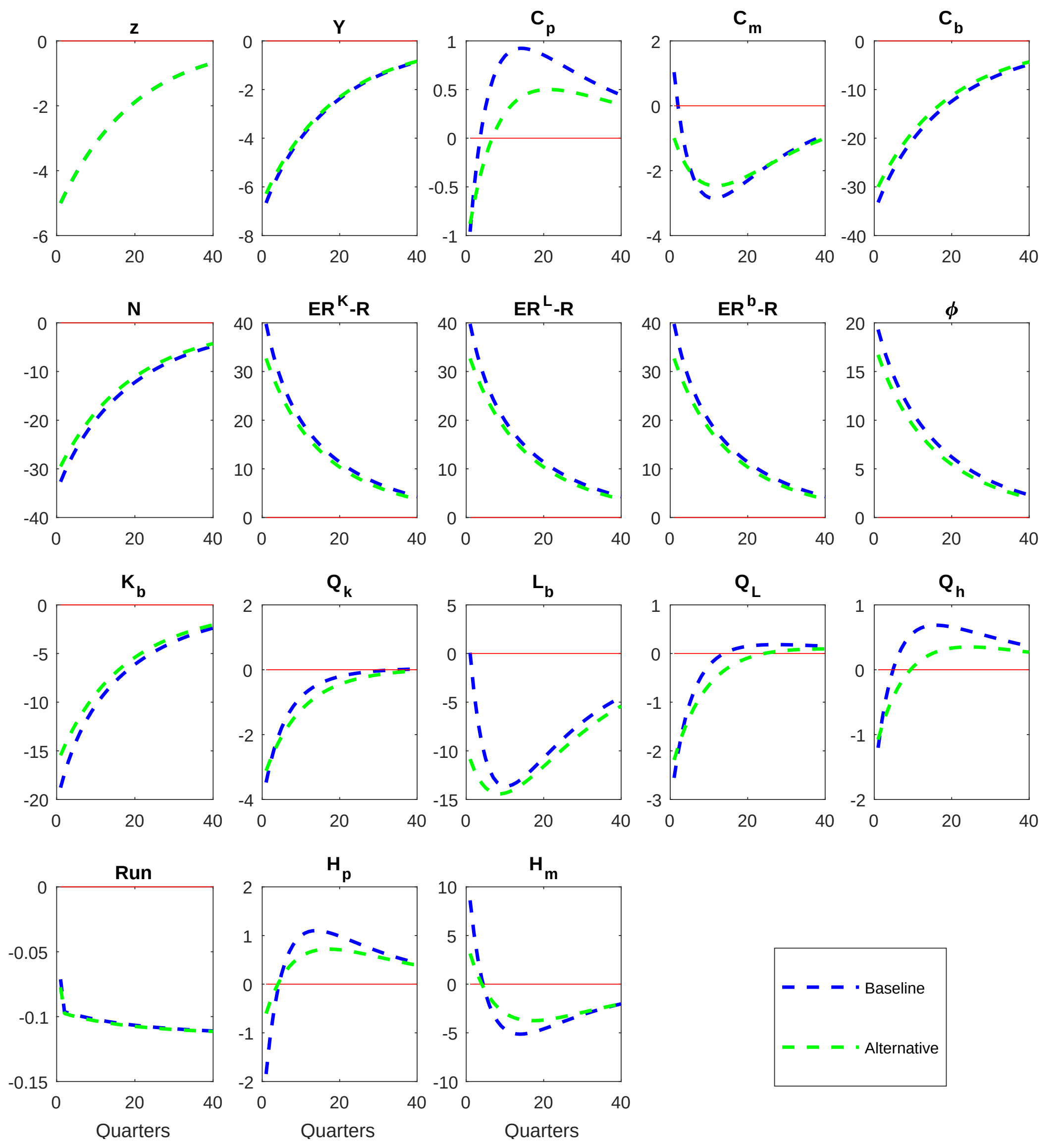
Spreads are in annual basis points. All of the remaining variables are in percentage deviations from their steady-state values.

