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Should the most efficient firm invest in its capacity? A value capture approach

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Should the most efficient firm invest in its capacity? A value capture approach*

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Abstract

We consider a game in which firms first install capacity (at no cost) that will constrain how much they can sell in the subsequent stage of the game. In opposition to the classic model (Kreps and Scheinkman, 1983) we suppose that the limited number of firms and buyers makes it such that prices are individually bargained, yielding a wide possible range of outcomes. The resulting biform game is then solved, and we study the circumstances under which we can observe some paradoxes. One such paradox is related to the confidence in the bargaining ability: a more confident firm could install less capacity, and a fully confident firm might still decide to install the minimum capacity. Another paradox is related to the cost advantage, as a more efficient firm might also decide to install less capacity. We show that the biggest contributor to these paradoxes is the strength of the bottom of the demand function: if these agents do value the good highly, it encourages investment in capacity as it increases the bargaining power of the firms, by which it can extract more from their buyers by threatening to sell to these buyers instead. The link between the competitiveness of the rival firm and market efficiency is also less intuitive than expected, as a more competitive rival firm might lead the efficient firm to install less capacity, increasing inefficiency.

1 Introduction

It is an empirical fact that firms in similar economic positions can have drastically different levels of profits, a fact that is counter to classic economic theories. A recent explanation provided using the value capture approach is that firms with market power often deal with firms (clients, suppliers, etc.) that also have market power. Thus, the terms of trade between them will be determined through bargaining. Results of these negotiations are difficult to predict, and will often depend on factors not easy to identify by outside observers, notably negotiation skills and industry norms. Instead of trying to predict the exact price on which they will agree, a recent approach (see for example Brandenburger and Stuart (1996, 2007), MacDonald and Ryall (2004), Adegbesan (2009), Obloj and Zemsky (2015), Montez et al. (2017), and the review of Gans and Ryall (2017)) consists in defining the range of potential prices they can agree on. To obtain that range, the notion of the core (Gillies, 1953), borrowed from cooperative game theory, is used. For all prices outside the range, at least one of the partners has incentives to move away from the partnership, as there are better deals attainable with other partners.

Our starting point is the paradox described by Gans and Ryall (2017) for a market with two consumers and two producers. These producers offer identical products but are limited by their capacity, initially such that they can only serve a single client. We suppose that one of the producers is more efficient than its rival, i.e. that it has a lower marginal cost of production. Gans and Ryall (2017) show that, under some conditions, expanding capacity is a bad idea for the more efficient firm, even if capacity itself is costless: both the lower and upper bounds of the profit range decrease with the expansion. This counter-intuitive phenomenon happens because the firm, by increasing its capacity, affects its negotiation power: pre-expansion it could put in competition the two consumers, which cannot be done anymore after the expansion, when the firm is trying to sell to both.

Compared to Gans and Ryall (2017), our contribution is threefold. First, we study two-stage model in which capacity is simultaneously chosen in strategic manner. Second, we add a third buyer, allowing for a wider variety of equilibria, including some in which at least one buyer is left unserved. Third, we expand and adapt the paradox idea to our setting, obtaining precise conditions for these to occur.

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Capacity decisions play a vital role in firms' strategy. There is a vast literature on non-cooperative, two-stage games in which firms make a capacity precommitment, starting with Kreps and Scheinkman (1983) who find that Bertrand competition in the second stage yields the Cournot outcomes. Moreno and Ubeda (2006) show a similar result for a more general price competition in the second stage. Other related papers include Davidson and Deneckere (1986), Osborne and Pitchik (1986) and Vives (1986). Closer to our approach is Stuart (2005), who studies a game of capacity precommitment followed by a second stage in which the value capture approach is used, and for which the equivalence with Cournot remains. In opposition to our approach, no price discrimination is allowed.

Moreover, the notion that capacity impacts bargaining power is not new, and some real-life examples include airports limiting slots to increase bids by airline companies (Fan and Odoni, 2002), retailers limiting shelf space to increase competition between manufacturers (Marx and Shaffer, 2010) and insurance companies limiting the number of drugs covered by their plans to increase competition among pharmaceutical companies (Huskamp et al., 2005).

To analyze the two - stage game in which firms choose capacity first and then split the surplus, we adopt a biform game, as described in Brandenburger and Stuart (2007). The noncooperative model is typically used to analyze firms strategic decisions while the cooperative model is used to model situations in which value capture plays a crucial role. The biform model offers both noncooperative and cooperative environments.

Our paradox results can serve as a warning for managers, as it highlights the importance of cost advantage and bargaining power and how they can affect the profitability of some decisions, often in counterintuive manners. Furthermore, it shows the impact of bilateral trades in which terms of trades are bargained: market efficiency changes in unexpected ways. In particular, in opposition to traditional models, the trailing firm becoming more competitive might lessen efficiency.

It is worth noting that depending on the capacity installed, the market is a special case of the (one-to-one) assignment problem (Shapley and Shubik, 1971) (if both capacity are one) or of the many-to-one assignment problem (Kelso and Crawford, 1982) otherwise. Both models are well-studied in the economics literature, from the cooperative and non-cooperative points of view.

The paper is divided as follows. Section 2 describes the model, biform games, as well as our equilibrium concepts and our definitions of the paradoxes. We then proceed by backward-induction, solving the second (cooperative) stage in Section 3, obtaining expected profits for all capacity combinations. The first (non-cooperative) stage in which firms simultaneously chose capacities is solved in Section 4. Analysis of the equilibria and occurrences of the paradoxes is provided in Section 5. Section 6 discusses some implications and Section 7 concludes.

2 The model

We consider a biform game played by two firms. The firms are identified as e (for efficient) and r (for rival). Firms produce an identical good or service. We have three potential consumers, labeled 1, 2 and 3, which either consume zero or one unit of the good/service.

Firms first simultaneously decide how much capacity to install, before competing for the business of the three buyers.

We first provide the general definition of biform games, following the definition of Brandenburger and Stuart (2007).

2.1 Biform games

A N-player biform game is a collection $(S^1, ..., S^n; V; \alpha^1, ..., \alpha^n)$ where $S^1, ..., S^n$ are finite set of strategies, V is a map from $S^1 \times ... \times S^n$ to the set of maps from 2^N to the reals, with $V(s^1, ..., s^n)(\emptyset) = 0$, and $0 \leq \alpha^i \leq 1$ for all $i = 1, ..., n$.

In the first (non-cooperative) stage, each agent chooses a strategy among their strategy set. In the second (cooperative) stage, the function V, which depends on the strategies played, describes how much value each group of agents can generate. From this, we compute the core, and find, for every agent, its minimal and maximal core allocation. Agent i then estimates her payoff to be her minimal core allocation plus the difference between her maximal and minimal core allocation, multiplied by α^i. The parameter can thus be interpreted as a confidence index: if α^i is large, the agent is confident that she will be able to extract close to her maximal core allocation.

Given these expected payoffs, firms choose their first-period strategies.

2.2 Market structure

In addition to the two firms, the market contains three buyers. The valuation of buyer i for the good/service is u_i, with $u_1 \geq u_2 \geq u_3$.[1]

Firms have constant marginal costs of production. We suppose that this marginal cost is normalized to zero for firm e and is $c_r > 0$ for firm r. Thus, c_r can be interpreted as the cost advantage of firm e. We assume that $u_3 \geq 0$ (firm e can credibly serve all three buyers) and $c_r \leq u_1$ (firm r can credibly serve at least the first buyer). We suppose that all these values are common knowledge.

To serve buyers, firms must have installed capacity in the first stage. In the first stage, firms simultaneously choose their capacity.

2.3 The non-cooperative stage: installation of capacities

We assume that capacity installation is costless, which implies that $k_i = 0$ is a (weakly) dominated strategy, allowing to simplify the exposition. Since $k_e \geq 1$ and $c_r > 0$, it is obvious that in any situation firm e will sell to at least one buyer, thus making $k_r = 3$ a weakly dominated strategy.

Thus, $S^e = \{1, 2, 3\}$ and $S^r = \{1, 2\}$. Buyers have no strategic choices.

2.4 The cooperative stage: generation of market value

In the second stage, firms and buyers observe the installed capacities and simultaneously negotiate with each other, under the constraint that firm i cannot sell to more than k_i buyers.

To simplify, we write $V(k_e, k_r)(S)$ as $V^{k_e,k_r}(S)$ for all S in $N = \{e, r, 1, 2, 3\}$. It represents the maximum value being generated by trades among members of S given the capacities.

In a typical cooperative problem, we would need to allocate the surplus among the participants. These allocations are constrained by stability conditions: if the combined allocations of a group of market participants is lower than the surplus they can jointly generate by trading only among themselves, then this group will secede from the grand coalition and trade among themselves. The set of allocations of the surplus that removes any incentives for groups to secede in such a manner is called the core (Gillies, 1953).

Formally, for each $S \subseteq N$, let $V^{k_e,k_r}(S)$ be the (maximal) surplus generated by the coalition S when capacities are k_e, k_r. Computations of these values are presented in Section 3. For a vector $x \in R_+^N$ and a coalition $S \subseteq N$, let $x_S = \sum_{i \epsilon S} x_i$. An allocation is $y \in R_+^N$ such that $y_N = V^{k_e,k_r}(N)$. A core allocation y is an allocation such that $y_S \geq V^{k_e,k_r}(S)$ for all $S \subseteq N$. We write $\pi_i^{\min}(k_e, k_r)$ to denote the minimal core allocation of firm i and $\pi_i^{\max}(k_e, k_r)$ to be its maximal core allocation when capacities are (k_e, k_r).

Given the confidence indexes α^e, α^r we obtain expected profits of

$$\begin{aligned} E\pi_i(k_e, k_r) &= \pi_i^{\min}(k_e, k_r) + \alpha^i \left(\pi_i^{\max}(k_e, k_r) - \pi_i^{\min}(k_e, k_r)\right) \\ &= (1 - \alpha^i)\pi_i^{\min}(k_e, k_r) + \alpha^i \pi_i^{\max}(k_e, k_r) \end{aligned}$$

2.5 Equilibrium

Using the expected profits computed in the previous section, we look for the Nash equilibrium when firms select their capacities simultaneously. Notice that in biform games the equilibria depend on the confidence indexes.

