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Welfare and Inequality with Hard-to-Tax Markets

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Abstract

Tax enforcement costs constrain the government's ability to observe economic activities, giving rise to hard-to-tax (HTT) markets. In this paper, we develop a Hotelling-type spatial model of sales taxation to analyze the welfare and distributional effects of the existence of HTT transactions. We show that an economy with HTT markets suffers from lower provision of public goods not only due to higher marginal cost of taxation, but also because (i) the planner might be concerned about the inequality in consumption caused by the unequal taxation across markets and (ii) the tax base might be over-extended to allow for a more inclusive taxation.

Keywords: Sales tax; Tax evasion; Hard-to-tax markets; Public good provision.

JEL Classification: H1, H21, H26.

1 Introduction

The existence of hard-to-tax (HTT) markets and tax evasion are well-known challenges faced by governments in both developed and developing countries. To avoid taxation, individuals and businesses purposely misreport or conceal transactions. The costs borne by the tax agency to enforce compliance impose a limit on its reach, making it hard to collect tax revenues in all markets. In this paper, we develop a model where the existence of HTT

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markets is endogenously determined by the government's inability to enforce tax collection. We analyze the welfare and distributional effects of the existence of HTT transactions when the planner is concerned about inequality across consumers.

We present a Hotelling-type linear economy model (Hotelling, 1929) of sales taxation where the cost of enforcement increases with the distance from the tax authority. In each market, there is a continuum of identical buyers and sellers, who cannot relocate. Markets within the government's reach are audited directly and sellers pay taxes accordingly. Markets beyond the cut-off distance are denoted as HTT markets, where the tax authority does not enforce tax compliance and transactions go unrecorded, rendering zero tax revenue.

The government's inability to audit all markets creates heterogeneity in an otherwise homogenous population of consumers. Consumers, who are ex-ante identical, are ex-post heterogenous because the ones in HTT markets are not burdened by sales tax, paying a lower price for the private good. After all, the beneficiaries of HTT transactions are those who directly participate in these activities (Alm, Martinez-Vazquez, and Schneider, 2004).

We show that the provision of the public good in an economy with HTT markets is reduced not only because of the higher marginal social cost of taxation (due to the limited reach of tax collection), but also because of concerns about the inequality across consumers caused by the unequal taxation. Moreover, the size of the economy that remains untaxed (i.e., the number of HTT markets) should be chosen by the planner to maximize welfare, but political or administrative pressures to make taxation more inclusive may lead to an extension of the tax audit to all markets where a positive net tax revenue can be obtained. This over-extension of the tax base can increase the degree of under provision of the public good depending on how it is implemented. Numerical results are presented to illustrate how tax rates, the provision of the public good, and social welfare are affected in the presence of HTT markets. The more the planner is concerned about equity issues in taxation, the lower is the level of public good provision. Practical measures to deal with these issues are discussed in the Conclusion.

HTT markets are related to underground economies and tax evasion. Although there are well-known examples of HTT markets, e.g., hotels, restaurants, household services, and construction, there is no precise definition (Musgrave, 1981; Tanzi and Jantscher, 1989; Das-Gupta, 1994; Terkper, 2003; Alm, Martinez-Vazquez, and Schneider, 2004). Our work builds on the tax evasion literature that focuses on interactions between buyers and sellers - e.g., Boadway, Marceau, and Mongrain (2002), Chang and Lai (2004), Gordon and Li (2009), Cuff et al. (2011), and Arbex and Mattos (2013), analyzing how an inequality-concerned planner can balance tax evasion with the optimal provision of public goods in the economy.

The remaining of the paper is organized as follows. In Section 2, we develop a model where HTT markets exist because of increasing tax audit costs. In Section 3, we characterize the optimal tax policies. Next, we present a numerical simulation to better understand how the economy is affected by the tax policies. Last, concluding remarks are offered.

2 A Model of Hard-to-Tax Markets

The underlying framework of our model builds on a spatial setting that follows Hotelling's (1929) linear economy model. The economy is a continuous line of unity length, with markets being uniformly distributed over the line and indexed by $x \in (0, 1]$. The tax authority is located at $x = 0$. In each market, there is a continuum of consumers, indexed by i, and a continuum of firms, indexed by j, where $i, j \in [0, 1]$. Firms produce a standard good with constant marginal cost and zero fixed cost. Markets are competitive. Consumers and firms cannot relocate and the price of the good in market x is p_x.