FIGURE 6 – After a Mortgage Shock – Run vs. No Run



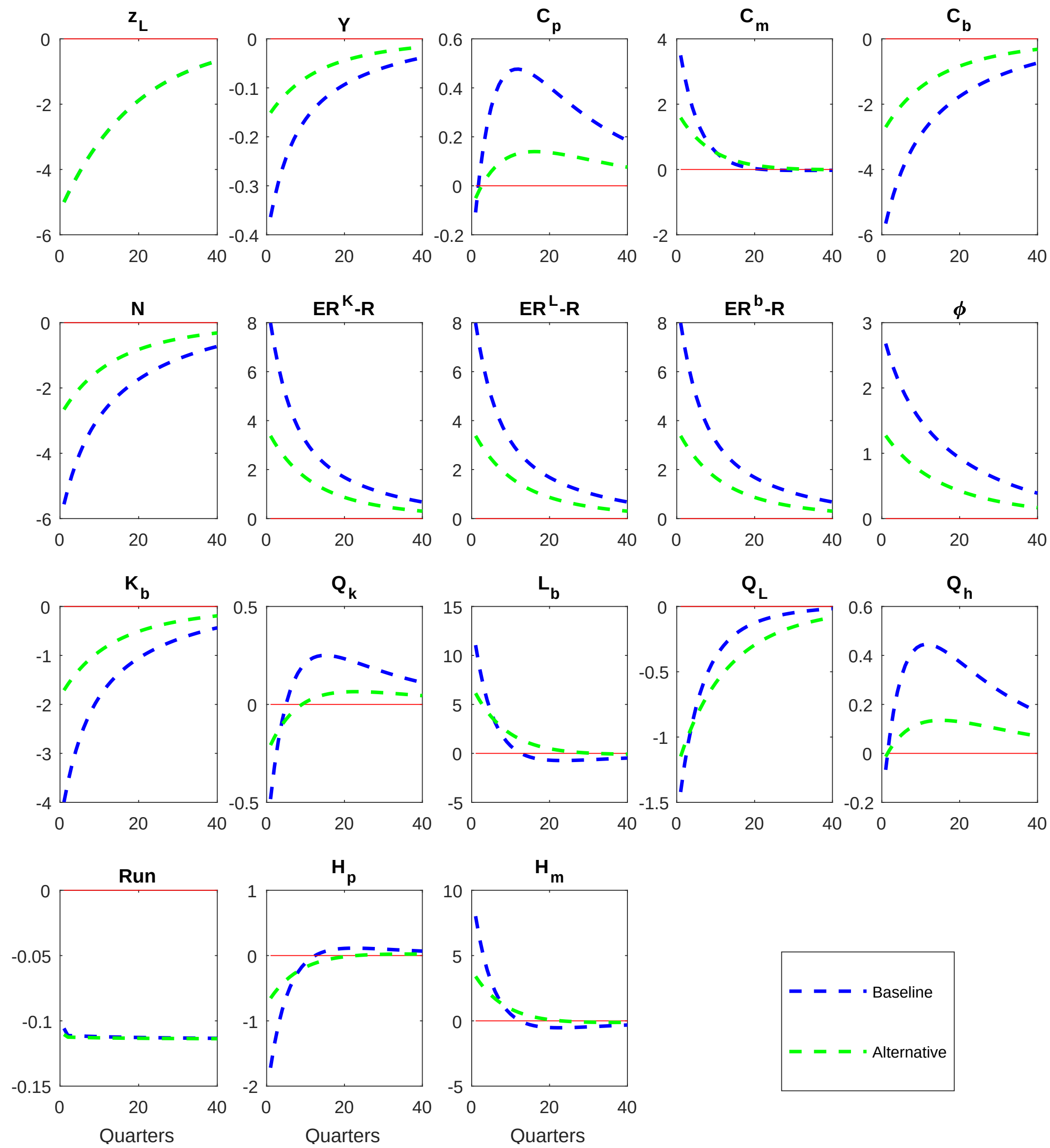
Spreads are in annual basis points. All of the remaining variables are in percentage deviations from their steady-state values.

