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Unemployment fluctuations, and optimal monetary policy in a small open economy

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Abstract

In this paper, we incorporate key ingredients of a small open economy into the New Keynesian model with unemployment of Galí (2011a,b) to discuss the design of the monetary policy. The main findings regarding the issue of monetary policy design can be summarized as threefold. First, the optimal policy is to seek to minimize variance of domestic price inflation, wage inflation, and the output gap if both domestic price and wage are sticky. Second, stabilizing unemployment rate is important to reduce the welfare loss incurred by both technology and labor supply shocks. Therefore, introducing the unemployment rate as an another argument into the Taylor-rule type interest rate rule will be welfare-enhancing. Last, controlling CPI inflation is the best when the policy is not allowed to respond to unemployment rate.

Keywords: Unemployment; Monetary policy; Small open economy

JEL Classification: E31; E58; F41.

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1 Introduction

The New Keynesian model with staggered price setting constitutes the core of the dynamic stochastic general equilibrium framework that has emerged in recent years. It also has been adopted by many central banks and policy institutions as an analytical tool. The model, however, is not perfect and has shortcomings. One of the main weakness is the lack of reference to unemployment and its fluctuations. This absence of unemployment and the frictions underlying it may be interpreted as suggesting that central banks need not consider unemployment and its fluctuations in the design of monetary policy.

Over the past few years, however, a growing number of researchers have developed an extension of the models that incorporate the labor market frictions and unemployment into the New Keynesian framework. The typical framework in the literature combines the nominal rigidities of New Keynesian model with real frictions introduced by embedding a labor market with search and matching. Walsh (2003, 2005) and Trigari (2009) studied the effects of monetary policy shocks within a model that incorporates labor market frictions with sticky prices and flexible wages. More recent contributions introduce different forms of nominal and real wage rigidities to analyze the implications of labor market frictions and unemployment for the design of monetary policy (e.g.,Blanchard and Galí, 2010; Faia, 2008, 2009; Thomas, 2008).

Recently, Galí (2011a, b) proposed different approach to introduce unemployment into the standard New Keynesian model. Galí's approach is the reinterpretation of the labor market of Erceg, Henderson, and Levin (2000). The main advantage of Galí's approach is that the equilibrium levels of unemployment, the labor force, and unemployment rate can be easily determined within a standard representative household framework. In the model, the presence of market power in the labor markets results in unemployment, but the fluctuation in the unemployment rate is due to the presence of nominal wage rigidities.

Despite a growing number of studies that deal with labor market frictions and unemployment in the New Keynesian framework, any extension of the model to a small open economy has not been researched yet. Therefore, it is critical to extend the New Keynesian model with unemployment to a small open framework. This provides the motivation for our work.

Based on Galí (2011a, b), We develop a small open economy version of the New Keynesian model that allows for unemployment in the labor market. The goods market side of the model is similar in structure developed in Clarida, Galí and Gertler (2002), and Galí and Monacelli (2005). Monopolistically competitive domestic producers set prices in staggered contracts. In the labor market, individual households supply differentiated labor services to domestic firms, and domestic firms combine this labor services to produce domestic goods. Furthermore, each household with monopoly power in the labor market sets the nominal wages in staggered contract with timing like that of Calvo (1983). In this study, following Galí (2011b), we treat labor as being indivisible in that each period a given individual works a fixed number of hours or does not work at all. As a result all variations in labor input take place in the form of variations in employment.

The resulting framework allows us to determine the equilibrium level of employment and hence the unemployment in a small open economy. Unemployment in this model results from market power in the labor markets, reflected in a positive wage markup. The fluctuations in the unemployment rate are associated with variations in average wage markup due to the presence of nominal wage rigidities. The wage markup also depends on the behavior of the terms of trade. Therefore, the terms of trade affects unemployment and its fluctuations through the change in the wage markup. This is a natural consequence of a small open economy framework.

Within this model, we analyze the equilibrium properties of unemployment in response to various shocks using a calibrated version of the model, when the central bank follows a conventional Taylor rule. In order to disentangle the role played by different shocks results are shown for four different shocks (technological, foreign income, labor supply, and monetary shocks). Unfortunately, we find that none of shocks cannot provide reasonable relative volatilities and correlations of real world labor market. However, both monetary and foreign income shocks are able to generate a countercyclical unemployment and a procyclical employment.

Next, we turn to the relation between unemployment and the design of optimal monetary policy in a small open economy. In order to find the optimal monetary policy we derive a second-order approximation to the average welfare losses experienced by households in the economy with both wage and price stickiness around a steady state with zero inflation. The resulting welfare function can be expressed in terms of the unconditional variances of the output gap, domestic price inflation, and wage inflation, and the optimal policy seeks to minimize a weighted average of these variances. In addition to the optimal policy, we study several types of alternative simple policy rule in which the domestic nominal interest rate responds to inflation as well as output gap. The first rule, which is referred to as a domestic inflation–based Taylor rule, requires that the domestic interest rate responds systematically to domestic inflation, whereas the second assumes that the domestic interest rate responds to CPI inflation. That rule is referred to as a CPI inflation–based Taylor rule. Also assumed is analogous rule for wage inflation (wage inflation–based Taylor rule). We begin with analysis of unemployment and other macro variables under the optimal monetary policy and compare it to that under alternative rules. We use the welfare function to evaluate the performance of alternative policy rules. From this exercise, we find that CPI inflation-based Taylor rule generates relatively small welfare losses incurred by both technology and labor supply shocks. The welfare gains of CPI inflation–based Taylor rule comes from the fact that it is able to stabilize unemployment rate by reducing the fluctuations in real wage. This result suggest that introducing the unemployment rate as an another argument into the Taylor-rule type interest rate rule will be welfare-enhancing.

We also compute optimal simple interest rate rules among the class of alternative policy rules considered above. Motivated by the previous finding, we study the impact of adding the unemployment rate to the alternative policy rules. From this exercise, we find that for all cases, the inflation coefficients are positive and above one, whereas the output coefficients are negative and small. The unemployment coefficients are negative and relatively larger than output coefficients in absolute value. We also find that the welfare losses are reduced significantly once the interest rate is allowed to respond to the unemployment rate. This result points to the desirability of unemployment stabilization in the monetary policy. The optimized simple rule for the specification of CPI inflation-based Taylor generates relatively small welfare lose when unemployment is not allowed in the policy.

The plan of this paper is as follows. We present the basic model in section 2, while section 3 describes the equilibrium conditions and dynamic system of the model. The implications and performance of alternative policy regimes are discusses in section 4. Section 5 studies the optimal operational interest rate rules. In section 6, we draw the main conclusions.