An equilibrium in this biform game is a pair (k_e^*, k_r^*) such that

$$\begin{aligned} E\pi_e(k_e^*, k_r^*) &> E\pi_e\left(k_e^{'}, k_r^*\right) \text{ for all } k_e' < k_e^* \\ E\pi_e(k_e^*, k_r^*) &\geq E\pi_e(k_e'', k_r^*) \text{ for all } k_e'' > k_e^* \\ E\pi_r(k_e^*, k_r^*) &> E\pi_r\left(k_e^*, k_r^{'}\right) \text{ for all } k_r' < k_r^* \\ E\pi_r(k_e^*, k_r^*) &\geq E\pi_r(k_e^*, k_r'') \text{ for all } k_r'' > k_r^* \end{aligned}$$

The asymmetry in the equilibrium conditions are to avoid multiplicity of equilibria: a firm only invests in a larger capacity if it strictly increases its expected profits. Without this, for example, in an equilibrium where firm e sells to all three buyers, firm r would be indifferent between installing 1 or 2 units of capacity, creating multiple uninteresting equilibria.

[1] More precisely, buyers have a quasi-linear utility function in money, with the willingness of buyer i to pay for the first unit is u_i and zero for all subsequent units.

2.6 The paradoxes

Gans and Ryall (2017) describe the paradox of a firm having lower maximal and minimal core allocation after having increased its capacity. The example that they give is a simpler version of our model, with only two buyers and the capacity of the rival firm set at one.

In our setting, given that firm e has three possible levels of capacities, it would be very demanding to define the paradox as the lower and upper bounds to be higher at capacity 1 than at both higher capacities. We keep the general message of Gans and Ryall's paradox in the following way. While it might be a decent strategy for a firm in a dominant position to restrict its capacity in order to extract more from its clients, we would still expect the following behaviors:

1. When the confidence in its bargaining ability increases, the efficient firm (weakly) increases its capacity.
2. In any situation, if we increase enough the confidence index in the bargaining ability, the efficient firm will install more than one unit of capacity.
3. When the cost advantage increases, the efficient firm (weakly) increases its capacity.
4. In any situation, if we increase enough the cost advantage, the efficient firm will install more than one unit of capacity.

We call violations of the first and third observations the **decreasing capacity paradox of the confidence index** and the **decreasing capacity paradox of cost advantage**, respectively**.** We call violations of the second and fourth observations the **flat capacity paradox of the confidence index** and the **flat capacity paradox of cost advantage.**

In mathematical terms, the decreasing capacity paradox of the bargaining index occurs when the equilibria are such that firm e plays k_e at α^e and $k_e' < k_e$ at $\alpha_e' > \alpha_e$ (all else constant). The flat capacity paradox of the confidence index occurs when firm e plays $k_e = 1$ when $\alpha_e = 1$. The decreasing capacity paradox of cost advantage occurs when firm e plays k_e at c_r and $k_e' < k_e$ at $c_r' > c_r$ (all else constant). The flat capacity paradox of cost advantage occurs when firm e plays $k_e = 1$ when $c_r = u_1$, its highest possible value.

3 Second stage: cooperative game

Let $V(s,b)$, with $s \in \{e,r\}$ and $b \in \{1,2,3\}$ be the value of matching seller s with buyer b. More precisely, $V(s,b) = \max\{u_b - c_s, 0\}$. Notice that $V(e,b) = u_b$ for all $b \in \{1,2,3\}$ and $V(r,1) = u_1 - c_r$.

Fix the capacities (k_e, k_r) and $S \subseteq N$. It is clear that if $S \cap \{e,r\} = \emptyset$ or if $S \cap \{1,2,3\} = \emptyset$, then $U^{k_e,k_r}(N) = 0$ as only one side of the market is present.

Otherwise, it is clear that the way to maximize surplus is to assign the highest-valued buyers to firm e (if present in S) until it has reached its capacity, then match any remaining buyers with r. Formally, let b_i^S be the i^{th} buyer in coalition S, according to the order $\{1,2,3\}$. Let B^S be the number of buyers in S. Then,

$$V^{k_e,k_r}(S) = \begin{cases} \sum_{i=1}^{B^S} V(e, b_i^S) \text{ if } e \in S \text{ and } B^S \leq k^e \\ \sum_{i=1}^{k_e} V(e, b_i^S) + \sum_{i=k_e+1}^{\min\{B^S, k_e+k_r\}} V(r, b_i^S) \text{ if } e \in S \text{ and } B^S > k^e \\ \sum_{i=1}^{\min\{B^S, k_r\}} V(r, b_i^S) \text{ if } e \notin S \end{cases}$$

We then obtain the minimal and maximal core allocations for e and r, allowing us to obtain $E\pi_i(k_e, k_r)$ for all combinations of capacities.

Using the convention $V(e,i) = V(r,i) = 0$ if $i > 3$, we can write the upper bounds for firm e as follows:

$$\pi_e^{\max}(k_e, k_r) = \sum_{i=1}^{k_e} \left[V(e,i) - V(r,i) + V(r, k_e + k_r)\right].$$

Detailed calculations are presented in the appendix.

It can be interpreted as follows: From each of its clients, firm e can extract $V(e,l) - V(r,l) + V(r, k_e + k_r)$. $V(e,l) - V(r,l)$ is the difference in surplus when buyer l buys from e instead of r. In addition, if firm r was already selling to somebody else (buyer $k_e + k_r$) that reduces of bargaining power of buyer l, allowing firm e to (potentially) extract the value of firm r with that buyer, $V(r, k_e + k_r)$.

Similarly, the lower bounds can be written as follows:

$$\pi_e^{\min}(k_e, k_r) = k_e \left[V(e, k_e+1) - V(r, k_e+1) + V(r, k_e+k_r+1) \right].$$

At worst, firm e is able to extract $V(e, k_e+1) - V(r, k_e+1)$ from each buyer, which is the extra surplus if it sells to the next available buyer. If there is a free buyer (which in our model only happens when $k_e = k_r = 1$) then this next available buyer has less bargaining buyer, allowing e to also extract $V(r, k_e+k_r+1)$. We can see that the upper bound becomes 0 when $k_e = 3$.

As for firm r, we can write the upper bounds as follows:

$$\pi_r^{\max}(k_e, k_r) = k_r \left[V(r, k_e+k_r) \right].$$

From each buyer, given that the firm has to fill all of its slots, the most it can extract is the value generated by the last of its buyers. We can already see that if there is extra capacity installed, the upper bound is zero.

The lower bound can be written as:

$$\pi_r^{\min}(k_e, k_r) = k_r \left[V(r, k_e+k_r+1) \right].$$

This lower bound is obtained when firm r has bargaining power because there is an extra buyer that doesn't buy from anybody. This occurs only when $k_e = k_r = 1$.

Putting it all together, we obtain that expected profits (π_e, π_r) at (k_e, k_r) can be described as

$$(\pi_e(k_e,k_r), \pi_r(k_e,k_r)) = \left(\begin{array}{c} \left[\begin{array}{c} \alpha^e \sum_{i=1}^{k_e} \left[V(e,i) - V(r,i) + V(r, k_e+k_r) \right] + \\ (1-\alpha^e) k_e \left[V(e, k_e+1) - V(r, k_e+1) + V(r, k_e+k_r+1) \right] \end{array} \right], \\ k_r \left(\alpha^r V(r, k_e+k_r) + (1-\alpha^r) V(r, k_e+k_r+1) \right) \end{array} \right)$$

4 First stage: choice of capacities

Using the expected profits formulas as defined above, we can then find conditions under which a particular combination of capacities is an equilibrium, by comparing with deviations in the capacity choices.

To obtain more precise conditions, we divide our analysis in three cases, depending on how many buyers can generate surplus with firm r. Computations are in the appendix, with only the results presented below.

We first discuss the case in which firm r can only sell to buyer 1: $V(r,2) = 0$. This is the case with the largest competitive advantage. Table 1 presents the equilibrium conditions for each pair of capacities.

	$k_r = 1$	$k_r = 2$
$k_e = 1$	$\alpha^e \leq \frac{u_2 - 2u_3}{2(u_2 - u_3)}$	$\varnothing$
$k_e = 2$	$\frac{u_2 - 2u_3}{2(u_2 - u_3)} < \alpha^e \leq \frac{2}{3}$	$\varnothing$
$k_e = 3$	$\alpha^e > \frac{2}{3}$	$\varnothing$

Table 1: Equilibrium conditions when $V(r,2) = 0$

In this case, in which r is not much of a rival for firm e, results are very much as expected, as there are no equilibria in which firm r installs two units of capacity, and capacity increases with the confidence index. This implies that minimal bounds are decreasing and maximal bounds are increasing with capacity.

We next move to a more competitive situation in which firm r can sell to buyer 2, but not buyer 3: $V(r,2) > V(r,3) = 0$. Then the equilibrium conditions are presented in Table 2.

	$k_r = 1$	$k_r = 2$
$k_e = 1$	$\left[\begin{array}{c} \alpha^e \leq \min \left[\frac{c_r - 2u_3}{3c_r - u_2 - 2u_3}, \frac{c_r}{\max(3c_r + u_3 - u_2, 0)} \right] \text{ if } 3c_r - u_2 - 2u_3 \geq 0 \\ \frac{c_r - 2u_3}{3c_r - u_2 - 2u_3} \leq \alpha^e \leq \frac{c_r}{\max(3c_r + u_3 - u_2, 0)} \text{ if } 3c_r - u_2 - 2u_3 \leq 0 \end{array} \right]$	$\varnothing$
$k_e = 2$	$\left[\begin{array}{c} \frac{c_r - 2u_3}{3c_r - u_2 - 2u_3} < \alpha^e \leq \frac{2}{3} \text{ if } 3c_r - u_2 - 2u_3 \geq 0 \\ \alpha^e < \min \left[\frac{c_r - 2u_3}{3c_r - u_2 - 2u_3}, \frac{2}{3} \right] \text{ if } 3c_r - u_2 - 2u_3 \leq 0 \end{array} \right]$	$\varnothing$
$k_e = 3$	$\alpha^e > \max \left[\frac{c_r}{\max(3c_r + u_3 - u_2, 0)}, \frac{2}{3} \right]$	$\varnothing$

Table 2: Equilibrium conditions when $V(r,2) > V(r,3) = 0$

With firm r competing with firm e for the business of the second buyer, we still have no equilibria in which firm r installs 2 units of capacity, and we again require a large confidence in the bargaining ability to install 3 units. A new feature is the possibility of flip-flops between one and two units of capacity, resulting from the fact that it is not clear if minimum and maximum bounds are larger with one or two units of capacity. More precisely, the interval for the profits of firm e when $(k_e, k_r) = (1, 1)$ is $[c_r, u_2]$ while when $(k_e, k_r) = (2, 1)$ it is $[2u_3, 2c_r]$.