FIGURE 7 – No Run Case – Technology Shock – High vs. Low HH Leverage



Spreads are in annual basis points. Run is expressed in absolute values. All of the remaining variables are in percentage deviations from their steady-state values. LTV: Baseline=0.85; Alterntive=0.75

FIGURE 8 – No Run Case – Mortgage Shock – High vs. Low HH Leverage



Spreads are in annual basis points. Run is expressed in absolute values. All of the remaining variables are in percentage deviations from their steady-state values. LTV: Baseline=0.85, Alternative=0.75

FIGURE 9 – After a Technology Shock – Effect of LTV on Run Variable

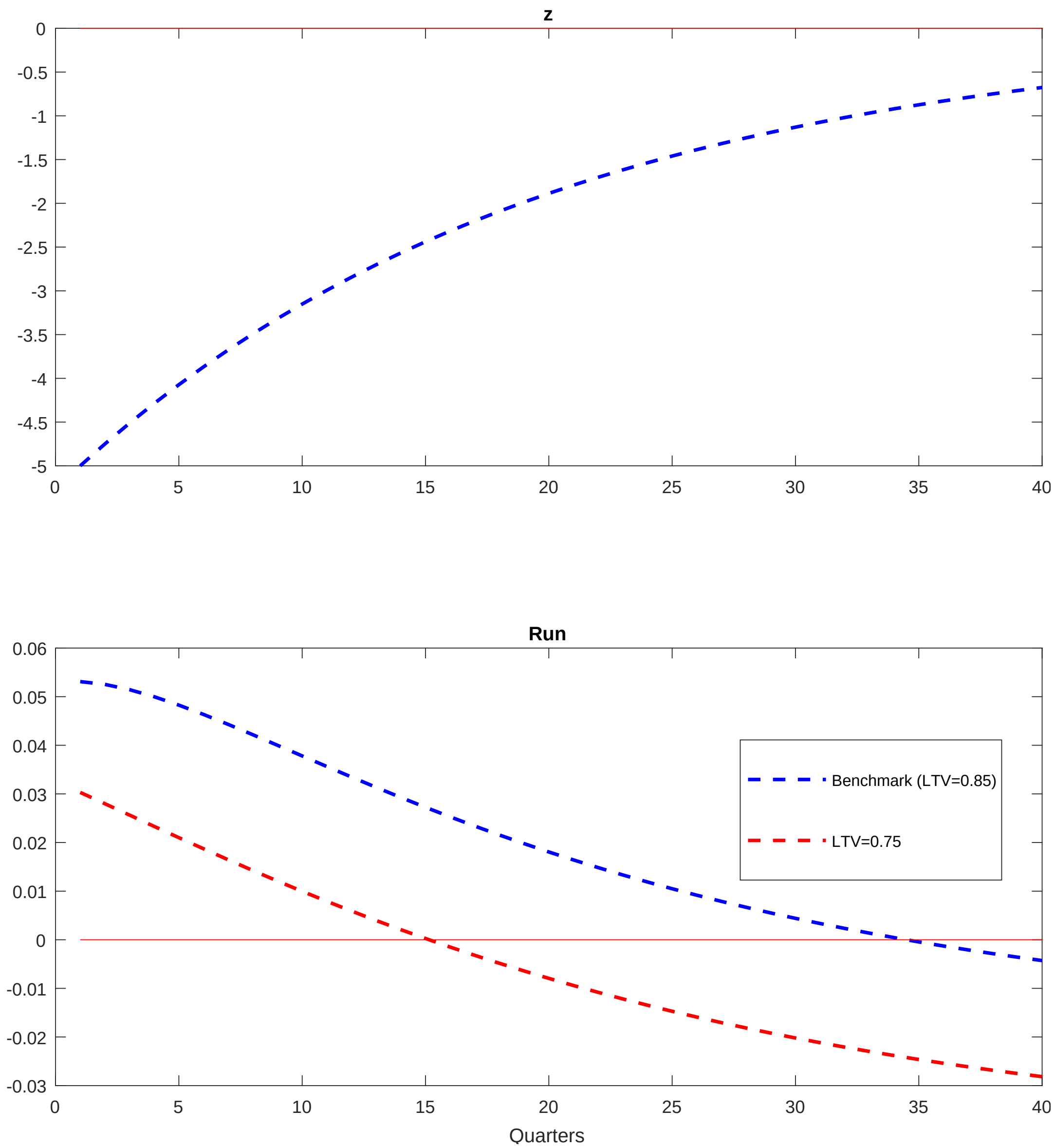