2 The model

In this section, we consider a variant of a dynamic New Keynesian (NK) model applied to a small open economy. The goods market side of model draws most of its structure from those of Galí and Monacelli (2005). To keep the analysis simple, we assume that there are two countries, home (H) and foreign (F). The two countries share the same preferences, technology, and market structure, but differ in size: it is assumed that the foreign country is a large economy, but the home country is small. Following Galí (2011a,b), we modify the labor market and introduce unemployment into the small open economy NK model. We also treats labor as being indivisible in a sense that each period individual works a fixed number of hours or does not work at all. Therefore, all variations in labor input take place in the form of variations in employment.

Next, we describe the problem facing households and firms in this environment.

2.1 Households

The home country is populated by a large number of identical households. Each household has a continuum of members represented by the unit square and indexed by a pair $(i, j) \in [0, 1] \times [0, 1]$. The index, $i \in [0, 1]$ represents the type of labor services and the index, $j \in [0, 1]$ determines the disutility from work, which is given by

$$\begin{array}{ll} \chi_t j^{\varphi} & \text{if she is employed,} \\ 0 & \text{otherwise,} \end{array}$$

where $\varphi \geq 0$ is the inverse of Frisch elasticity of labor supply and $\chi_t > 0$ is an exogenous labor supply shock.

The household's period utility is given by the integral of its member's period utilities and can be written as

$$U(C_t, \{N_t(i)\}; \chi_t) \equiv \log C_t - \chi_t \int_0^1 \int_0^{N_t(i)} j^{\varphi} dj di$$
$$= \log C_t - \chi_t \int_0^1 \frac{N_t(i)^{1+\varphi}}{1+\varphi} di,$$

where $N_t(i) \in [0,1]$ is the fraction of members specialized in type i labor who are employed in period t, and C_t is a composite consumption index defined by

$$C_t \equiv \left[(1-\delta)^{\frac{1}{\eta}} (C_{H,t})^{\frac{\eta-1}{\eta}} + \delta^{\frac{1}{\eta}} (C_{F,t})^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{1-\eta}},$$

where, $C_{H,t}$ denotes an index of consumption of domestic goods given by

$$C_{H,t} \equiv \left[\int_0^1 C_{H,t}(z)^{\frac{\epsilon_P - 1}{\epsilon_P}} dz \right]^{\frac{\epsilon_P}{\epsilon_P - 1}},$$

and $C_{F,t}$ is an index of imported goods from the foreign country given by

$$C_{F,t} \equiv \left[\int_0^1 C_{F,t}(z^*)^{\frac{\epsilon_P - 1}{\epsilon_P}} dz^* \right]^{\frac{\epsilon_P}{\epsilon_P - 1}},$$

where $z, z^* \in [0,1]$ denote the good varieties produced by monopolistically competitive firms at home and abroad, respectively. Notice that parameter $\epsilon_P > 1$ denotes the elasticity of substitution between varieties of domestic and foreign goods, and parameter $\delta \in [0,1]$ is related to the share of imported goods in domestic consumption. It can also be interpreted as an index of openness. Parameter $\eta > 0$ measures the substitutability between domestic and foreign goods. It is also assumed that $\xi_t \equiv \log \chi_t$ follows the $AR(1)$ process:

$$\xi_t = \rho_\xi \xi_{t-1} + \varepsilon_t^\xi,$$

where $\rho_\xi \in [0,1]$ and ε_t^ξ is a white noise process with zero mean and variance σ_ξ^2.

A typical household seeks to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\log C_t - \chi_t \int_0^1 \frac{N_t(i)^{1+\varphi}}{1+\varphi} di \right],$$

subject to a sequence of budget constraints

$$P_t C_t + E_t\{Q_{t,t+1} B_{t+1}\} \leq B_t + \int_0^1 W_t(i) N_t(i) di + T_t, \tag{1}$$

where B_t is the purchase of a nominally riskless, internationally tradable, one-period discount bond paying one monetary unit, Q_t is the price of that bond, $W_t(i)$ is the nominal wage for type i labor, T_t denotes lump sum component of income (which includes transfers/taxes, and lump sum profits accruing from ownership of monopolistic firms), and

$$P_t \equiv \left[(1-\delta) (P_{H,t})^{1-\eta} + \delta (P_{F,t})^{1-\eta} \right]^{\frac{1}{1-\eta}},$$

is the consumer price index (CPI) with the domestic price index ($P_{H,t}$) and a price index for goods imported from foreign country ($P_{F,t}$) given by

$$P_{H,t} \equiv \left[\int_0^1 P_{H,t}(z)^{1-\epsilon_p} dz \right]^{\frac{1}{1-\epsilon_p}},$$

and

$$P_{F,t} \equiv \left[\int_0^1 P_{F,t}(z^*)^{1-\epsilon_p} dz^* \right]^{\frac{1}{1-\epsilon_p}}.$$

We assume that the household has access to a complete set of contingent claims traded internationally. The riskless short-term nominal interest rate, R_t, is given by

$$E_t \{Q_{t,t+1}\} = R_t^{-1}.$$

2.1.1 Optimal Wage Setting

Consider a household resetting its nominal wage in period t and let $\bar{W}_t$ denote the newly set wage. Under the assumption of full consumption risk sharing across households, all households resetting their wage in any given period will choose the same wage. The household will choose $\bar{W}_t$ in order to maximize

$$E_t \left\{ \sum_{k=0}^{\infty} (\beta\theta_w)^k \left[\log C_{t+k|t} - \chi_{t+k} \frac{N_{t+k|t}^{1+\varphi}}{1+\varphi} \right] \right\}, \tag{2}$$

where $C_{t+k|t}$ and $N_{t+k|t}$ respectively denote the composite consumption of domestic and imported goods and labor supply in period $t+k$ of a household that last reset its wage in period t.

Maximization of (2) is subject to the sequence of labor demand schedules and budget constraints that are effective while $\bar{W}_t$ remains in place.,

$$N_{t+k|t} = \left(\frac{\bar{W}_t}{W_{t+k}} \right)^{-\epsilon_w} \int_0^1 N_{t+k}(z) dz, \tag{3}$$

$$P_{t+k} C_{t+k|t} + E_t \left\{ Q_{t,t+k+1} B_{t+k+1|t} \right\} \leq B_{t+k|t} + \bar{W}_t N_{t+k|t} + T_{t+k},$$

for $k = 0, 1, 2, ...$ where $X_{t+k|t}$ denotes the value of X in period $t+k$ of a household that last reset its wage in period t. The remaining variables are defined as above.