Finally, we examine the case for which firm r is a serious competitor who can sell to all buyers: $V(r, 3) > 0$:. Then, the equilibrium conditions are given in Table 3

	$k_r = 1$	$k_r = 2$
$k_e = 1$	$\left[\begin{array}{c} \alpha^r \leq \frac{u_3-c_r}{\max(3u_3-u_2-2c_r,0)} \\ \alpha^e \leq \min\left[\frac{u_3-2c_r}{3u_3-u_2-2c_r}, \frac{u_3}{\max(3c_r+u_3-u_2,0)}\right] \text{ if } 3u_3-u_2-2c_r \geq 0 \\ \frac{u_3-2c_r}{3u_3-u_2-2c_r} \leq \alpha^e \leq \frac{u_3}{\max(3c_r+u_3-u_2,0)} \text{ if } 3u_3-u_2-2c_r \leq 0 \end{array}\right]$	$\left[\begin{array}{c} \alpha^r > \frac{u_3-c_r}{\max(3u_3-u_2-2c_r,0)} \\ \frac{c_r}{u_3-c_r} \leq \alpha^e \leq \frac{c_r}{\max(4c_r-u_3,0)} \end{array}\right]$
$k_e = 2$	$\left[\begin{array}{c} \frac{u_3-2c_r}{3u_3-u_2-2c_r} < \alpha^e \leq \frac{2c_r}{\max(5c_r-2u_3,0)} \text{ if } 3u_3-u_2-2c_r \geq 0 \\ \alpha^e < \min\left[\frac{u_3-2c_r}{3u_3-u_2-2c_r}, \frac{2c_r}{\max(5c_r-2u_3,0)}\right] \text{ if } 3u_3-u_2-2c_r \leq 0 \end{array}\right]$	$\varnothing$
$k_e = 3$	$\alpha^e > \max\left[\frac{u_3}{\max(3c_r+u_3-u_2,0)}, \frac{2c_r}{\max(5c_r-2u_3,0)}\right]$	$\varnothing$

Table 3: Equilibrium conditions when $V(r, 3) > 0$

While $k_r = 1$ is a dominant strategy when $V(\{r, 3\}) = 0$, when it is strictly positive firm r might prefer to install 2 units of capacity when e installs a single unit and when its confidence is high. This opens up the possibility of an equilibria in which $k_e = 1$ and $k_r = 2$.

Interestingly, the way firm e evaluates installing one unit versus two depends on how many units of capacity firm r installs. If r installs a single unit it behaves in a similar manner to when $V(\{r, 2\}) > V(\{r, 3\}) = 0$: the capacity it prefers when it has a low confidence depends on the respective rankings of the minimum core bounds. If $3u_3 > u_2$ (the value of buyer 3 is not small compared to the value of buyer 2) and c_r is small (the cost advantage is small), the lower bound is higher bound is larger for capacity 1. When these inequalities are satisfied, the lower bound is higher as it is better to leave buyer 3 unserved and use him as leverage over buyer 1 than it is to install a second unit and sell also to buyer 2, with little leverage as the small value of c_r means that buyers do not particularly care to be served by e or r. In other cases, the lower bound is higher with 2 units of capacity.

With $k_r = 2$, the logic is different: if firm e installs a second unit, we will have extra capacity installed, which destroys the bargaining power of both firms. It is in fact easy to see that in that case, $\pi_e^{\min} = \pi_e^{\max} = c_r$: firm e's confidence has no impact, as its payoff is fixed. By opposition, with $k_e = 1$, the lower bound is c_r and the upper bound is u_3. It's thus only when $u_3 > 2c_r$, i.e. buyer 3 has a high valuation, twice as large as the cost of firm r, that installing a single unit becomes an alternative.

5 Analysis

We first examine the existence of an equilibria in pure strategies. We obtain the following result.

Theorem 1. *Suppose that $V(r, 3) > 0$, $3u_3 - u_2 - 2c_r > 0$ and $\alpha^r > \frac{u_3-c_r}{3u_3-u_2-2c_r}$.*

If $\alpha^e \leq \min\left[\frac{u_3-2c_r}{3u_3-u_2-2c_r}, \frac{u_3}{\max(3c_r+u_3-u_2,0),}, \frac{c_r}{u_3-c_r}\right]$ or $\frac{c_r}{\max(4c_r-u_3,0)} < \alpha^e \leq \min\left[\frac{u_3-2c_r}{3u_3-u_2-2c_r}, \frac{u_3}{\max(3c_r+u_3-u_2,0),}\right]$, there is no pure-strategy equilibria. Otherwise, there exists a unique pure-strategy equilibrium.

The existence of an equilibrium in pure strategies is not guaranteed: suppose that if firm r installs a single unit of capacity, firm e prefers to install a single unit. If the confidence of firm r is high, it will deviate to install 2 units. But if the confidence of firm e does not fall in the prescribed range, it will respond to firm r installing 2 units by installing 2 or 3 units of capacity. Then, firm r deviates by installing a single unit of capacity. And given that, firm e changes its capacity back to one, and the cycle of deviations restarts.

Notice that for firm r to be tempted to install 2 units of capacity, we need a very specific set of circumstances. First, it needs to be able to sell to buyer 3 ($V(r, 3) > 0$). Second, the difference in value put on the object by buyers 2 and 3 must be lower than the surplus it can generate with buyer 3 ($3u_3 - u_2 - 2c_r > 0$, which can be rewritten as $2(u_3 - c_r) > u_2 - u_3$. Finally, its confidence index must be high.

It is easy to verify that in other cases, there is a single equilibrium in pure strategies. In particular, if $k_e > 1$, the dominant strategy for firm r is to install a single unit of capacity.

Notice also, from Section 4.1, that if firm r is not a viable rival, being only able to sell to buyer 1 ($V(r, 2) = 0$), then not only is installing a single unit of capacity a dominant strategy for firm r, but the optimal capacity for firm e is increasing in α^e. This is the intuitive behavior with respect to α^e. The presence of firm e crowds firm r out of the market. Firm e expands its operation beyond the first consumer only if it is confident in its negotiation abilities.

We next examine the conditions under which we obtain our paradoxes, starting with the paradoxes of the confidence index.

Theorem 2. *The flat capacity paradox of the confidence index occurs if and only if we are in one of these situations:*
i) $u_3 > c_r$ *and* $u_2 \geq \max[3c_r, 2u_3]$
ii) $u_2 > c_r \geq u_3$ *and* $u_2 - u_3 \geq 2c_r$

The flat capacity paradox of the confidence index occurs when the valuation of the second buyer is large, compared to the valuation of the third buyer and to the cost advantage. Then, if it installs a single unit, it can extract a large surplus from buyer 1, by threatening (credibly) to sell to buyer 2 instead. If it installs two units, the threat to sell to the third buyer instead of the second one is much less effective, resulting in firm e preferring to install a single unit of capacity.

Theorem 3. *The decreasing capacity paradox of the confidence index occurs if and only if we are in one of these situations:*
i) $c_r < u_3 < \min\left[\frac{u_2}{2}, 2c_r\right]$
ii) $u_3 \leq c_r < \min\left[\frac{u_2}{2}, 2u_3\right]$

The first case occurs when firm r can sell to the third buyer, but where the third buyer has a much lower value than the second buyer, and in which the cost advantage is relatively large. Then, when the confidence is low, firm e installs 2 units of capacity, as it allows to extract a decent amount from buyers 1 and 2, as selling to buyer 3 is a moderately effective threat. As confidence grows, firm e installs a single unit of capacity in the hope that it will be able to extract a larger amount from buyer 1, using the sale to buyer 2 as a threat.

The second case is similar, but is such that firm r cannot sell to buyer 3. That puts firm e in a better position when it installs two units of capacity, as selling to buyer 3 is a better threat, as it sits idle. But giving the large difference in value between buyers 2 and 3, as firm e gains in confidence it installs a single unit, convinced that it will be able to extract more from buyer 1.

We next move to the paradoxes of cost advantage.

Theorem 4. *i) The flat capacity paradox of cost advantage occurs if and only if* $u_2 \geq 2u_3$.
ii) The decreasing capacity paradox of cost advantage occurs if and only if $u_2 \geq 2u_3$.

If the value of the second buyer is more than twice as large as the value of the third buyer, when confidence is low, firm e reduces the capacity installed when the cost advantage increases. With a very large cost advantage, firm e prefers to extract as much as possible from buyer 1 by putting him in competition with buyer 2. With a smaller cost advantage, such that firm r can sell to the second buyer, firm e prefers to install 2 units, as it would lose the bargaining power over buyer 1, as buyer 2, used to create bargaining power, would now buy from firm r.

To illustrate how the equilibria varies with the parameters, we fix $u_3 = 1$, and initially also $u_2 = 1.8$. We then examine the equilibria as functions of c_r and α^e. The results are in Figure 1.