FIGURE 10 – After a Mortgage Shock – Effect of LTV on Run Variable

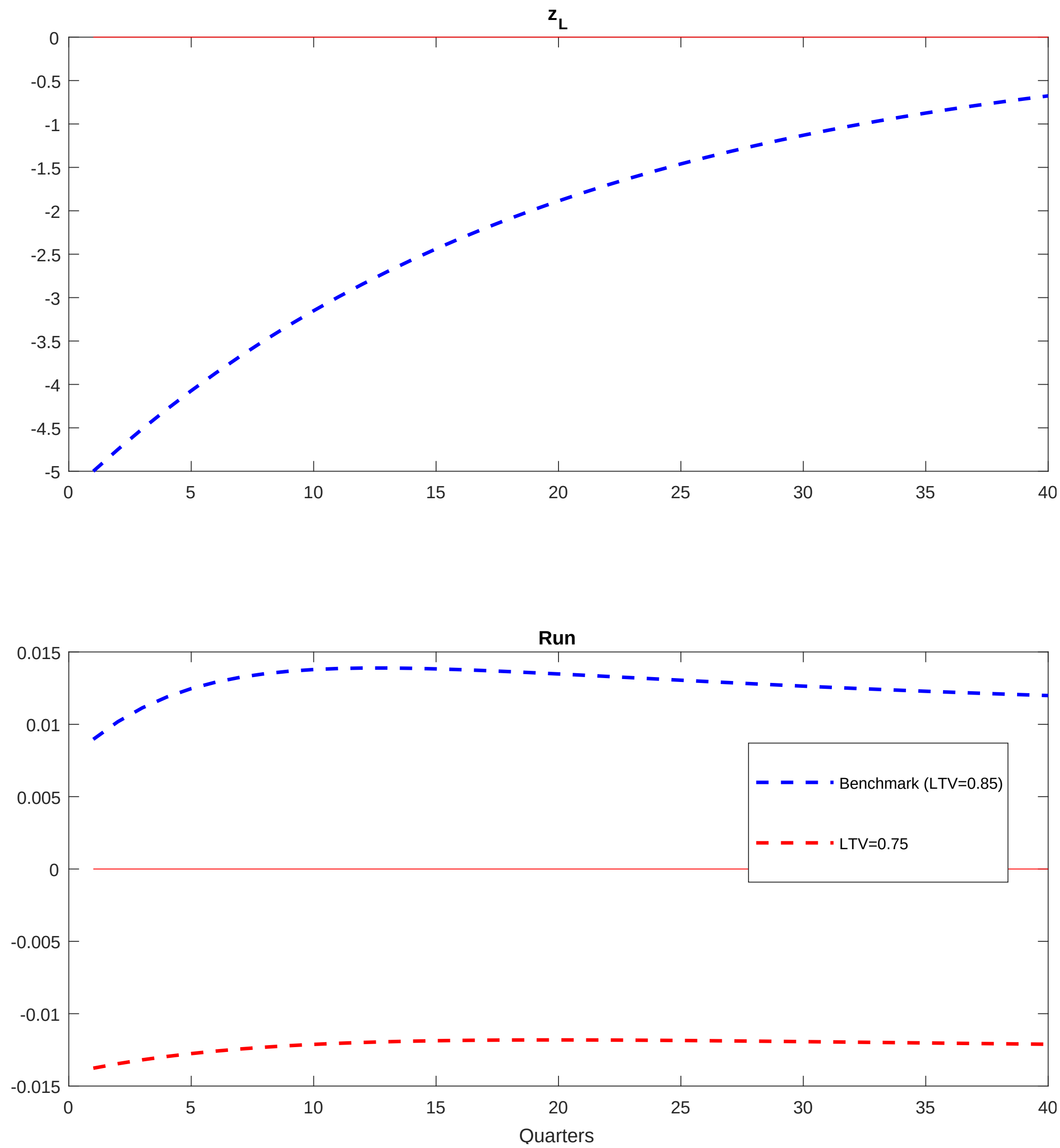
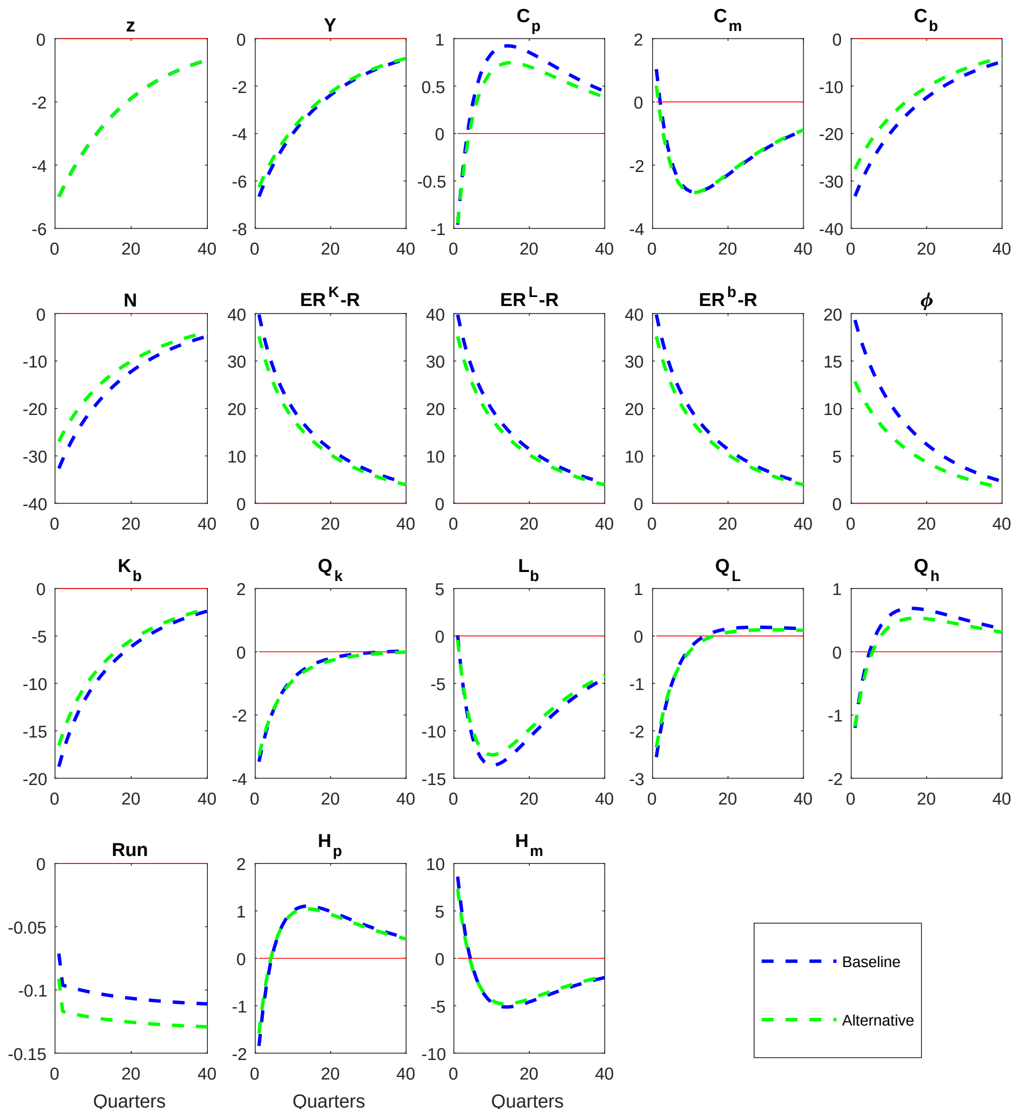
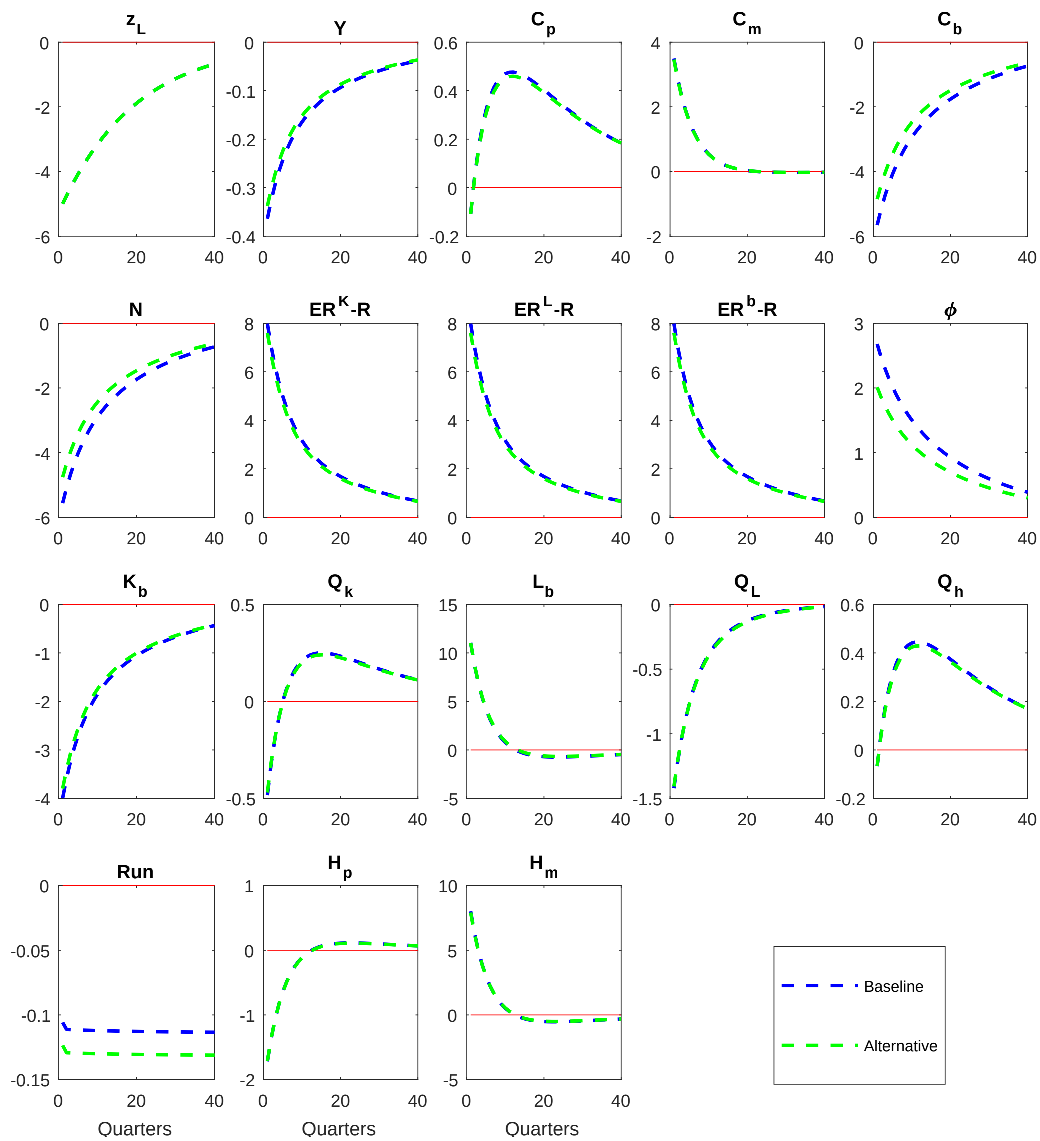