The first-order condition is given by

$$\sum_{k=0}^{\infty} (\beta\theta_w)^k E_t \left\{ \frac{N_{t+k|t}}{C_{t+k|t}} \left(\frac{\bar{W}_t}{P_{t+k}} - \frac{\epsilon_w}{\epsilon_w - 1} MRS_{t+k|t} \right) \right\} = 0, \tag{4}$$

where $MRS_{t+k|t} \equiv \chi_{t+k} C_{t+k} N_{t+k|t}^{\varphi}$ denotes the marginal rate of substitution between consumption and labor supply in period $t+k$ for the household resetting the wage in period t. Log-linearizing (4) around the zero inflation steady state yields

$$\bar{w}_t = \mu^w + (1 - \beta\theta_w) \sum_{k=0}^{\infty} (\beta\theta_w)^k E_t \left\{ mrs_{t+k|t} + p_{t+k} \right\} \tag{5}$$

where $\mu^w \equiv log\left(\frac{\epsilon_w}{\epsilon_w - 1}\right)$, which corresponds to the log of the optimal or desired wage mark-up.

Let us define the economy's average marginal rate of substitution as $MRS_t \equiv \chi_t C_t N_t^{\varphi}$, where $N_t \equiv \int_0^1 N_t(i)di$ is the aggregate employment rate. Then, the (log) marginal rate of substitution in period $t+k$ for a household that last reset its wage in period t can be written as

$$\begin{aligned} mrs_{t+k|t} &= mrs_{t+k} + \varphi\left(n_{t+k|t} - n_{t+k}\right) \\ &= mrs_{t+k} - \epsilon_w \varphi\left(\bar{w}_t - w_{t+k}\right) \end{aligned}$$

where the last equality makes use of (3). Hence, we can rewrite (5) as

$$\bar{w}_t = \frac{1-\beta\theta_w}{1+\epsilon_w\varphi}\sum_{k=0}^{\infty}(\beta\theta_w)^k E_t\left\{\mu^w + mrs_{t+k} + \epsilon_w\varphi w_{t+k} + p_{t+k}\right\} \tag{6}$$

2.1.2 Other Optimality Conditions

In addition to the optimal wage setting condition, the solution to the household's problem also yields the optimal demand for each goods

$$C_{H,t}(z) = \left[\frac{P_{H,t}(z)}{P_{H,t}}\right]^{-\epsilon_p} C_{H,t}; \quad C_{F,t}(z^*) = \left[\frac{P_{F,t}(z^*)}{P_{F,t}}\right]^{-\epsilon_p} C_{F,t}, \tag{7}$$

The optimal allocation of expenditures between domestic and imported goods is also given by

$$C_{H,t} = (1-\delta)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t; \quad C_{F,t} = \delta\left(\frac{P_{F,t}}{P_t}\right)^{-\eta} C_t,$$

The household's intertemporal optimality condition is given by

$$\beta\left[\frac{C_t}{C_{t+1}}\frac{P_t}{P_{t+1}}\right] = Q_{t,t+1}, \tag{8}$$

Equation (8) is a standard Euler equation for intertemporal consumption decision and represents the expectational IS curve.

Taking conditional expectations of both sides of (8) and rearranging with the riskless short-term nominal interest rate, we obtain a standard stochastic Euler equation

$$\beta R_t E_t\left\{\frac{C_t}{C_{t+1}}\frac{P_t}{P_{t+1}}\right\} = 1. \tag{9}$$

For future reference, we write (9) in log-linearized form as

$$c_t = E_t\{c_{t+1}\} - \left[r_t - E_t\left\{\pi_{c,t+1}\right\} - \rho\right], \tag{10}$$

where lower case letters denote the log-deviations of the respective variables from their steady states, $\pi_{c,t+1} \equiv p_{t+1} - p_t$ is CPI inflation, and c_t denotes total aggregate consumption; finally, $r_t \equiv -\log Q_{t+1}$ is the nominal yield on the one-period bond.

2.2 Unemployment

In this section, we introduce unemployment into the standard small open economy NK model and discuss its relation with the wage markup. The model draws most of its structure from those of Galí (2011b). An individual specialized in type i labor with disutility of work $\chi_t j^{\varphi}$ will be willing to work in period t if and only if

$$\frac{W_t(i)}{P_t} \geq \chi_t C_t j^{\varphi},$$

where the term on the right side represents the disutility of work in terms of marginal utility of consumption. Let $L_t(i)$ denote type i labor supply or participation. Then, for the marginal supplier of type i labor

$$\frac{W_t(i)}{P_t} = \chi_t C_t L_t(i)^{\varphi}. \tag{11}$$

Taking log of (11) and integrating over i, we can derive the following approximation:

$$w_t - p_t = c_t + \varphi l_t + \xi_t, \tag{12}$$

where $w_t \simeq \int_0^1 w_t(i)di$ and $l_t \simeq \int_0^1 l_t(i)di$ is the first-order approximation of aggregate labor force or participation around its symmetric steady state. Equation (12) can be interpreted as an aggregate labor supply or participation condition.

We define the unemployment rate u_t as the log difference between the labor force and employment:

$$u_t \equiv l_t - n_t. \tag{13}$$

The average wage mark-up can be defined as

$$\mathcal{M}_t^w = \frac{W_t/P_t}{MRS_t}.$$

Then, using the definition of marginal rate of substitution, we write the average wage mark-up in log–linearized form

$$\mu_t^w \equiv (w_t - p_t) - (c_t + \varphi n_t + \xi_t), \tag{14}$$

where μ_t^w is a log of the average wage mark-up. Combining (14) with (12) and (13), we can obtain the following simple relation between the wage markup and the unemployment rate:

$$\mu_t^w = \varphi u_t. \tag{15}$$

Equation (15) shows that unemployment fluctuation is a consequence of variations in the wage markup, which are the result of nominal wage rigidities. Now, we can define the natural rate of unemployment, u_t^n, the unemployment rate that would prevail in the absence of nominal wage rigidities, as

$$u^n = \frac{\mu^w}{\varphi}. \tag{16}$$

Equation (16) implies that since the optimal wage markup is constant, a natural rate is also constant over time. It also shows that there exists positive unemployment even in the absence of wage rigidities as long as $\mu^w > 0$. The presence of market power in the labor market, reflected in a positive optimal wage markup, accounts for the existence of positive unemployment.