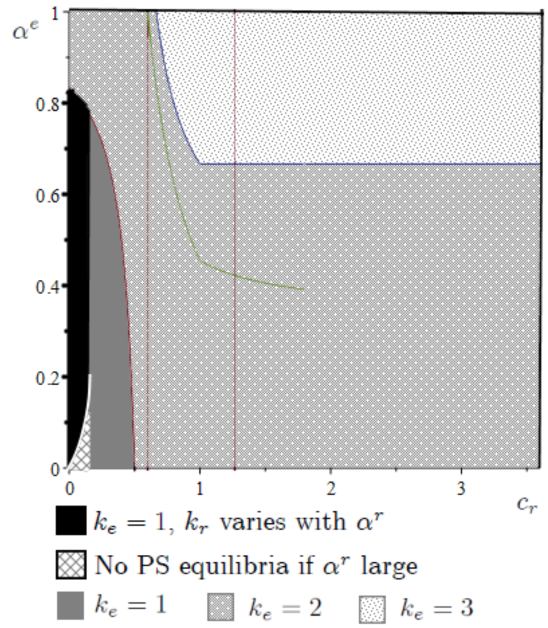

Figure 1: Equilibrium capacity for firm e as a function of c_r and α^e, when $u_2 = 1.8$ and $u_3 = 1$

This case is quite intuitive, with no paradox occurences. Firm e installs a single unit of capacity only when it has a tiny cost advantage over firm r, and low confidence in its bargaining ability, and it installs three units of capacity only if it has a large cost advantage over firm r and high confidence in its bargaining abilities. It is however noteworthy that for very small cost differences, firm r might be tempted to install two units of capacity. When the cost difference is very small, this might result in an equilibrium where firm r has two units of capacity and firm e a single one, requiring high confidence for r and low confidence for e. As the cost advantage increases however, firm e might fight this. It results in a zone in which if firm r is confident enough in its abilities, it will want to install two units of capacities when firm e installs one, but if firm e has low confidence in its bargaining ability, it counteracts with the installment of a second unit, resulting in a cycle as described in Theorem 1.

Constrast this with the case in which there is a bigger difference in valuations between the second and third consumers: $u_2 = 2.25$. Then, we obtain a completely different picture, as depicted in Figure 2.

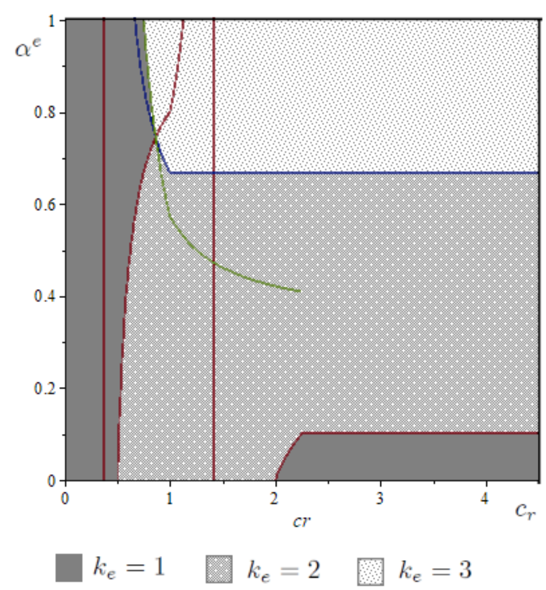

Figure 2: Equilibrium capacity for firm e as a function of c_r and α^e, when $u_2 = 2.25$ and $u_3 = 1$

Then, with low cost differences, not only do we always have a pure strategy equilibrium, that equilibrium is such that $k_e = 1$, regardless of the confidence level of firm e. This is the flat capacity paradox of the confidence index, as described in Theorem 2, part i). For values of c_r around 0.8, we have an occurence of the decreasing capacity paradox of the confidence index, as in Theorem 3: for low confidence level firm e installs capacity of 2, for high confidence it installs capacity of 3, and in between it installs capacity of 1. To better understand what is going on, the following table gives the minimum and maximum profits for all capacity levels, when $c_r \leq u_3 = 1$.

k_e	$\pi_e^{\min}$	$\pi_e^{\max}$
1	1	2.25
2	$2c_r$	2
3	0	$3c_r$

Thus, for $c_r \leq 0.5$, capacity 1 is clearly dominant: it has higher lower and upper bounds. With $0.5 \leq c_r \leq 0.75$, firm e still has higher lower and upper bounds with capacity 1 than with capacity 3. However, the lower bound is largest with capacity 2, resulting in it being played when confidence is low. For $0.75 \leq c_r \leq 1$, then capacity 3 has a larger upper bound, meaning that a very confident firm e will select that capacity. Still, there exists medium levels for which capacity 1 is chosen. In particular, suppose $c_r = 0.8$. Then, firm e installs capacity 2 if $\alpha^e < 0.705$ and capacity 3 if $\alpha^e > 0.87$. In between, the firm installs capacity 1.

Notice also that for very low confidence levels, firm e installs capacity 1 if c_r is small, capacity 2 when c_r has a medium value, but goes back to capacity 1 when c_r is large. Those are occurences of both the flat capacity and decreasing capacity paradoxes of cost advantage, as decribed in Theorem 4.

If $u_2 - u_3$ increases, as in Figure 3 where $u_2 = 4.25$, then $k_e^* = 1$ occurs much more frequently, and we have multiple instances of the confidence index and cost advantage paradoxes.

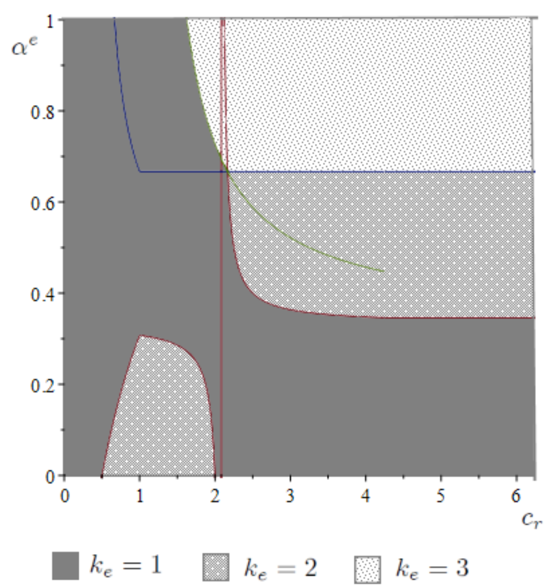

Figure 3: Equilibrium capacity for firm e as a function of c_r and α^e, when $u_2 = 4.25$ and $u_3 = 1$

First, we can see that firm e installs a single unit of capacity when $\alpha^e = 1$ and c_r roughly smaller than 1.7. This is the flat capacity paradox of the confidence index, and it covers both parts i) and ii) of Theorem 2. We also see that if $0.5 \leq c_r \leq 2$, firm e installs 2 units of capacity if the confidence is low enough, but reduces that capacity to 1 for larger values. Those are examples of the decreasing capacity paradox of the confidence index, covering both parts i) and ii) of Theorem 3. Finally, it is easy to see that both the flat capacity and decreasing capacity paradoxes of cost advantage occur more often in this example.

6 Implications

Our results show that the paradoxes can occur quite frequently. We discuss in this section the implications of the paradoxes.

6.1 Managerial decisions

The idea that a firm with a cost advantage should expand to capitalize on this advantage seems like a no-brainer. But this is without taking into account the impacts on the bargaining powers of the agents involved.

In general, we show that unless a firm has an important cost advantage over its rival and it has strong bargaining abilities, it will install less than the maximum capacity. This is because the bargaining position might worsen: we must consider what are the outside options for the buyers and for firm e. Expanding capacity might simultaneously improve the outside options for the buyers and worsen those of firm e. Even just one of these outcomes might be enough to discourage investment.

6.2 Efficiency of the market

In our simple model, social welfare would be maximized if the efficient firm installed 3 units of capacity, yielding welfare of $u_1 + u_2 + u_3$. In situations where firm e installs a single unit of capacity, welfare loss is $u_2 + u_3$ if firm r cannot sell to the second buyer and $c_r + u_3$ otherwise. If firm e installs two units, the welfare loss is u_3 if firm r cannot sell to the third buyer and c_r otherwise.

Instances of the paradoxes are thus such that the inefficiency grows as the confidence index and/or the cost advantage increases. The inefficiency might remain very large even if the confidence and/or the cost advantage reaches their maximum value.

One way to reduce the paradox is to generate stronger demand at the bottom of the demand function (u_3). While this has a direct, intuitive effect by increasing the potential amount to extract if firm e installs three units of capacity, it also has a less intuitive effect on the bargaining power. When u_2 is large compared to u_3, firm e finds it more profitable to install a single unit and extract more from buyer 1, as selling to buyer 2 is a good threat that gives the firm some leverage. By opposition, when it installs two units of capacity, the firm is left with a weak hand as selling to the third buyer is a weak threat.

We can also see in the examples the impact of competition, by comparing equilibria when c_r is very large (for which firm e has no competition) to when c_r is close to zero (for which r is a very effective competitor). In almost all cases, a lower c_r (and thus more competition) means less capacity installed and higher welfare losses. The exception is when we have the decreasing capacity paradox of cost advantage, in which case the welfare loss might (temporarily) decrease as competition increases. Still, this result that inefficiency increases as competition increases is contrary to what is obtained in traditional models.

In a similar manner, the impact of policies that help dominant firms establish their bargaining power (setting industry standards on surplus sharing through government contracts, or setting minimum prices, for example) is unclear, because of the flat capacity and decreasing capacity paradoxes of the confidence index.

7 Conclusion and extensions

We have studied a biform game in which firms select their capacities, before bargaining with potential buyers. The model is meant to represent situations in which we have a handful (or less) of buyers and sellers, resulting in a market where terms of trade depend on individual bargaining abilities.

Many counterintuitive results appear. Notably, we have many instances in which a firm that sees its confidence in its bargaining increase decides to install less capacity. Even firms convinced that they can extract as much as possible from their buyers might prefer to install a single unit of capacity. In addition, more competition might lead to more inefficiency.

These results are because firms take into account the impact of their capacity decision on their bargaining power. To managers, this is an example showing that such decisions can have important impacts, and that these impacts are not necessarily intuitive.

While the paper has allowed for a new, important distinction between capacity expansion that crowd out a rival from one that merely pushes the rival towards a less lucrative part of the market, it seems like the major insights of this paper are robust to extensions related to further increases in the number of buyers, or adding firms.