Sales of goods to consumers are subject to a proportional tax rate τ, the only tax policy available. Firms should remit tax payments, but they attempt to evade if enforcement is imperfect. We also assume that concealment cost is zero, so that a firm remits tax payments only if transactions are audited. The probability of collecting tax revenue is $\theta \in [0, 1]$, which implies that the expected tax rate is $\Theta = \theta\tau$. The tax authority incurs a cost t per unit of distance to audit transactions, i.e., a cost tx to audit the sale of one unit of the good in market x. This cost imposes a limit on how far in the linear economy the authority should attempt to collect taxes.[1]

Let $\bar{x}$ be the cut-off distance such that for $x \in (0, \bar{x}]$, denoted as region I, the tax authority audit transactions, obtaining a net expected tax rate $(\Theta - tx)$ per unit sold. For $x \geq \bar{x}$ (region II), there is no tax audit or collection. Hence, the increasing auditing costs to reach more distant markets give rise to untaxed markets (region II).

Consumers and firms make optimal choices taking tax enforcement policies and prices as given. Each consumer i at location x has a fixed non-consumable endowment $L_{i,x}$ which is transformed into consumable goods by firms. Each firm produces output $Q_{j,x}$ using a production technology with constant marginal return equal to one. Firm j located in market x chooses how much to produce to maximize its expected profit $\Pi_{j,x}$, determined as follows:

$$\Pi^{I}_{j,x} = p^{I}(1-\Theta)Q_{j,x} - Q_{j,x} \quad \text{if} \quad x \in (0, \bar{x}], \tag{1}$$

$$\Pi^{II}_{j,x} = p^{II}Q_{j,x} - Q_{j,x} \quad \text{if } x \in (\bar{x}, 1]. \tag{2}$$

where $\Pi^{I}_{j,x}$ and $\Pi^{II}_{j,x}$ represent the expected profit of firm j in market x, and p^{I} and p^{II} are the prices in regions I and II, respectively. Profit maximization and perfect competition imply that equilibrium prices are $p^{I} = (1-\Theta)^{-1}$ and $p^{II} = 1$.[2] Thus, the expected tax payment in taxed markets is Θp^{I} per unit and firms make zero profit in equilibrium.

Each consumer derives utility from the consumption of a private good (c) and a pure

[1] Geographical distance imposes a traveling cost to audit firms (Almeida and Carneiro, 2012), but distance can be interpreted more generally as the degree of difficulty to access transaction information. In practice the decrease in the effectiveness of tax collection can also arise due to a diminishing probability θ, but we simplify by assuming that only the audit cost is affected by x.

[2] In audited markets, tax is collected by the tax authority with probability θ, including fines for tax evasion. Hence, firms expect to pay a share θ of the nominal tax rate.

public good (g) as follows:

$$u^I(c^I_{i,x}, g) = w(c^I_{i,x}) + v(g) \quad \text{if } \; x \in (0, \bar{x}], \tag{3}$$

$$u^{II}(c^{II}_{i,x}, g) = w(c^{II}_{i,x}) + v(g) \quad \text{if } x \in (\bar{x}, 1]. \tag{4}$$

Utility is assumed to be additively separable, where w and v are increasing and strictly concave functions. Given the consumers' endowments and equilibrium prices, consumption of the private good in regions I and II are respectively

$$c^I_{i,x} = L_{i,x}/p^I = (1-\Theta)L_{i,x} \text{ if } x \in (0, \bar{x}], \tag{5}$$

$$c^{II}_{i,x} = L_{i,x}/p^{II} = L_{i,x} \text{ if } x \in (\bar{x}, 1]. \tag{6}$$

Hence, consumers in region I are clearly worse off as their private consumption is lower while everybody has equal access to the public good.

The provision of the public good g is determined by a technology with constant marginal rate of transformation between the public and the private good equal to 1, such that

$$g = R^I(\Theta) = \int_0^{\bar{x}} (\Theta - tx)\, L_x dx, \tag{7}$$

where L_x is the aggregate consumer endowment in market x and $R^I(\Theta)$ is the net tax revenue collected from taxed markets (region I).