2.3 Firm

2.3.1 Technology

Next, we consider the production side of the economy. The market for domestic goods in the home country is populated by a continuum of domestic firms acting as monopolistic competitors indexed by $z \in [0,1]$, whose total is normalized to unity. Each domestic firm produces a differentiated good with a technology represented by the production function

$$Y_t(z) = A_t N_t(z)^{1-\alpha}, \tag{17}$$

where $a_t \equiv \log A_t$ follows the AR(1) process $a_t = \rho_a a_{t-1} + \varepsilon_t^a$, and $N_t(z)$ is an index of labor input used by firm i and defined by

$$N_t(z) \equiv \left[\int_0^1 N_t(z,i)^{1-\frac{1}{\epsilon_w}} di\right]^{\frac{\epsilon_w}{\epsilon_w - 1}}, \tag{18}$$

where $N_t(z,i)$ denotes the quantity of type-i labor employed by firm z in period t, and parameter ϵ_w denotes the elasticity of substitution among labor varieties. We also assume a continuum of labor types, indexed by $i \in [0,1]$.

Let $W_t(i)$ denote the nominal wage for type-i labor effective in period t, for all i. As mentioned above, wages are set by workers of each type and taken as given by firms. Given the wages at any point in time, cost minimization yields a corresponding set of demand schedules for each firm z and labor type i, given the firm's total employment $N_t(z)$

$$N_t(z,i) = \left(\frac{W_t(i)}{W_t}\right)^{-\epsilon_w} N_t(z), \tag{19}$$

for all $z, i \in [0,1]$, where

$$W_t \equiv \left[\int_0^1 W_t(i)^{1-\epsilon_w} di\right]^{\frac{1}{1-\epsilon_w}}, \tag{20}$$

is a aggregate nominal wage index.

Furthermore, it is assumed that each firm receives a subsidy of τ percent of its wage bill. Then, the real marginal cost in terms of domestic good prices (in log term) will be the same across domestic firms and given by

$$mc_t = \nu + w_t - p_{H,t} - a_t + \alpha n_t, \tag{21}$$

where $\nu \equiv \log(1-\tau) - \log(1-\alpha)$.

For future reference, we derive an approximate aggregate production function in relation to aggregate employment. Hence, notice that

$$N_t \equiv \int_0^1 N_t(z)dz = \frac{Y_t D_t}{A_t},$$

where $D_t \equiv \int_0^1 \frac{Y_t(z)}{Y_t}dz$. Around the perfect foresight steady state, equilibrium variation in $d_t \equiv \log D_t$ are of second order (See Galí and Monacelli,2005). Thus, up to a first order approximation, we have an aggregate production relation

$$y_t = a_t + (1-\alpha)n_t.$$

2.3.2 Price-setting

We now turn to the pricing decisions of domestic firms. Following Calvo (1983), we assume that a fraction of $1-\theta_p$ of (randomly selected) domestic firms set new prices each periods, with an individual firm's probability of re-optimizing in any given period being independent of the time elapsed since it last reset its price. As shown in Galí and Monacelli (2005), optimal price-setting strategy for the typical firm resetting its price in period t can be approximated by the (log-linear) rule

$$\bar{p}_{H,t} = \mu^p + (1-\beta\theta_p)\sum_{k=0}^{\infty}(\beta\theta_p)^k E_t\left\{\frac{1-\alpha}{1-\alpha+\alpha\epsilon_p}mc_{t+k} + p_{H,t}\right\}, \tag{22}$$

where $\bar{p}_{H,t}$ denotes the (log) of newly set domestic prices, and $\mu^p \equiv \log \mathcal{M}^p = \log\left(\frac{\epsilon_p}{\epsilon_p - 1}\right)$, which corresponds to the log of the optimal price mark-up in a flexible price equilibrium or in the steady state.

2.3.3 Foreign country

We assume that the foreign economy is large so that

$$P^*_{F,t} = P^*_t,$$
$$C^*_t = Y^*_t.$$

With complete international securities markets, a symmetric set of first-order conditions holds for the foreign country:

$$\beta\left\{\frac{C^*_t}{C^*_{t+1}}\frac{P^*_t}{P^*_{t+1}}\right\} = Q^*_{t,t+1}, \tag{23}$$

where $\frac{1}{E_t\left[Q^*_{t,t+1}\right]} = R^*_t$ is the riskless short-term foreign nominal interest rate. Goods produced in the home country are sold to foreigners. The foreign country's demand for the home country's output z is given by

$$C^*_{H,t}(z) = \alpha\left[\frac{P^*_{H,t}(z)}{P^*_{H,t}}\right]^{-\epsilon_p} C^*_{H,t},$$

and the optimal allocation of expenditures for domestic goods is given by

$$C_{H,t}^{*} = \alpha \left(\frac{P_{H,t}^{*}}{P_t^{*}} \right)^{-\eta} C_t^{*}.$$

3 Equilibrium

3.1 Aggregate Demand

Goods market clearing in the home country requires

$$\begin{aligned} Y_t(z) =& C_{H,t}(z) + C_{H,t}^{*}(z) \\ =& \left(\frac{P_{H,t}(z)}{P_{H,t}} \right)^{-\epsilon} \left[(1-\delta) \left(\frac{P_t}{P_{H,t}} \right) C_t + \delta \left(\frac{P_t^{*}}{P_{H,t}^{*}} \right) C_t^{*} \right]. \end{aligned} \tag{24}$$

Define aggregate output as

$$Y_t \equiv \left[\int_0^1 Y_t(z)^{\frac{\varepsilon_p - 1}{\varepsilon_p}} dz \right]^{\frac{\varepsilon_p}{\varepsilon_p - 1}}.$$

Substituting (24) into the definition of aggregate domestic output index together with the international risk-sharing condition, $C_t = \mathcal{Q}_t C_t^{*}$, yields

$$Y_t = \left(\frac{P_{H,t}}{P_t} \right)^{-\eta} C_t \left[(1-\delta) + \delta \mathcal{Q}_t^{\eta} \right], \tag{25}$$

where $\mathcal{Q} = \frac{\epsilon_t P_t^{*}}{P_t}$ is the real exchange rate.[1]

The first order log-linear approximation to (25) around the steady state:

$$\begin{aligned} y_t &= c_t - \eta \left(p_{H,t} - p_t \right) + \delta \left(\eta - 1 \right) q_t \\ &= c_t + \delta \sigma s_t, \end{aligned} \tag{26}$$

where the last equality follows from the definition of the terms of trade, $s_t = p_{F,t} - p_{H,t}$ and the real exchange rate $q_t = (1-\delta) s_t$, and while $\sigma \equiv \eta + (1-\delta)(\eta - 1)$.