Considerations of non-linear cost functions would further complicate the model, and would involve interesting tradeoff between effects on scale and effects on bargaining power.

Another possible extension of the model would allow for differentiation between the products sold by the firm.

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A Appendix

A.1 Derivation of core bounds

We start with the upper bounds.

By definition, $y_{\{r,k_{e+1},k_e+k_r\}} = \sum_{j=1}^{k_r} V(r, k_e + j)$.

For all $i = 2, ..., k_e$, we have $y_{\{r,i,k_{e+1},k_e+k_r\}} \geq V(r,i) + \sum_{j=1}^{k_r-1} V(r, k_e + j)$. Combining, we obtain that $y_i \geq V(r,i) - V(r, k_e + k_r)$.

Thus, we must have that

$$y_{\{r,1,2,3\}} \geq V(r,1) + \sum_{j=1}^{k_r-1} V(r, k_e + j) + \sum_{i=2}^{k_e} (V(r,i) - V(r, k_e + k_r))$$

Since $y_N = \sum_{i=1}^{k_e} V(e,i) + \sum_{i=1}^{k_r} V(r, k_e + i)$, it implies that

$$\begin{aligned}
y_e &\leq \sum_{i=1}^{k_e} V(e,i) + \sum_{i=1}^{k_r} V(r, k_e + i) - V(r,1) - \sum_{i=1}^{k_r-1} V(r, k_e + i) - \sum_{i=2}^{k_e} (V(r,i) - V(r, k_e + k_r)) \\
&= \sum_{i=1}^{k_e} (V(e,i) - V(r,i)) + V(r, k_e + k_r) + \sum_{i=2}^{k_e} V(r, k_e + k_r) \\
&= \sum_{i=1}^{k_e} (V(e,i) - V(r,i) + V(r, k_e + k_r))
\end{aligned}$$

We also have that $y_{\{e,2,3\}} \geq \sum_{i=1}^{k_e} V(e, i+1)$ and $y_{\{r,1,k_e+1\}} \geq V(r,1) + \sum_{i=1}^{k_r-1} V(r, k_e + i)$, adding and substracting y_N, we obtain

$$\begin{aligned}
y_{k_e+1} &\geq \sum_{i=1}^{k_e} V(e, i+1) + V(r,1) + \sum_{i=1}^{k_r-1} V(r, k_e + i) - \sum_{i=1}^{k_e} V(e,i) - \sum_{i=1}^{k_r} V(r, k_e + i) \\
&= \sum_{i=1}^{k_e} [V(e, i+1) - V(e,i)] + V(r,1) - V(r, k_e + k_r) \\
&= u_{k_e+1} - u_1 + u_1 - c_r - V(r, k_e + k_r) \\
&= u_{k_e+1} - c_r - V(r, k_e + k_r)
\end{aligned}$$

Since we also have the constraint $y_{\{k_e+1\}} \geq 0$, we can write this constraint as $y_{\{k_e+1\}} \geq V(r, k_e+1) - V(r, k_e+k_r)$. Since we have that $y_{\{r,k_e+1,k_e+k_r\}} = \sum_{i=1}^{k_r} V(r, k_e + i)$.

Notice that if $k_r = 1$, then we obtain $y_{\{k_e+1\}} \geq 0$ and thus $y_r \leq V(r, k_e + 1) = k_r V(r, k_e + k_r)$.

If $k_r = 2$, then we obtain $y_{\{k_e+1\}} \geq V(r, k_e + 1) - V(r, k_e + 2)$ and thus

$$\begin{aligned}
y_r &\leq V(r, k_e + 1) + V(r, k_e + 2) - V(r, k_e + 1) + V(r, k_e + 2) \\
&= 2V(r, k_e + 2) = k_r V(r, k_e + k_r)
\end{aligned}$$

Let y' such that $y'_e = \sum_{i=1}^{k_e} (V(e,i) - V(r,i) + V(r, k_e + k_r))$, $y'_r = k_r V(r, k_e + k_r)$, $y'_i = V(r,i) - V(r, k_e + k_r)$ for all $i = 1, ..., k_e + 1$ and $y'_i = 0$ for all $i > k_e + 1$. We can verify that y' is a core allocation, confirming that $\pi^e_{\max} = \sum_{i=1}^{k_e} (V(e,i) - V(r,i) + V(r, k_e + k_r))$ and $\pi^r_{\max} = k_r V(r, k_e + k_r)$.

We now move to the lower bounds.

First, we once again use the fact that $y_{\{r,k_e+1,k_e+k_r\}} = \sum_{i=1}^{k_r} V(r, k_e+i)$. We next verify that $y_{\{e,r,1,k_e+1,k_e+k_r\}} \geq V(e,1) + \sum_{i=1}^{k_e-1} V(e, k_e + 1) + \sum_{i=1}^{k_r} V(r, 2k_e + i - 1)$.

Combining these two results, we obtain that

$$y_{\{e,1\}} \geq V(e,1) + \sum_{i=1}^{k_e-1} V(e, k_e + 1) + \sum_{i=1}^{k_r} V(r, 2k_e + i - 1) - \sum_{i=1}^{k_r} V(r, k_e + i).$$

We also have that $y_{\{e,r,2,3\}} \geq \sum_{i=1}^{k_e} V(e,i+1) + \sum_{i=1}^{k_r} V(r,k_e+i+1)$. Summing up with the inequality on $y_{\{e,1\}}$ above and substracting y_N, we obtain

$$y_e \geq \begin{array}{c} V(e,1) + \sum_{i=1}^{k_e-1} V(e,k_e+1) + \sum_{i=1}^{k_r} V(r,2k_e+i-1) - \sum_{i=1}^{k_r} V(r,k_e+i) \\ + \sum_{i=1}^{k_e} V(e,i+1) + \sum_{i=1}^{k_r} V(r,k_e+i+1) - \sum_{i=1}^{k_e} V(e,i) - \sum_{i=1}^{k_r} V(r,k_e+i) \end{array}.$$

Using the fact that $V(e,1) + \sum_{i=1}^{k_e} V(e,i+1) - \sum_{i=1}^{k_e} V(e,i) = V(e,k_e+1)$, we simplify it to

$$y_e \geq \begin{array}{c} V(e,k_e+1) + \sum_{i=1}^{k_e-1} V(e,k_e+1) + \sum_{i=1}^{k_r} V(r,2k_e+i-1) \\ + \sum_{i=1}^{k_r} V(r,k_e+i+1) - 2\sum_{i=1}^{k_r} V(r,k_e+i) \end{array}$$

Suppose that $k_e = 1$. It then simplifies to

$$\begin{aligned} y_e &\geq \begin{array}{c} V(e,2) + \sum_{i=1}^{k_r} V(r,i+1) \\ + \sum_{i=1}^{k_r} V(r,i+2) - 2\sum_{i=1}^{k_r} V(r,i+1) \end{array} \\ &= V(e,2) + \sum_{i=1}^{k_r} [V(r,i+2) - V(r,i+1)] \\ &= V(e,2) + V(r,k_r+2) - V(r,2) \\ &= k_e [V(e,k_e+1) - V(r,k_e+1+V(r,k_e+k_r+1))] \end{aligned}$$

Suppose that $k_e = 2$. It then simplifies to

$$\begin{aligned} y_e &\geq \begin{array}{c} V(e,3) + V(e,3) + \sum_{i=1}^{k_r} V(r,3+i) \\ + \sum_{i=1}^{k_r} V(r,3+i) - 2\sum_{i=1}^{k_r} V(r,2+i) \end{array} \\ &= 2V(e,3) - 2V(r,3) \\ &= k_e [V(e,k_e+1) - V(r,k_e+1+V(r,k_e+k_r+1))] \end{aligned}$$

Suppose that $k_e = 3$. Then, since all terms involve $V(e,i)$ or $V(r,i)$ with $i > 3$, it then simplifies to $y_e \geq 0$, which can also be written as $k_e [V(e,k_e+1) - V(r,k_e+1+V(r,k_e+k_r+1))]$.

Thus, in all 3 cases we obtain $y_e \geq k_e [V(e,k_e+1) - V(r,k_e+1+V(r,k_e+k_r+1))]$.

It is obvious that since buyer k_e+k_r+1 is not served, $y_{\{k_e+k_r+1\}} = 0$ in any core allocation. But $y_{\{r,k_e+k_r+1\}} \geq V(r,k_e+k_r+1)$. Combining, we obtain that $y_r \geq V(r,k_e+k_r+1) = k_r V(r,k_e+k_r+1)$.

Consider the allocation y'' such that $y''_e = k_e [V(e,k_e+1) - V(r,k_e+1) + V(r,k_e+k_r+1))]$, $y''_r = k_r V(r,k_e+k_r+1)$, $y''_i = V(e,i) - V(e,k_e+1) + V(r,k_e+1) - V(r,k_e+k_r+1)$ for all $i = 1,...,k_e$, $y''_i = V(r,i) - V(r,k_e+k_r+1)$ for all $i = k_e+1,...,k_e+k_r$ and $y''_i = 0$ for all $i > k_e+k_r$. Since we can verify that y'' is a core allocation, we have confirmed that $\pi_e^{\min} = k_e [V(e,k_e+1) - V(r,k_e+1+V(r,k_e+k_r+1))]$ and $\pi_r^{\min} = k_r V(r,k_e+k_r+1)$.