FIGURE 11 – No Run Case – Technology Shock – High vs. Low Bank Leverage



Spreads are in annual basis points. Run is expressed in absolute values. All of the remaining variables are in percentage deviations from their steady-state values. Baseline: ϕ_{ss}=10; Alternative:ϕ_{ss}=8.8

FIGURE 12 – No Run Case – Mortgage Shock – High vs. Low Bank Leverage



Spreads are in annual basis points. Run is expressed in absolute values. All of the remaining variables are in percentage deviations from their steady-state values. Baseline: ϕ_{ss}=10; Alternative:ϕ_{ss}=8.8

FIGURE 13 – After a Technology Shock – Effect of Bank Leverage on Run Variable

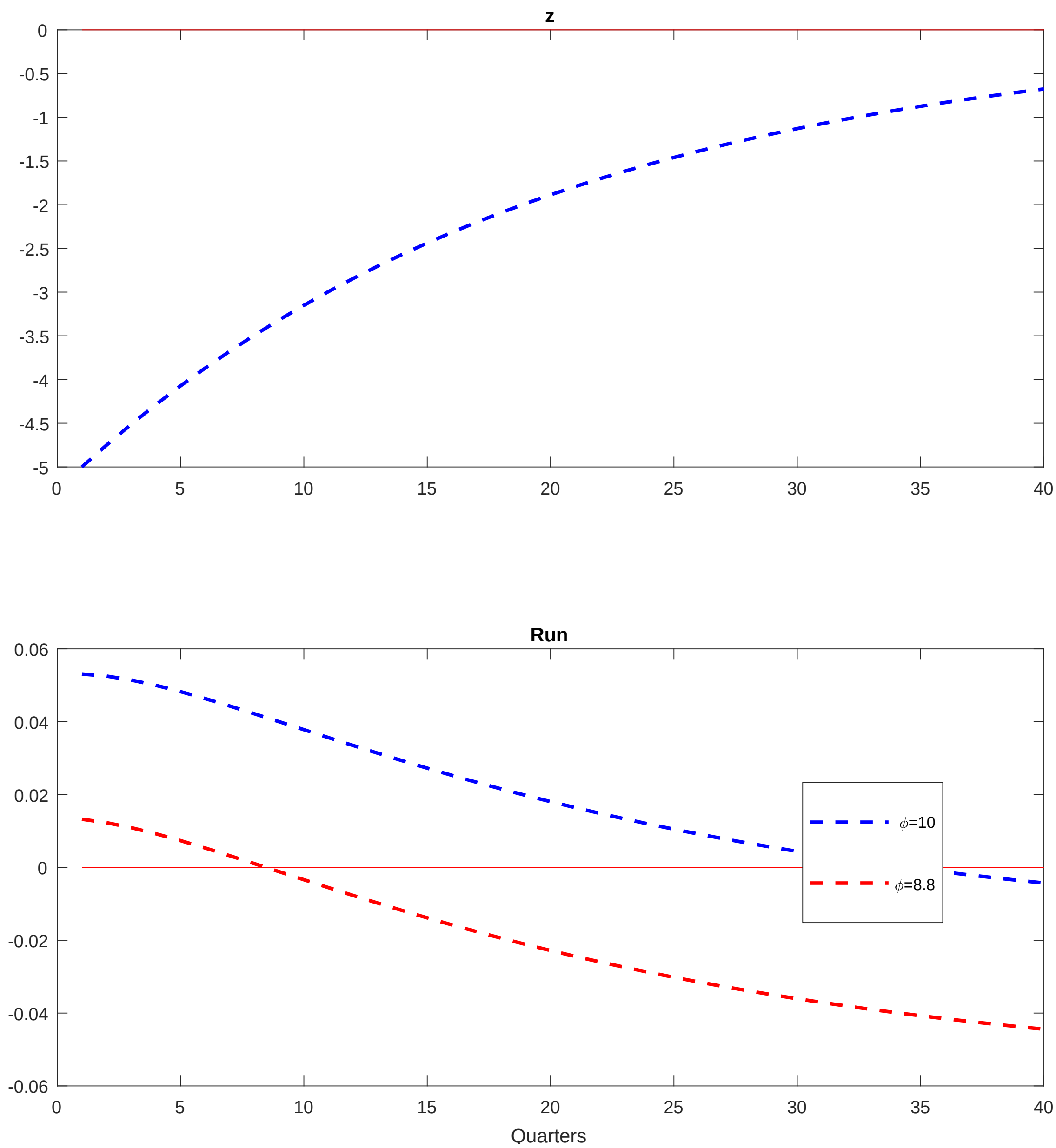
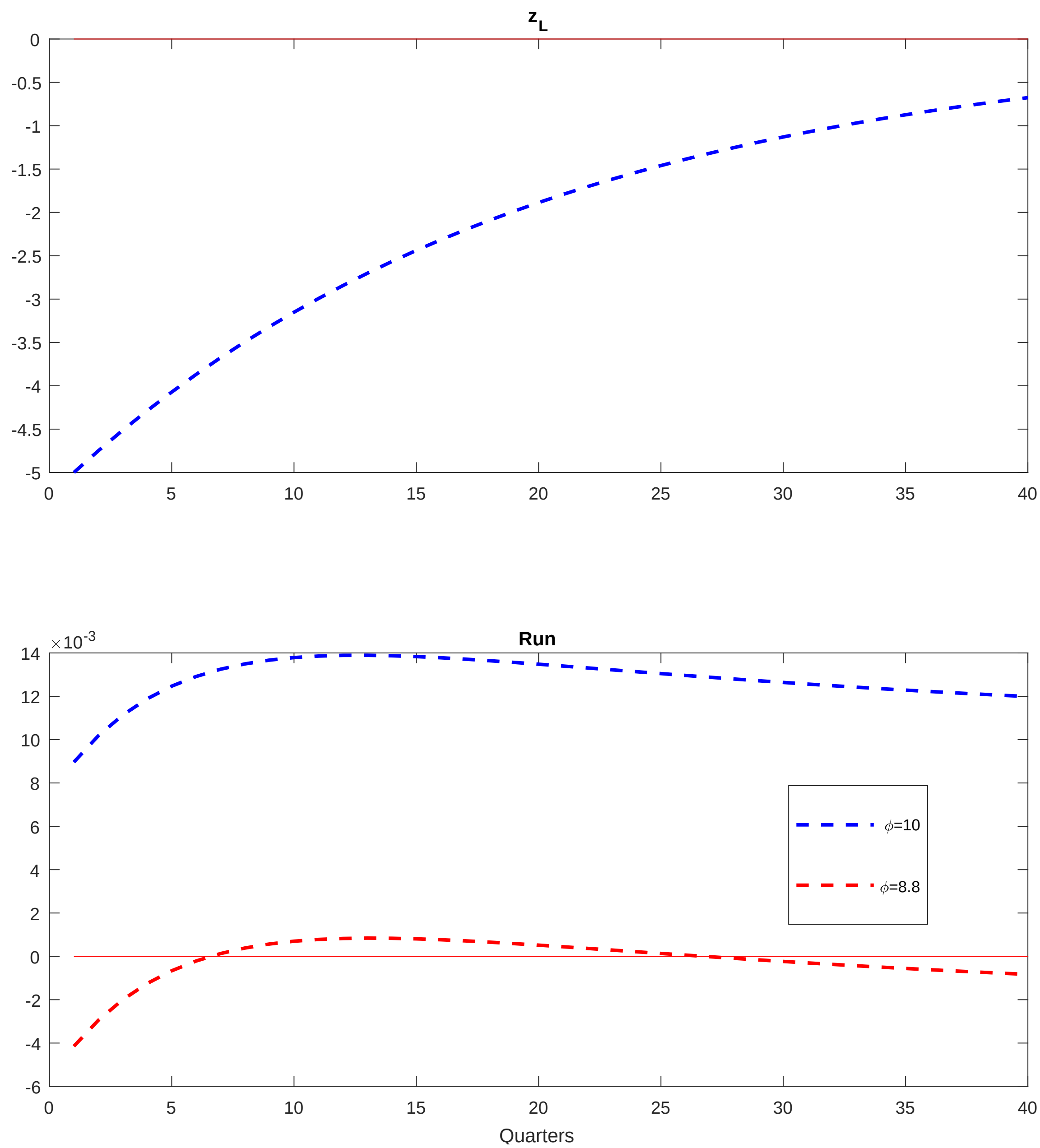


FIGURE 14 – After a Mortgage Shock – Effect of Bank Leverage on Run Variable



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