3 Policy Choices

Assume all consumers and firms are identical ($L_{i,x} = L$, $Q_{j,x} = Q$), with uniform distribution over $x \in (0, 1]$. Since there is a measure one of consumers and firms in each market, the aggregate endowment, production, and consumption of the private good in market x are respectively $L_x = L$, $Q_x = Q_{j,x} = Q$, $c^I_x = c^I_{i,x}$ and $c^{II}_x = c^{II}_{i,x}$. The constant returns technology implies that $Q_x = L$ for all x.

Since consumers have identical endowments, they get the same utility within each market. Hence, the aggregate utility of consumers in region I and II are as follows:

$$U^I = \int_0^{\bar{x}} u^I(c_x^I, g)dx = \bar{x}\left[w(c^I) + v(g)\right], \tag{8}$$
$$U^{II} = \int_{\bar{x}}^1 u^{II}(c_x^{II}, g)dx = (1-\bar{x})\left[w(c^{II}) + v(g)\right]. \tag{9}$$

The planner chooses the effective tax rate (Θ) and the reach of the tax auditing efforts ($\bar{x}$) taking into account the aggregate utility in the economy and the inequality created by the unequal taxation across markets:

$$\max_{\{\Theta,\bar{x}\}} \quad U^I + U^{II} - \gamma D\left(\bar{x}, c^I, c^{II}\right) \tag{10}$$
$$\text{subject to (7).}$$

The function $D\left(\bar{x}, c^I, c^{II}\right)$ represents the concern of the planner about the disparity in utility across individuals due to differences in private consumption and γ is a scalar that represents the degree of that concern.[3] For instance, if $\gamma = 0$, the planner has no concern about the inequality across consumers. In the remaining of the paper, assume a separable $D(.)$ function, with $D'_{\bar{x}} \geq 0$ because a greater $\bar{x}$ implies that more consumers would have their private consumption reduced. On the other hand, when $c^{II} > c^I$, inequality decreases if c^I rises, but increases if c^{II} rises (thus, $D'_{c^I} < 0$ and $D'_{c^{II}} > 0$). However, $D = 0$ if there is no inequality, which happens in this model only when $\bar{x} = 1$ (i.e., when all markets are taxed).

[3] Similar inequality concern functions are considered by Kanbur, Keen, and Tuomala (1994), Wayne (2001), Blomquist and Micheletto (2006), Greco (2011), and Arbex, Mattos, and Trudeau (2012).

3.1 Optimal Tax Policies

Under the simplifying assumption that $L_{i,x} = L$, equation (7) becomes $g = L\left[\bar{x}\Theta - t\bar{x}^2/2\right]$. Hence, (10) implies the following condition for an interior solution of $\bar{x}$:

$$[\Theta - t\bar{x}]Lv'(g) = [w(c^{II}) - w(c^{I})] + \gamma D'_{\bar{x}}. \tag{11}$$

In words, the reach of tax audit should be expanded to the point where the marginal social utility of the provision of the public good (the left-hand side of (11)) equals the marginal social cost of private consumption loss (the first term in the right-hand side of (11)) and to the increased inequality (the last term in the right-hand side of (11)). The condition implies that if $t = 0$, all markets should be taxed ($\bar{x} = 1$) because the tax burden must be shared equally given the strict concavity of the $w(.)$ function. However, if t is large enough, it is not worth auditing distant markets ($\bar{x} < 1$). An interesting consequence is that the right-hand side of (11) is positive, implying that the net tax revenue $[\Theta - t\bar{x}] > 0$ at the optimal $\bar{x}$. That is, welfare is maximized by keeping some markets untaxed even if it is still possible to collect a positive net tax revenue from additional markets. Hence, forcing the tax authority to audit as many markets as feasible can lead to welfare reduction (this case is discussed in the next subsection).