A.2 Derivation of equilibria conditions

Using the formulas found in Section 3, we compare expected profits. We start with firm e. Capacity 1 is better than capacity 2 if

$$\begin{array}{ccc} \begin{array}{c} \alpha^e [V(e,1) - V(r,1) + V(r,k_r+1)] + \\ (1-\alpha^e) [V(e,2) - V(r,2) + V(r,k_r+2)] \end{array} & \geq & \begin{array}{c} \alpha^e \sum_{l=1}^{2} [V(e,l) - V(r,l) + V(r,k_r+2)] + \\ (1-\alpha^e) 2 [V(e,3) - V(r,3)] \end{array} \\ \begin{array}{c} \alpha^e V(r,k_r+1) + \\ (1-\alpha^e) [V(e,2) - V(r,2) + V(r,k_r+2)] \end{array} & \geq & \begin{array}{c} \alpha^e [V(e,2) - V(r,2) + 2V(r,k_r+2)] + \\ (1-\alpha^e) 2 [V(e,3) - V(r,3)] \end{array} \\ \begin{array}{c} V(e,2) - V(r,2) + V(r,k_r+2) \\ -2V(e,3) + 2V(r,3) \end{array} & \geq & \alpha^e \left[\begin{array}{c} 2V(e,2) - 2V(r,2) + 2V(r,3) \\ -2V(e,3) - V(r,k_r+1) + 3V(r,k_r+2) \end{array} \right] \end{array}$$

Capacity 1 is better than capacity 3 if

$$\begin{array}{ccc} \begin{array}{c} \alpha^e [V(e,1) - V(r,1) + V(r,k_r+1)] + \\ (1-\alpha^e) [V(e,2) - V(r,2) + V(r,k_r+2)] \end{array} & \geq & \alpha^e \sum_{l=1}^{3} [V(e,l) - V(r,l)] \\ \begin{array}{c} \alpha^e V(r,k_r+1) + \\ (1-\alpha^e) [V(e,2) - V(r,2) + V(r,k_r+2)] \end{array} & \geq & \alpha^e \left[\begin{array}{c} V(e,2) - V(r,2) \\ +V(e,3) - V(r,3) \end{array} \right] \\ V(e,2) - V(r,2) + V(r,k_r+2) & \geq & \alpha^e \left[\begin{array}{c} 2V(e,2) - 2V(r,2) - V(r,k_r+1) \\ +V(e,3) - V(r,3) + V(r,k_r+2) \end{array} \right] \end{array}$$

Capacity 2 is better than capacity 3 if

$$\begin{aligned} \alpha^e \sum_{l=1}^{2} \left[V(e,l) - V(r,l) + V(r,k_r+2)\right] + \\ (1-\alpha^e) 2 \left[V(e,3) - V(r,3)\right] &\geq \alpha^e \sum_{l=1}^{3} \left[V(e,l) - V(r,l)\right] \\ 2\alpha^e V(r,k_r+2) + \\ (1-\alpha^e) 2 \left[V(e,3) - V(r,3)\right] &\geq \alpha^e \left[V(e,3) - V(r,3)\right] \\ 2\left[V(e,3) - V(r,3)\right] &\geq \alpha^e \left[3V(e,3) - 3V(r,3) - 2V(r,k_r+2)\right] \end{aligned}$$

Switching now to firm r, it prefers capacity 1 to capacity 2 if

$$\begin{aligned} \alpha^r V(r,k_e+1) + (1-\alpha^r) V(r,k_e+2) &\geq 2\alpha^r V(r,k_e+2) \\ V(r,k_e+2) &\geq \alpha^r \left[3V(r,k_e+2) - V(r,k_e+1)\right] \end{aligned}$$

Next, we examine each potential equilibria by combining the conditions above, and looking at the different cases depending if $V(r,2)$ and $V(r,3)$ are positive or zero.

A.2.1 $k_e = 1,\ k_r = 1$

Firm r prefers 1 to 2 if

$$V(r,3) \geq \alpha^r \left(3V(r,3) - V(r,2)\right)$$

Firm e prefers 1 to 2 if

$$\left(V(e,2) + 3V\left(\{r,3\}\right) - 2V(e,3) - V(r,2)\right) \geq \alpha^e \left(5V(r,3) + 2V(e,2) - 3V(r,2) - 2V(e,3)\right)$$

Firm e prefers 1 to 3 if

$$V\left(\{r,3\}\right) + V\left(\{e,2\}\right) - V(\{r,2\}) \geq \alpha^e \left(2V(e,2) + V(e,3) - 3V(r,2)\right)$$

Suppose that $V(r,2) = 0$. The first condition becomes $0 \geq 0$, satisfied.

The second condition becomes

$$\begin{aligned} (1-\alpha^e)\left(V(e,2) - 2V(e,3)\right) &\geq \alpha^e V(e,2) \\ V(e,2) - 2V(e,3) &\geq \alpha^e \left(2V(e,2) - 2V(e,3)\right) \\ \frac{V(e,2) - 2V(e,3)}{2V(e,2) - 2V(e,3)} &\geq \alpha^e \end{aligned}$$

or $\alpha^e \leq \frac{u_2 - 2u_3}{2(u_2 - u_3)}$

The third condition becomes

$$\begin{aligned} (1-\alpha^e) V\left(\{e,2\}\right) &\geq \alpha^e \left(V(e,2) + V(e,3)\right) \\ V\left(\{e,2\}\right) &\geq \alpha^e \left(2V(e,2) + V(e,3)\right) \\ \frac{V\left(\{e,2\}\right)}{2V(e,2) + V(e,3)} &\geq \alpha^e \end{aligned}$$

or $\alpha^e \leq \frac{u_2}{2u_2 + u_3}$

It is easy to verify that

$$\frac{V\left(\{e,2\}\right)}{2V(e,2) + V(e,3)} \geq \frac{V(e,2) - 2V(e,3)}{2V(e,2) - 2V(e,3)}$$

and therefore we have the equilibrium if $\frac{V(e,2) - 2V(e,3)}{2V(e,2) - 2V(e,3)} \geq \alpha^e$ or $\alpha^e \leq \frac{u_2 - 2u_3}{2(u_2 - u_3)}$

Suppose that $V(r,2) > V(r,3) = 0$**.** The first condition becomes $0 \geq -\alpha^r V(r,2)$, always satisfied.

The second condition becomes

$$\begin{aligned}(1-\alpha^e)\left(V(e,2) - 2V(e,3) - V(r,2)\right) &\geq \alpha^e\left(V(e,2) - 2V(r,2)\right)\\ V(e,2) - 2V(e,3) - V(r,2) &\geq \alpha^e\left(2V(e,2) - 2V(e,3) - 3V(r,2)\right)\end{aligned}$$

or

$$c_r - 2u_3 \geq \alpha^e\left(3c_r - u_2 - 2u_3\right)$$

Not clear what the signs of $3c_r - u_2 - 2u_3$ or $c_r - 2u_3$ are. If $3c_r - u_2 - 2u_3 \geq 0$, the condition becomes

$$\alpha^e \leq \frac{c_r - 2u_3}{3c_r - u_2 - 2u_3}.$$

(In particular, if $c_r - 2u_3 \leq 0$, the condition is impossible to satisfy). If $3c_r - u_2 - 2u_3 \leq 0$, it becomes

$$\alpha^e \geq \frac{c_r - 2u_3}{3c_r - u_2 - 2u_3}$$

(in particular, in that case, if $c_r - 2u_3 > 0$ the condition is always satisfied).

The third condition becomes

$$\begin{aligned}(1-\alpha^e)\left(V(\{e,2\}) - V(\{r,2\})\right) &\geq \alpha^e\left(V(e,2) + V(e,3) - 2V(r,2)\right)\\ V(\{e,2\}) - V(\{r,2\}) &\geq \alpha^e\left(2V(e,2) + V(e,3) - 3(V(r,2))\right)\end{aligned}$$

or

$$c_r \geq \alpha^e\left(3c_r + u_3 - u_2\right)$$

Not clear what the sign of $3c_r + u_3 - u_2$ is. But $c_r \geq 0$. If $3c_r + u_3 - u_2 < 0$, the condition is satisfied. Thus, we can write the condition as

$$\alpha^e \leq \frac{c_r}{\max\left(3c_r + u_3 - u_2, 0\right)}.$$

Suppose that $V(r,3) > 0$ The first condition simplifies to

$$u_3 - c_r \geq \alpha^r\left(3u_3 - u_2 - 2c_r\right)$$

Not clear what the sign of $3u_3 - u_2 - 2c_r$ is. But $u_3 - c_r \geq 0$. If $3u_3 - u_2 - 2c_r < 0$, the condition is satisfied. Thus, we can write the condition as

$$\alpha^r \leq \frac{u_3 - c_r}{\max\left(3u_3 - u_2 - 2c_r, 0\right)}.$$

The second condition simplifies to

$$u_3 - 2c_r \geq \alpha^e\left(3u_3 - u_2 - 2c_r\right)$$

Not clear what the signs of $3u_3 - u_2 - 2c_r$ or $u_3 - 2c_r$ are. If $3u_3 - u_2 - 2c_r \geq 0$, the condition becomes

$$\alpha^e \leq \frac{u_3 - 2c_r}{3u_3 - u_2 - 2c_r}.$$

If $3u_3 - u_2 - 2c_r \leq 0$, it becomes

$$\alpha^e \geq \frac{u_3 - 2c_r}{3u_3 - u_2 - 2c_r}$$

The third condition simplifies to

$$u_3 \geq \alpha^e\left(3c_r + u_3 - u_2\right)$$

Not clear what the sign of $3c_r + u_3 - u_2$ is. But $u_3 \geq 0$. If $3c_r + u_3 - u_2 < 0$, the condition is satisfied. Thus, we can write the condition as

$$\alpha^e \leq \frac{u_3}{\max\left(3c_r + u_3 - u_2, 0\right)}.$$

A.2.2 $k_e = 2,\ k_r = 1$

Firm r always prefers 1 to 2.

Firm e prefers 2 to 1 if

$$\alpha^e \left(5V(r,3) + 2V(e,2) - 3V(r,2) - 2V(e,3)\right) > \left(V(e,2) + 3V(\{r,3\}) - 2V(e,3) - V(r,2)\right)$$

Firm e prefers 2 to 3 if

$$2V(e,3) - 2V(\{r,3\}) \geq \alpha^e \left(3V(e,3) - 5V(r,3)\right)$$

Suppose that $V(r,2) = 0$. The first condition becomes

$$\alpha^e > \frac{V(e,2) - 2V(e,3)}{2V(e,2) - 2V(e,3)}$$

or $\alpha^e > \frac{u_2 - 2u_3 - u_3}{2(u_2 - u_3)}$.