For foreign country, $y_t^{*} = c_t^{*}$. Hence, a condition similar to (26) for foreign country can be written as

$$c_t^{*} = y_t^{*} + (1-\delta) s_t. \tag{27}$$

Combining (26) with (27), we obtain

$$y_t = y_t^{*} + \phi s_t, \tag{28}$$

where $\phi \equiv (1-\delta) + \delta\sigma > 0$.

Finally, combining (26) with Euler equation (10), we get

$$y_t = E_t\{y_{t+1}\} - \psi^{-1} \left[r_t - E_t\{\pi_{H,t+1}\} + \delta \phi^{-1} (\sigma - 1) E_t \left\{ \Delta y_{t+1}^{*} \right\} - \rho \right], \tag{29}$$

where $\psi^{-1} \equiv 1 + \delta \phi^{-1} (1 - \sigma)$.

[1] ϵ is a nominal exchange rate, which is defined as the price of foreign currency in terms of domestic currency.

3.2 Supply side

As shown in Galí and Monacelli (2005), the (log-linearized) optimal price-setting condition (22) can be combined with the (log-linearized) difference equation describing the evolution of domestic prices to yield an equation determining domestic inflation as a function of deviations of marginal cost from its steady state value

$$\pi_{H,t} = \beta E_t \{\pi_{H,t+1}\} - \lambda_{p_H} \hat{\mu}_t^{p_H}, \tag{30}$$

where $\hat{\mu}_t^{p_H} \equiv \mu_t^w - \mu^{p_H} \equiv -\hat{mc}_t$ and $\lambda_{p_H} \equiv \frac{(1-\theta_{p_H})(1-\beta\theta_{p_H})}{\theta_{p_H}(1-\alpha+\alpha\epsilon_p)}(1-\alpha)$.

Let $\hat{\mu}_t^w \equiv \mu_t^w - \mu^w$ denote the deviation of the economy's (log) average wage markup as $\mu_t^w \equiv w_t - mrs_t$ from its steady state level μ^w. Then (6) can be rewritten as

$$w_t = \beta\theta_w E_t \{\bar{w}_{t+1}\} + (1-\beta\theta_w)\left[w_t - (1-\epsilon_w\varphi)^{-1}\hat{\mu}_t^w\right]. \tag{31}$$

Given the assumption that all households which are able to adjust their wage at time t will choose the same wages, the aggregate real wage index will evolve according to

$$w_t = \left[\theta_w w_{t-1}^{1-\epsilon_w} + (1-\theta_w)\bar{w}_t^{1-\epsilon_w}\right]^{\frac{1}{1-\epsilon_w}}. \tag{32}$$

A first-order Taylor expansion of (32) around the zero inflation steady state yields

$$w_t = \theta_w w_{t-1} + (1-\theta_w)\bar{w}_t, \tag{33}$$

Combining (31) with(33) results in

$$\pi_{W,t} = \beta E_t\{\pi_{W,t+1}\} - \lambda_w \hat{\mu}_t^w, \tag{34}$$

where $\pi_{W,t} = w_t - w_{t-1}$, and $\lambda_w = \frac{(1-\theta_w)(1-\beta\theta_w)}{\theta_w(1+\epsilon_w\varphi)}$. Note that this wage inflation equation has a formal analogous to (30), equation describing the dynamics of domestic price inflation.

Equations (34), (15), and (16) can be combined to derive a relation between wage inflation and unemployment:

$$\pi_{W,t} = \beta E_t\{\pi_{W,t+1}\} - \lambda_w \varphi\,(u_t - u^n),$$

Galí (2011b) refers to this equation as the New Keynesian wage Phillips curve and also provides empirical evidence using postwar US data.

3.3 Equilibrium dynamics

In this section, we derive linearized equilibrium dynamics for the domestic price and wage inflation in term of output gap $\tilde{y}_t = y_t - y_t^n$ analogous to that of the recent standard New Keynesian small open economy model.

First, we introduce a new variable, the real wage gap $\tilde{\omega}_t^R$, and define formally as

$$\tilde{\omega}_t^R \equiv \omega_t^R - (\omega_t^R)^n,$$

where $(\omega_t^R)^n$ is the natural real wage, i.e., the real wage that would prevail in the flexible prices and wages equilibrium. The real marginal cost is

$$\begin{aligned} mc_t &= \nu + w_t - p_{H,t} - a_t + \alpha n_t \\ &= \nu + \omega_t^R + \delta s_t - y_t + n_t, \end{aligned} \tag{35}$$

where last equality makes use of $p_t = p_{H,t} + \delta s_t$ and aggregate production relation. Then the natural real wage can be expressed as

$$(\omega_t^R)^n = -\nu - \mu^p - \delta s_t^n + y_t^n - n_t^n, \tag{36}$$

where s_t^n, y_t^n, and n_t^n are the terms of trade, output, and employment at the natural level.

Next, we relate the average price markup to the output and real wage gaps. Using the fact that $\hat{\mu}_t^{p_H} = -(mc_t - mc_t^n)$,

$$\begin{aligned} \hat{\mu}_t^{p_H} &= \left[(y_t - y_t^n) - (n_t - n_t^n) - \left(\omega_t^R - (\omega_t^R)^n\right) - \delta(s_t - s_t^n)\right] \\ &= -\tilde{\omega}_t^R - \delta \tilde{s}_t \\ &= -\tilde{\omega}_t - \frac{\delta}{\phi}\tilde{y}_t. \end{aligned} \tag{37}$$

Hence, combining (30) and (37) yields

$$\pi_{H,t} = \beta E_t\left\{\pi_{H,t+1}\right\} + \lambda_{p_H}\tilde{w}_t^R + \kappa_{p_H}\tilde{y}_t, \tag{38}$$

where $\kappa_{p_H} = \left(\frac{\alpha+\delta\phi^{-1}}{1-\alpha}\right)\lambda_{p_H}$. Eq.(38) represents equation for domestic price inflation, which is similar to the equation for price inflation of Erceg, Henderson, and Levin (2000).