Similarly to equation (10), the optimal tax rate Θ follows:

$$\bar{x}v'(g) = \bar{x}w'(c^{I}) - \gamma D'_{c^{I}}. \tag{12}$$

Thus, the marginal social benefit of raising Θ, which comes from increased g (left-hand side of (12)), should equal the marginal social cost, which is due to lower c and increased inequality (right-hand side of (12)). To understand how the existence of HTT markets affects the optimal tax rate, it is helpful to rewrite (12) as:

$$\frac{v'(g)}{w'^{I})} = 1 - \frac{\gamma D'_{c^I}}{\bar{x} w'^{I})}. \tag{13}$$

First, note that if $t = 0$ all markets would be taxed ($\bar{x} = 1$). Thus, without inequality, (13) would simply be $v'(g)/w'^{I}) = 1$, so that the marginal rate of substitution (MRS) between the public and the private goods equals one, which is the marginal rate of transformation (MRT). Therefore, the Samuelson condition is satisfied.

However, when t is large enough, $\bar{x} < 1$. As a result, the provision of the public good g will be reduced for a few reasons. First, consider the case where $\gamma = 0$ (no inequality concern). The optimal tax rate condition (13) becomes $v'(g)/w'^{I}) = 1$. Although this relation indicates an efficient tax rate given the $\bar{x}$ chosen, the resulting g level is smaller compared to the case where $t = 0$. With fewer consumers sharing the tax burden and with part of the tax revenue spent on the auditing cost, c^I decreases faster as more g is provided, inducing a smaller g (due to the strict concavity of the $w(c^I)$ function). Now, turn to the case where $\gamma > 0$. Because $D'_{c^I} < 0$, the optimality condition (13) implies that $v'(g)/w'^{I}) > 1$, that is, $MRS_{g,c} > MRT_{g,c}$, indicating under provision of the public good as the planner restrains taxation to avoid greater inequality among consumers. From (13), it also seems that a greater degree of concern about inequality (indicated by γ) tends to lead to greater under provision of g, although it is not possible to show unambiguously that the resulting g decreases because the optimal Θ and $\bar{x}$ are dependent on γ. In Section 4, the numerical exercise shows a case where g decreases with γ.

3.2 An Administrative Rule For $\bar{x}$

Alternatively, the tax authority might be forced to audit all markets where it is possible to collect positive net tax revenues. Such administrative rule may not be optimal from the point of view of welfare maximization, but it is a possible attempt to make the tax policy more inclusive, with the burden shared among as many people as feasible.

The rule forces the tax audit to be extended up to where the net tax revenue $(\Theta - t\bar{x}) = 0$. In contrast, the optimal $\bar{x}$, given by (11), implied $(\Theta - t\bar{x}) > 0$. Thus, the administrative rule leads to a larger $\bar{x}$, given by $\bar{x} = \Theta/t$. Consequently, the optimal Θ follows:

$$\frac{v'(g)}{w'^{I})} = 1 - \frac{\gamma D'_{c^I}}{\bar{x}w'^{I})} + \frac{1}{\bar{x}w'^{I})L}\frac{\partial \bar{x}}{\partial \Theta}\left\{\left[w(c^{II}) - w(c^{I})\right] + \gamma D'_{\bar{x}}\right\}. \tag{14}$$

Note that $D'_{c^I} < 0$, $\left[w(c^{II}) - w(c^{I})\right] > 0$, and $D'_{\bar{x}} \geq 0$. This means that, compared to (13), there are two additional positive terms in (14). Therefore, a larger Θ expands the tax base ($\partial\bar{x}/\partial\Theta > 0$ when $\bar{x} = \Theta/t$), so that the reduction of private consumption is greater. This suggests that the administrative rule leads evolves under-taxation compared to our benchmark case (equation (13)). Our numerical exercise reinforces that a reduction in g in addition to lower social welfare when administrative rule for $\bar{x}$ is in place.

4 Numerical Results

In this numerical analysis, we assume utility given by $u = ln(c) + \alpha ln(g)$, where $\alpha > 0$ is the relative weight of the utility derived from the public good. The planner's inequality aversion is given by the quadratic function $D\left(\bar{x}, c^I, c^{II}\right) = \left[\bar{x}\left(c^{II} - c^I\right)\right]^2$. This function states that the planner is increasingly concerned about the asymmetric reduction in private consumption caused by the tax policy (notice that $c^{II} - c^I = \Theta$). The planner's objective function is, thus, $W = \bar{x}ln(c^I) + (1-\bar{x})ln(c^{II}) + \alpha ln(g) - \gamma\left[\bar{x}\left(c^{II} - c^I\right)\right]^2$.