The second condition becomes

$$2V(e,3) \geq \alpha^e 3V(e,3)$$
$$\frac{2}{3} \geq \alpha^e$$

We thus need

$$\frac{u_2 - 2u_3 - u_3}{2\left(u_2 - u_3\right)} < \alpha^e \leq \frac{2}{3}$$

Suppose that $V(r,2) > V(r,3) = 0$. The first condition becomes

$$\alpha^e \left(3c_r - u_2 - 2u_3\right) > c_r - 2u_3$$

It is not clear what the signs of $3c_r - u_2 - 2u_3$ and $c_r - 2u_3$ are. If $3c_r - u_2 - 2u_3 \geq 0$, then we can write the condition as

$$\alpha^e > \frac{c_r - 2u_3}{3c_r - u_2 - 2u_3}.$$

If $3c_r - u_2 - 2u_3 < 0$, then the condition becomes

$$\alpha^e < \frac{c_r - 2u_3}{3c_r - u_2 - 2u_3}$$

In particular, if $c_r - 2u_3 > 0$ the condition is impossible to satisfy.

The second condition remains at $\frac{2}{3} \geq \alpha^e$.

Suppose that $V(r,3) > 0$. The first condition simplifies to

$$\alpha^e \left(3u_3 - u_2 - 2c_r\right) > u_3 - 2c_r$$

It is not clear what the signs of $3u_3 - u_2 - 2c_r$ and $u_3 - 2c_r$ are. If $3u_3 - u_2 - 2c_r \geq 0$, then we can write the condition as

$$\alpha^e > \frac{u_3 - 2c_r}{3u_3 - u_2 - 2c_r}.$$

If $3u_3 - u_2 - 2c_r < 0$, then the condition becomes

$$\alpha^e < \frac{u_3 - 2c_r}{3u_3 - u_2 - 2c_r}$$

In particular, if $u_3 - 2c_r > 0$ the condition is impossible to satisfy.

The second condition simplifies to

$$2c_r \geq \alpha^e \left(5c_r - 2u_3\right)$$

Not clear what the sign of $5c_r - 2u_3$ is. Notice that the LHS is always positive. Thus, we can write the condition as

$$\alpha^e \leq \frac{2c_r}{\max\left(5c_r - 2u_3, 0\right)}.$$

A.2.3 $k_e = 3$ and $k_r = 1$

Firm r always prefers 1 to 2.

Firm e prefers 3 to 1 if

$$\alpha^e \left(2V(e,2) + V(e,3) - 3V(r,2)\right) > V\left(\{r,3\}\right) + V\left(\{e,2\}\right) - V(\{r,2\})$$

Firm e prefers 3 to 2 if

$$\alpha^e \left(3V(e,3) - 5V(r,3)\right) > 2V(e,3) - 2V\left(\{r,3\}\right)$$

Suppose that $V(r,2) = 0$. Condition 1 simplifies to

$$\alpha^e > \frac{V\left(\{e,2\}\right)}{2V(e,2) + V(e,3)}$$

or $\alpha^e > \frac{u_2}{2u_2+u_3}$.

Condition 2 simplifies to $\alpha^e \geq \frac{2}{3}$.

Since $\frac{2}{3} \geq \frac{u_2}{2u_2+u_3}$, the only constraint is $\alpha^e > \frac{2}{3}$.

Suppose that $V(r,2) > V(r,3) = 0$. The first condition becomes

$$\alpha^e \left(2V(e,2) + V(e,3) - 3(V(r,2)\right) > V\left(\{e,2\}\right) - V(\{r,2\})$$

or

$$\alpha^e \left(3c_r + u_3 - u_2\right) > c_r$$

It is not clear what the sign of $3c_r + u_3 - u_2$ is. But the RHS is positive. If $(3c_r + u_3 - u_2)$ is negative, we can't satisfy the constraint, thus we can write the condition as

$$\alpha^e > \frac{c_r}{\max\left(3c_r + u_3 - u_2, 0\right)}.$$

The second condition remain at $\alpha^e > \frac{2}{3}$

Suppose that $V(r,3) > 0$. The first condition simplifies to

$$\alpha^e \left(3c_r + u_3 - u_2\right) > u_3$$

It is not clear what the sign of $3c_r + u_3 - u_2$ is. But the RHS is positive. If $(3c_r + u_3 - u_2)$ is negative, we can't satisfy the constraint, thus we can write the condition as

$$\alpha^e > \frac{u_3}{\max\left(3c_r + u_3 - u_2, 0\right)}.$$

The second condition simplifies to

$$\alpha^e \left(5c_r - 2u_3\right) > 2c_r$$

Not clear what the sign of $5c_r - 2u_3$ is. But the RHS is positive. If $(5c_r - 2u_3)$ is negative, we can't satisfy the constraint, thus we can write the condition as

$$\alpha^e > \frac{2c_r}{\max\left(5c_r - 2u_3, 0\right)}.$$

A.2.4 $k_e = 1,\ k_r = 2$

Firm r prefers 2 to 1 if

$$\alpha^r \left(3V(r,3) - V(r,2)\right) > V(r,3)$$

The condition can only be satisfied if $V(r,3) > 0$. We then need

$$\alpha^r \left(3u_3 - u_2 - 2c_r\right) > u_3 - c_r$$

It is not clear what the sign of $3u_3 - u_2 - 2c_r$ is. The RHS is positive. If $(3u_3 - u_2 - 2c_r)$ is negative, we can't satisfy the constraint. We can thus write the constraint as

$$\alpha^r > \frac{u_3 - c_r}{\max\left(3u_3 - u_2 - 2c_r, 0\right)}$$

Firm e prefers 1 to 2 if

$$V(e,2) + 2V(r,3) - V(r,2) - 2V(e,3) \geq \alpha^e \left(2V(e,2) + V(r,3) - 2V(r,2) - 2V(e,3)\right)$$

With $V(r,3) > 0$, it simplifies to

$$\alpha^e \geq \frac{c_r}{u_3 - c_r}.$$

Firm e prefers 1 to 3 if

$$V(e,2) - V(r,2) \geq \alpha^e \left(2V(e,2) - 2V(r,3) - 2V(r,2) + V(e,3)\right)$$

With $V(r,3) > 0$, it simplifies to

$$c_r \geq \alpha^e(4c_r - u_3)$$

The LHS is positive, but the sign of $(4c_r - u_3)$ is unclear. If negative, the inequality is satisfied. We can thus rewrite it as

$$\alpha^e \leq \frac{c_r}{\max(4c_r - u_3, 0)}.$$

Combining, we need

$$\frac{c_r}{u_3 - c_r} \leq \alpha^e \leq \frac{c_r}{\max(4c_r - u_3, 0)}.$$

B Proofs

B.1 Proof of Theorem 2

Theorem. *The flat capacity paradox of the confidence index occurs if and only we are in one of these situations:*
i) $u_3 > c_r$ *and* $u_2 \geq \max\left[3c_r, 2u_3\right]$
ii) $u_2 > c_r \geq u_3$ *and* $u_2 - u_3 \geq 2c_r$

Proof. The flat capacity paradox of the confidence index occurs if $k_e^* = 1$ when $\alpha^e = 1$.We first show that the paradox occurs when either of the two conditions is satisfied.

i) $u_3 > c_r$ and $u_2 \geq \max\left[3c_r, 2u_3\right]$: If $3u_3 - u_2 - 2c_r \leq 0$, then we have that $k_e^* = 1$ if $\frac{u_3 - 2c_r}{3u_3 - u_2 - 2c_r} \leq \alpha^e \leq \frac{u_3}{\max(3c_r + u_3 - u_2, 0)}$. We need the lower bound to be smaller than 1, which is the case since $u_2 \geq 2u_3$. We need the upper bound to be larger or equal to 1. If $3c_r + u_3 - u_2 \leq 0$, the condition is satisfied. Otherwise, the upper bound is larger than 1 since $u_2 \geq 3c_r$.

If $3u_3 - u_2 - 2c_r \geq 0$, then we have that $k_e^* = 1$ if $\alpha^e \leq \min\left[\frac{u_3 - 2c_r}{3u_3 - u_2 - 2c_r}, \frac{u_3}{\max(3c_r + u_3 - u_2, 0)}\right]$. In the same way as above, $\min\left[\frac{u_3 - 2c_r}{3u_3 - u_2 - 2c_r}, \frac{u_3}{\max(3c_r + u_3 - u_2, 0)}\right] \geq 1$ if and only if $u_2 \geq \max\left[3c_r, 2u_3\right]$.

ii) $u_2 > c_r \geq u_3$ and $u_2 - u_3 \geq 2c_r$. If $3c_r - u_2 - 2u_3 \geq 0$, then we have that $k_e^* = 1$ if $\alpha^e \leq \min\left[\frac{c_r - 2u_3}{3c_r - u_2 - 2u_3}, \frac{c_r}{\max(3c_r + u_3 - u_2, 0)}\right]$. We have that $\frac{c_r - 2u_3}{3c_r - u_2 - 2u_3} \geq 1$ if and only if $u_2 \geq 2c_r$. We have that $\frac{c_r}{\max(3c_r + u_3 - u_2, 0)} \geq 1$ if and only if $u_2 - u_3 \geq 2c_r$. Thus, $\min\left[\frac{c_r - 2u_3}{3c_r - u_2 - 2u_3}, \frac{c_r}{\max(3c_r + u_3 - u_2, 0)}\right] \geq 1$ if and only if $u_2 - u_3 \geq 2c_r$.

If $3c_r - u_2 - 2u_3 \leq 0$, then we have that $k_e^* = 1$ if $\frac{c_r - 2u_3}{3c_r - u_2 - 2u_3} \leq \alpha^e \leq \frac{c_r}{\max(3c_r + u_3 - u_2, 0)}$. We need the lower bound to be no larger than one, and the upper bound to be no smaller than 1. As above, it is true if and only if $u_2 - u_3 \geq 2c_r$.