Similarly, relate the average wage markup to the output and real wage gaps as

$$\begin{aligned} \hat{\mu}_t^w &= \omega_t^R - [c_t + \varphi n_t + \xi_t] - \mu^w \\ &= \tilde{\omega}_t^R - (1-\delta)\phi^{-1}\tilde{y}_t - \varphi \tilde{n}_t \\ &= \tilde{\omega}_t^R - \left[(1-\delta)\sigma^{-1} + \frac{\varphi}{1-\alpha}\right]\tilde{y}_t, \end{aligned} \tag{39}$$

where last equality makes use of (28) and aggregate production relation. Therefore, we can derive the following wage inflation equation

$$\pi_{W,t} = \beta E_t\left\{\pi_{W,t+1}\right\} + \kappa_w \tilde{y}_t - \lambda_w \tilde{w}_t, \tag{40}$$

where $\kappa_w = \lambda_w\left[(1-\delta)\sigma^{-1} + \frac{\varphi}{1-\alpha}\right]$. Combining (15), and (39) lead to the following equation describing relation between the unemployment rate and the output and wage gaps as:

$$\tilde{u}_t = \varphi^{-1}\left\{\tilde{\omega}_t^R - \left[(1-\delta)\phi^{-1} + \frac{\varphi}{1-\alpha}\right]\tilde{y}_t\right\}. \tag{41}$$

Note that there is an identity relating the change in the real wage gap to domestic price inflation, wage inflation, and the natural wage

$$\tilde{\omega}_t^R \equiv \tilde{w}_{t-1}^R + \pi_{W,t} - \pi_{H,t} - \delta \triangle s_t - \triangle (w_t^R)^n.$$

The dynamic IS equation for the small open economy can be obtained by rewriting (26) in terms of output gap as

$$\tilde{y}_t = E_t \{\tilde{y}_{t+1}\} - \psi^{-1} \left[r_t - E_t\{\pi_{H,t+1}\} - \bar{rr}_t \right], \tag{42}$$

where $\psi^{-1} = 1 + \delta\phi^(- 1)(1 - \sigma)$, and

$$\bar{rr}_t \equiv \rho + \psi E_t\{\triangle y_{t+1}^n\} - \delta(\sigma - 1)\phi^{-1} E_t\{\triangle y_{t+1}^*\}$$

is the small open economy's natural rate of interest.

In order to close the model, we specify how the interest rate is determined. This is done by assuming a Taylor-type interest rule of the form

$$r_t = \rho + \phi_\pi \pi_{C,t} + \phi_{\tilde{y}} \tilde{y}_t + \nu_t, \tag{43}$$

where $\pi_{C,t} = \pi_{H,t} + \delta \triangle s_t$ CPI inflation, and where ν_t is an exogenous monetary policy component, which is assumed to follow an AR(1) process:

$$\nu_t = \rho_\nu \nu_{t-1} + \varepsilon_t^\nu,$$

where $\rho_\nu \in [0, 1]$ and ε_t^ν is a white noise process with zero mean and variance σ_ν^2.

Finally, and for the future reference, in the labor market equilibrium under flexible prices, $(\omega_t^R)^n = mrs_t^n$ for all t. Thus, the natural level of unemployment in the open economy is given by

$$n_t^n = -(\nu - \mu^{p_H})\Gamma^{-1} - \Gamma^{-1}\xi_t + (\Omega - \alpha\phi^{-1})\Gamma^{-1}(a_t - y_t^*), \tag{44}$$

where $\Gamma \equiv \frac{1}{\alpha+\varphi+(1-\Omega)(1-\alpha)+(1-\alpha)\delta\phi^{-1}}$, and $\Omega \equiv \frac{\sigma\delta}{1-\delta+\delta\sigma}$. Then, some straightforward algebra yields the following expression for the natural values of the output, and wages:

$$(\omega_t^R)^n = -(\nu - \mu^{p_H}) + \frac{1}{1-\alpha} a_t + \delta\phi^{-1} y_t^* - \frac{\alpha + (1-\alpha)\delta\phi^{-1}}{1-\alpha} y_t^n,$$
$$y_t^n = a_t + (1-\alpha) n_t^n.$$

4 Dynamic responses

4.1 Calibration

This section studies how the presence of unemployment together with nominal wage rigidities influence the economy's response to different types of shocks for a calibrated version of the small open economy model developed in the previous section. The setting chosen for many of the parameters is standard. In the baseline calibration of the model, one period corresponds to one quarter of a year. We set $\sigma = \eta = 1$, which is consistent with the case considered in the next section. The discount factor β is set to 0.99, which generates a real interest rate of around 4% per annum. We set the value of δ (degree of openness) to 0.4, which corresponds to the import/GDP ratio in Canada. Parameter α, the degree of decreasing returns to labor, is set to 0.25. The elasticity of substitution among goods ϵ_p is set to 9. This implies that at the steady state, the price markup is 12.5 percent, and with the calibration of α, labor income share is 2/3. The domestic price and wage contract duration parameters are set as $\theta_{p_H} = \theta_w = 0.75$, implying the average contract duration of one year in a way consistent with much of the micro–evidence. Galí (2011b) argues that the introduction of unemployment into the standard New Keynesian model poses some restriction on the calibration of the inverse Frisch elasticity of labor supply, φ, and the elasticity of substitution among labor services, ϵ_w, since the average markup is related to the natural rate of unemployment as $\frac{\epsilon_w}{\epsilon_w - 1} = \exp \varphi u^n$. Therefore, we set $\varphi = 5$, implying that the labor supply elasticity is taken as 1/5 and $u^n = 0.05$, implying $\epsilon_w = 4.52$. The values of average wage markup, then, is 28 percent. The specification of the interest rate rules follows Taylor (1993): $\phi_y = 0.125$ and $\phi_\pi = 1.5$. The persistency parameter of monetary shocks is chosen as $\rho_\nu = 0.5$ with $\sigma_\nu = 0.0075$. Finally, we follow Galí and Monacelli (2005), and Galí (2011b) to specify the exogenous processes as follows:

$$
\begin{aligned}
a_t &= 0.66 a_{t-1} + \varepsilon_t^a, \ \sigma_a = 0.0071, \\
y_t^* &= 0.86 y_{t-1}^* + \varepsilon_t^{y^*}, \sigma_{y^*} = 0.0078, \\
\xi_t &= 0.90 \xi_{t-1} + \varepsilon_t^{\xi}, \ \sigma_{\xi^*} = 0.0075,
\end{aligned}
$$

where ε_t^a, $\varepsilon_t^{y^*}$ and ε_t^{ξ} are white noises with variances σ_a, σ_{y^*}, and σ_ξ, respectively.

4.2 Impulse Responses

Figure 1 illustrates the dynamic responses of main variables (output gap, output, domestic and CPI inflation, real wage rate, the rate of depreciation, unemployment rate and labor force) to the four exogenous shocks–technology, monetary, labor supply and foreign income shocks–under the baseline calibration.