In the example presented here, α is kept constant at 0.10, which implies a marginal rate of substitution between the public good (g) and the private consumption good (c) equal to $0.10c/g$. HTT markets exist in the economy with a tax audit cost $t = 0.25$, which implies that a quarter of the production in the farthest market ($x = 1$) would be needed to cover the audit cost for that market. Endowment is normalized to one ($L_x = L = 1$).

Results are presented in Table 1. When there is no tax audit cost ($t = 0$), we obtain the efficient allocation with $g = 0.091$. Next, consider the case with $t = 0.25$ and the tax base ($\bar{x}$) chosen optimally by the tax authority. The provision of the public good is now much smaller than before ($g = 0.068$ when $\gamma = 0$). If the planner has concerns about the inequality across consumers ($\gamma > 0$), the chosen $\bar{x}$ and Θ become smaller as the degree of concern (γ) increases. A lower $\bar{x}$ implies that fewer consumers are burdened by taxation, while a lower Θ implies that consumers who still bear the burden will suffer less.[4] Since tax revenue decreases, the public good provision g drops even more.

Last, consider the case where the tax authority audits all markets where it can collect a positive net tax revenue. This administrative rule forces the tax base ($\bar{x}$) to be much larger, but at a lower tax rate (Θ). As a result, holding γ constant, g decreases even more.

5 Conclusion

The difficulty to audit all transactions in the economy leads to the existence of hard-to-tax (HTT) markets where little or no tax is collected. In this paper, we develop a model where increasing tax collection costs lead to the existence of HTT markets, thus reducing the provision of the public good. Moreover, inequality in private consumption arises due to the existence of untaxed markets, which can lead to policy concerns, thus restraining taxation and leading to an even smaller public good provision. Last, we show that forcing the tax authority to make tax collection more inclusive can aggravate the under provision of the public good. In practice, these problems might be mitigated by greater tax differentiation across markets together with more effective auditing methods and/or income transfers through other policy instruments to compensate for the unequal tax burden distribution. In recent years, governments in many countries have adopted measures such as monetary rewards (tax reduction, prizes, and rebates) and incentives to the use of electronic payments to monitor transactions (Eurofound, 2013; Schneider, 2013). However, evasion might benefit the poor

[4] This result depends on the form of the function D. For instance, the more the planner is concerned about the tax burden on each consumer rather than on the number of taxed consumers, the more likely it would choose a larger tax base $\bar{x}$ to allow for a lower tax rate Θ.

more than the rich in many cases. If so, measures that make taxation more inclusive may need to be balanced with policy changes to preserve the progressiveness of the tax system.

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Table 1 - Tax reach, tax rate, consumption, and welfare

γ	$\bar{x}$	Θ	g	c^I	c^{II}	$U^I + U^{II}$	γD	W
No tax audit cost ($t = 0$)								
-	1	0.091	0.091	0.909	-	-0.335	-	-0.335
Optimal $\bar{x}$ ($t = 0.25$)								
0	0.233	0.321	0.068	0.679	1.000	-0.359	0.000	-0.359
2	0.197	0.320	0.058	0.680	1.000	-0.360	0.008	-0.368
4	0.176	0.319	0.052	0.681	1.000	-0.363	0.013	-0.375
6	0.161	0.318	0.048	0.682	1.000	-0.365	0.016	-0.381
8	0.150	0.318	0.045	0.682	1.000	-0.368	0.018	-0.386
10	0.141	0.318	0.042	0.682	1.000	-0.370	0.020	-0.390
Administrative $\bar{x} = \Theta/t$ ($t = 0.25$)								
0	0.595	0.149	0.044	0.851	1.000	-0.408	0.000	-0.408
2	0.535	0.134	0.036	0.866	1.000	-0.410	0.010	-0.420
4	0.499	0.125	0.031	0.875	1.000	-0.413	0.016	-0.429
6	0.475	0.119	0.028	0.881	1.000	-0.417	0.019	-0.436
8	0.456	0.114	0.026	0.886	1.000	-0.420	0.022	-0.442
10	0.440	0.110	0.024	0.890	1.000	-0.423	0.024	-0.447

Fixed parameters: $(\alpha, L) = (0.10, 1)$.