We next show that in all other situations, $k_e^* > 1$ when $\alpha^e = 1$. Given that the proof that conditions i) and ii) are sufficient also shows that they are necessary if $u_2 > c_r$, it remains to show that no other cases exists when $u_2 \leq c_r$. If $c_r \leq u_2$, then $k_e^* = 3$ if $\alpha^e > \frac{2}{3}$, completing the proof. ☐

B.2 Proof of Theorem 3

Theorem. *The decreasing capacity paradox of the confidence index occurs if and only we are in one of these situations:*

i) $c_r < u_3 < \min\left[\frac{u_2}{2}, 2c_r\right]$

ii) $u_3 \leq c_r < \min\left[\frac{u_2}{2}, 2u_3\right]$

Proof. From the conditions in the previous section, it is obvious that we can only have the decreasing capacity paradox of the confidence index if we have a condition that firm e chooses $k_e^* = 1$ for α^e larger than a bound, itself larger than 0 but smaller than one.

If $u_3 > c_r$, this can occur if $3u_3 - u_2 - 2c_r \leq 0$ and $\frac{u_3-2c_r}{3u_3-u_2-2c_r} \leq \alpha^e \leq \frac{u_3}{\max(3c_r+u_3-u_2,0)}$. Suppose that $3u_3 - u_2 - 2c_r \leq 0$. Then, for $0 < \frac{u_3-2c_r}{3u_3-u_2-2c_r} \leq 1$, we need $u_3 < 2c_r$ and $u_2 \geq 2u_3$. Adding the two constraints, we obtain $3u_3 - u_2 - 2c_r < 0$ as needed.

If $u_3 \leq c_r < u_2$, then it can occur if $3c_r - u_2 - 2u_3 \leq 0$ and $\frac{c_r-2u_3}{3c_r-u_2-2u_3} \leq \alpha^e \leq \frac{c_r}{\max(3c_r+u_3-u_2,0)}$. Suppose that $3c_r - u_2 - 2u_3 \leq 0$. Then, for $0 < \frac{c_r-2u_3}{3c_r-u_2-2u_3} \leq 1$, we need $c_r < 2u_3$ and $u_2 \geq 2c_r$. Adding the two constraints, we obtain $3c_r - u_2 - 2u_3 < 0$ as needed. □

B.3 Proof of Theorem 4

Theorem. *i) The flat capacity paradox of cost advantage occurs if and only* $u_2 - u_3 \geq u_3$.

ii) The decreasing capacity paradox of cost advantage occurs if and only $u_2 - u_3 \geq u_3$.

Proof. i) We need to show that $k_e^* = 1$ when $c_r = u_1$ and some $\alpha^e \in [0, 1]$. But in the case $c_r = u_1$, we have $k_e^* = 1$ if and only if $\alpha^e \leq \frac{u_2-2u_3}{2(u_2-u_3)}$. We need $\frac{u_2-2u_3}{2(u_2-u_3)} \geq 0$, which we have if and only if $u_2 - u_3 \geq u_3$.

ii) We first show that the condition is sufficient. Suppose that $c_r = u_3$. Then, since $3c_r - u_2 - 2u_3 = u_3 - u_2 \leq 0$, then we have $k_e^* = 2$ if $\alpha^e \leq \min\left[\frac{c_r-2u_3}{3c_r-u_2-2u_3}, \frac{2}{3}\right]$. Given that $c_r = u_3$, $\min\left[\frac{c_r-2u_3}{3c_r-u_2-2u_3}, \frac{2}{3}\right] = \min\left[\frac{u_3}{u_2-u_3}, \frac{2}{3}\right] > 0$. Thus, at $\alpha^e = 0$, $k_e^* = 2$.

Suppose that $c_r = u_2$. Then, we have $k_e^* = 1$ if $\alpha^e \leq \frac{u_2-2u_3}{2(u_2-u_3)}$. Since $u_2 - u_3 \geq u_3$, then $\frac{u_2-2u_3}{2(u_2-u_3)} \geq 0$. Thus, at $\alpha^e = 0$, $k_e^* = 1$. Thus, moving from $c_r = u_3$ to $c_r = u_2$, we reduce the equilibrium capacity from 2 to 1.

We prove the necessity of the condition by looking at other cases. It is easy to verify that if $k_e^* = 3$ and c_r increases, we cannot have that capacity is reduced. We thus focus on cases where we reduce capacity from 2 to 1.

Suppose first that $c_r < u_3$. If $u_2 \leq 3u_3$, then $3u_3 - u_2 - 2c_r \geq 0$ when c_r is small enough, and $k_*^e = 1$ if $\alpha^e \leq \min\left[\frac{u_3-2c_r}{3u_3-u_2-2c_r}, \frac{u_3}{\max(3c_r+u_3-u_2,0)}\right]$. We would have the decreasing capacity paradox of cost advantage if $\min\left[\frac{u_3-2c_r}{3u_3-u_2-2c_r}, \frac{u_3}{\max(3c_r+u_3-u_2,0)}\right] \leq 1$ and $\min\left[\frac{u_3-2c_r}{3u_3-u_2-2c_r}, \frac{u_3}{\max(3c_r+u_3-u_2,0)}\right]$ increasing in c_r. It is easy to see that $\frac{u_3}{\max(3c_r+u_3-u_2,0)}$ is non-increasing in c_r and that $\frac{u_3-2c_r}{3u_3-u_2-2c_r}$ is increasing in c_r if and only $u_2 \leq 2u_3$. But in that case $\frac{u_3-2c_r}{3u_3-u_2-2c_r} \geq 1$.

Notice that at $c_r = u_3$, $3u_3 - u_2 - 2c_r \leq 0$, in which case $k_*^e = 1$ if $\frac{u_3-2c_r}{3u_3-u_2-2c_r} \leq \alpha^e \leq \frac{u_3}{\max(3c_r+u_3-u_2,0)}$. Given that $\frac{u_3-2c_r}{3u_3-u_2-2c_r}, \frac{u_3}{\max(3c_r+u_3-u_2,0)}$ are decreasing in c_r,we would have decreasing capacity paradox of cost advantage if $\min\left[\frac{u_3-2c_r}{3u_3-u_2-2c_r}, \frac{u_3}{\max(3c_r+u_3-u_2,0)}\right]$ at $c_r = 0$ is larger than $\frac{u_3}{\max(3c_r+u_3-u_2,0)}$ at $c_r = \frac{3u_3-u_2}{2}$. That would be true, with $\frac{u_3}{\max(3c_r+u_3-u_2,0)}$ smaller than 1 if $u_2 \leq \frac{9u_3}{5}$. But if $u_2 \leq 2u_3$, $\frac{u_3-2c_r}{3u_3-u_2-2c_r}$ evaluated at $c_r = \frac{3u_3-u_2}{2}$ is larger than 1, meaning that there is no α^e for which $k_e^* = 1$.

If $u_2 \geq 3u_3$, then $3u_3 - u_2 - 2c_r \leq 0$ for all $0 \leq c_r \leq u_3$, in which case $k_*^e = 1$ if $\frac{u_3-2c_r}{3u_3-u_2-2c_r} \leq \alpha^e \leq \frac{u_3}{\max(3c_r+u_3-u_2,0)}$. There would be the decreasing capacity paradox of cost advantage if $\frac{u_3}{\max(3c_r+u_3-u_2,0)}$ is increasing in c_r or $\frac{u_3-2c_r}{3u_3-u_2-2c_r}$ decreasing in c_r. The first part is clearly untrue, while the second is true if and only if $u_2 \geq 2u_3$. But in that case $\frac{u_3-2c_r}{3u_3-u_2-2c_r} \geq 1$.

Suppose next that $u_2 > c_r \geq u_3$. Notice first that at $c_r = u_3$, $3c_r - u_2 - 2u_3 \leq 0$, and thus we have $k_e^* = 1$ if $\frac{u_3-2c_r}{3u_3-u_2-2c_r} \leq \alpha^e \leq \frac{u_3}{\max(3c_r+u_3-u_2,0)}$. Again, we would have the decreasing capacity paradox of cost advantage if the lower bound is decreasing in c_r but larger than zero or if the upper bound is increasing in c_r and smaller than one. It is easy to see that the upper bound is always decreasing. The lower bound is decreasing and initially above zero if $u_2 > 4u_3$ and $c_r \leq 2u_3$, a case included in our existence results above.

When we reach $c_r = \frac{u_2+2u_3}{3}$, $3c_r - u_2 - 2u_3 = 0$ and the condition for $k_e^* = 1$ changes to $\alpha^e \leq \min\left[\frac{c_r-2u_3}{3c_r-u_2-2u_3}, \frac{c_r}{\max(3c_r+u_3-u_2,0)}\right]$. If $c_r < 2u_3$, the condition at $c_r = \frac{u_2+2u_3}{3}$ becomes $\alpha^e < -\infty$, and thus impossible to satisfy. If $c_r > 2u_3$, it becomes $\alpha^e \leq \frac{u_2+2u_3}{9u_3}$. We would have the decreasing capacity paradox of cost advantage if that upper bound was larger than the upper bound at $c_r = u_2$. But that is impossible, since that upper bound is $\frac{c_r}{\max(3c_r+u_3-u_2,0)}$, and that function is decreasing in c_r. We would also have the paradox if $\alpha^e \leq \frac{u_2+2u_3}{9u_3}$ but $k_e^* = 1$ for $\alpha^e = 0$ at $c_r = u_2$, a case already covered in the existence part. Finally, we would have the paradox if the upper bound was increasing in c_r, with the bound between 0 and 1. The function $\frac{c_r}{\max(3c_r+u_3-u_2,0)}$ is decreasing in c_r. The function $\frac{c_r-2u_3}{3c_r-u_2-2u_3}$ is increasing in c_r if $u_2 \leq 4u_3$. The condition $2u_3 \leq u_2 \leq 4u_3$ is part of of existence conditions. Suppose that $u_2 < 2u_3$. To have $\frac{c_r-2u_3}{3c_r-u_2-2u_3} \geq 0$ we require $c_r \geq 2u_3$. Since $u_2 > c_r$, we obtain $u_2 > 2u_3$, a contradiction.

Suppose that $c_r \geq u_2$. Notice first that bounds are continuous at $c_r = u_2$. Then bounds do not depend on c_r.

Putting all together, we have examined all cases, and the paradox only occurs when $u_2 \leq 2u_3$. $\square$