First described are the dynamic effects of a domestic productivity shock. This is illustrated by the circled lines in Figure 1. It shows that both output and unemployment rate rise, but labor force declines slightly to an improvement in technology. The increase in the unemployment rate, hardly muted by the small decline in the labor force, is result of the drop in employment. These findings are consistent with much of empirical evidence, which has been found in many studies (see Blanchard and Quad, 1989; Galí, 1999; Galí and Rabanal, 2004). Notice that domestic inflation declines as would be expected, but CPI inflation rises before it starts to decline. The initial increase of CPI inflation is manly due to the movement of nominal exchange rate. The shock leads the central bank to reduce the domestic interest rate. Given

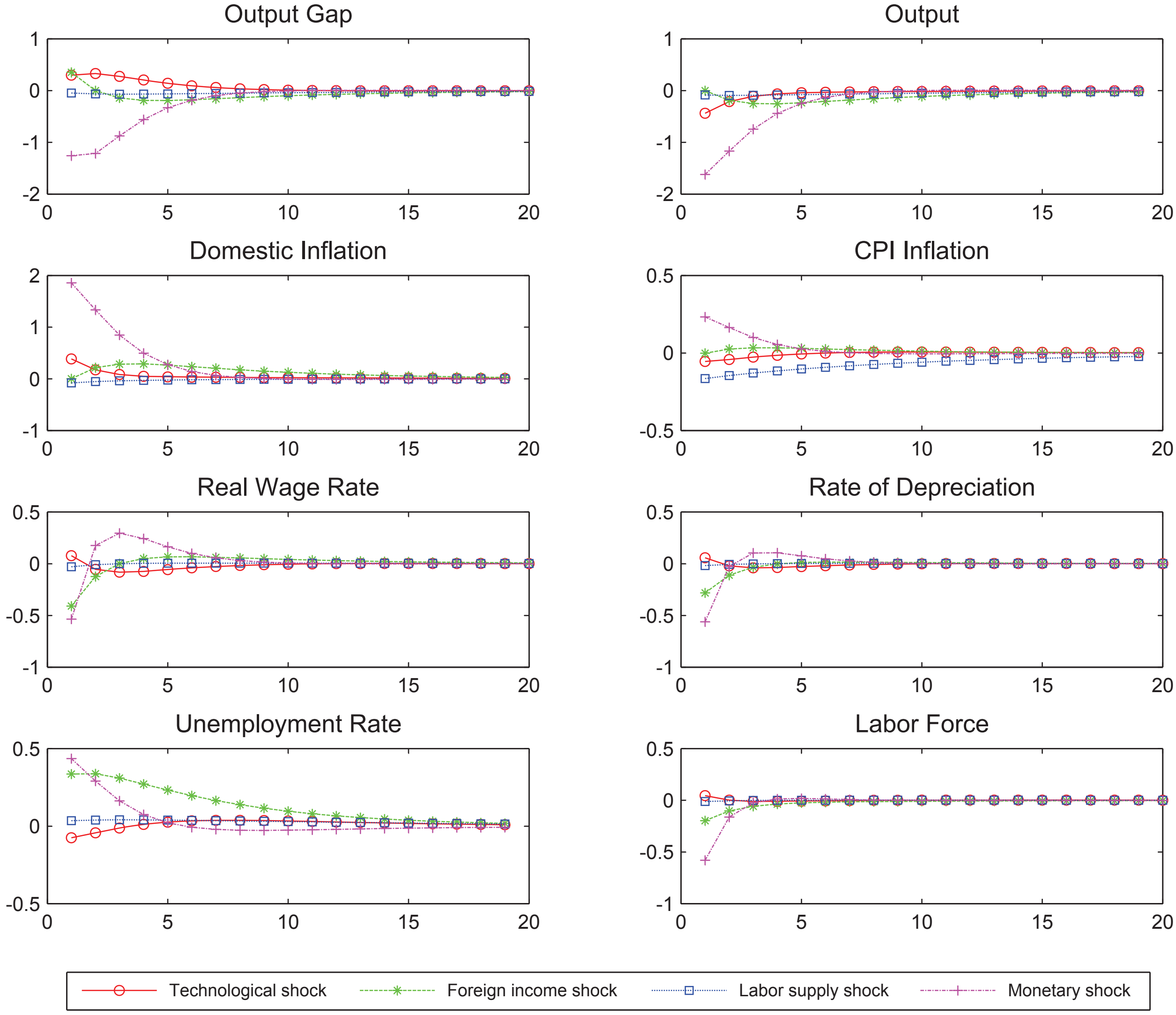

Figure 1: Impulse responses

the constancy of the world nominal interest rate, uncovered parity implies an initial nominal depreciation followed by a future appreciation. The effect of the decrease in domestic inflation on CPI inflation is muted by the initial nominal depreciation. Notice also that the real wage rises gradually after a substantial decrease. With nominal wage rigidities, the initial increase of CPI inflation leads to a decline in real wage. As CPI inflation decreases, real wage increases slightly over time.

The dynamic responses of the same variables to a foreign income shock is illustrated by the dashed lines with star. Note that the increase in foreign income improves the domestic terms of trade that results in an appreciation of domestic currency and decrease in CPI inflation. These changes are substantial. The increase in foreign income also leads to rise in both output and domestic inflation, but leads to decline in the unemployment rate. The labor force also declines due to an wealth effect. Note also that changes in output, unemployment, labor force, and domestic inflation are relatively small compared to changes in the exchange rate and CPI inflation. The reason is that the effect of the foreign income shock on these variables is muted by a large appreciation of domestic currency. The decline in CPI inflation together with lower unemployment rate leads to a rise in real wage rate.

Given the constancy of real wage and consumption, (12) implies that an increase in ξ_t contracts the labor force by $1/\varphi$. From Figure 1, we can see that the drop in the labor force leads the unemployment rate to decline, putting upward pressure on nominal wages and hence, although small, the domestic inflation rises gradually. The shock, therefore, leads to a persistent increase in the domestic interest rate, which results in the contraction of output. The interest rate hike also leads to a nominal appreciation followed by depreciation. Note that CPI inflation declines, while the real wage rate increases as a result of the upward pressure of both unemployment and CPI inflation on nominal wage.

Finally, the dashed line with cross in figure 1 shows the responses to a contractionary monetary shock. The shock consists of an increase of 0.25 percentage points in the exogenous component of the interest rate rule, which, in the absence of an endogenous change induced by the response of inflation or the output gap, would lead to an impact increase of one percentage point in the annualized nominal interest rate. As would be expected, output declines, but unemployment rate increases substantially due to the contraction in consumption resulting from the interest rate hike. Note that the labor force rises, only by a small amount, due to the decreased in consumption. Notice also that the nominal exchange rate overshoot the long run value in response to the tightening of monetary policy. The movement of CPI mainly follows the response of the nominal exchange rate. The counter cyclical response of the real wage is due to the decline in CPI inflation.

4.3 Second Moments

For the quantitative evaluation of the model, the table 1 reports, after detrended by applying Hodrick-Prescott filter with smoothing parameter of 1600, the standard deviations (relative output) and the correlations with output of unemployment, employment, the labor force, the real wage and CPI inflation generated by the calibrated model conditional on each of four shocks.

Table 1: Second Moments

	Technology		Foreign Income		Labor Supply		Monetary	
	$\rho(x,y)$	$\frac{\sigma(x)}{\sigma(y)}$	$\rho(x,y)$	$\frac{\sigma(x)}{\sigma(y)}$	$\rho(x,y)$	$\frac{\sigma(x)}{\sigma(y)}$	$\rho(x,y)$	$\frac{\sigma(x)}{\sigma(y)}$
Labor Force	-0.98	0.16	-0.99	0.19	0.89	2.57	-0.98	0.19
Unemployment	0.75	1.91	-0.99	1.54	0.63	1.42	-0.99	1.53
Employment	-0.79	2.03	0.99	1.35	0.99	1.29	0.99	1.33
CPI Inflation	-0.12	0.19	-0.81	0.59	0.23	0.28	0.47	0.33
Real Wage	-0.84	0.28	-0.19	1.05	-0.98	0.57	- 0.99	0.37

The cyclical properties of the labor market are well known and common to many countries, but are summarized here to be compared with the model: Employment is more volatile than unemployment, but labor force is least volatile. All three variables are less volatile than GDP. Unemployment is highly countercyclical, but employment and labor force are procyclical, but latter is only mildly. The real wage is only mildly procyclical or almost acyclical with moderate volatility. CPI inflation is also moderately procyclical.[2].

The corresponding second moments of the calibrated model conditional on the technological shock are reported on the first panel of Table 1. As seen in the impulse response analysis, technology shock can be seen to generate a countercyclical employment and a procyclical unemployment rate. Both employment and unemployment rate are much more volatile than output. The cyclical properties of the labor market implied are not consistent with the date. The real wage is highly countercyclical, and CPI inflation is also countercyclical, but only mildly.

The next panel reports the second moments generated by a demand shock, shock to the foreign income. As in the data, employment is highly procyclical and unemployment rate is highly countercyclical. However, both are more volatile that the output. The labor force shows a high countercyclicality, which is not consistent with the procyclicality of the labor force in the data. The real wage is mildly procyclical, but CPI inflation is countercyclical.

In the next panel in the Table 1, the second moments generated by labor supply shock are shown. The distinguishing patterns generated by labor supply shocks are high volatility and procyclicality of employment, unemployment rate and labor force. This is not surprising. The real wage is highly procyclical, while CPI inflation is countercyclical.

The last panel shows the conditional statistics of same variables generated by a contractionary monetary shock. The monetary shock generates almost same patterns of employment

[2]Galí (2011 b) reports the standard deviations relative GDP and the correlations with GDP of the unemployment rate, employment, the labor force, the real wage and inflation for both US and the euro area

and unemployment rate as a foreign income shock. The noticeable difference is that the real wage is now highly countercyclical, but CPI inflation is procyclical.

From the analysis, we can see that the cyclical properties of the calibrated model are not consistent with the stylized facts of the labor market to suggest that the none of these shock alone is not able to replicate the relative volatilities and cyclicalities observed in the data. The demand-driven shocks, monetary and foreign income shocks, are capable of generating a counter-cyclical unemployment rate and procyclical employment rate. This finding is similar to the closed economy model of Galí (2011b). Main difference is the responses of real wage and CPI inflation to the shocks. The real wage is highly countercyclical to the technology and monetary shocks, which is in the sharp contrast with Galí (2011b). The main reason is that the CPI is more flexible than nominal wages and domestic prices. Thus, the countercyclical movement of CPI overwhelms the cyclical movement of the nominal wage rate and leads the real wage rate to move countercyclically. The flexility of CPI comes from the movement of nominal exchange rate. Thus monetary shocks generate a procyclical CPI inflation through domestic currency depreciation. The foreign income shocks, however, appreciate domestic currency such that CPI inflation moves countercyclically.

The analysis in this section suggests that demand shocks, monetary and foreign income shocks, are able to generate realistic cyclical fluctuations of labor market, both a countercyclical unemployment and a procyclical employment, but shocks from real sides (technological and labor supply shocks) fail to generating related correlations observed in the data. None of shocks considered cannot provide reasonable relative volatilities and correlations of real world labor market.

5 Monetary policy design in a small open economy with unemployment

This section explores the implication of the existence of unemployment in a small open economy, as modeled in section 2, for the conduct of monetary policy. Let us first consider the efficient allocation, i.e., the equilibrium allocation in the absence of any form of rigidities. In order to keep the analysis as simple as possible, we restrict ourselves to the special case of $\sigma = \eta = 1$.

The efficient allocation corresponds to the solution of a sequence of static social planner's problem of the form

$$\max U\left(C_t, \{N_t(i)\}; \chi_t\right) ,$$

subject to (i) the technological constraint $Y_t(z) = A_t N_t(z)^{1-\alpha}$, (ii) the index of labor input (18) used by firm i, (iii) the market clearing condition (25), and (iv) a consumption/output possibilities set (the international risk-sharing condition, $C_t = \mathcal{Q}_t C_t^*$). The efficient allocation must satisfy

$$W_t(i) = W_t, \forall i,$$

$$N_t(z,i) = N_t, \forall i, z$$

$$-\frac{U_N(C_t, N_t)}{U_C(C_t, N_t)} = MPN_t,$$

where $MPN_t = (1-\alpha)(1-\delta)\frac{C_t}{N_t}$. Then, the efficient allocation can be characterized by $C_t N_t^{\varphi} = (1-\alpha)(1-\delta)C_t/N_t$.

Notice that the flexible price and wage equilibrium satisfies

$$-\frac{U_N(C_t, N_t)}{U_C(C_t, N_t)} = MPN_t \frac{C_t}{Y_t} \frac{1}{(1-\tau)\mathcal{M}^w \mathcal{M}^p}.$$

Then, at the flexible prices equilibrium

$$C_t N_t^{\varphi} = (1-\alpha)\frac{C_t}{N_t}\frac{1}{(1-\tau)\mathcal{M}^w \mathcal{M}^p}.$$

Therefore, by setting $(1-\tau) = \frac{1}{(1-\delta)\mathcal{M}^w \mathcal{M}^p}$ the condition for the efficient allocation is also satisfied, thus guaranteeing the efficiency of the flexible price equilibrium allocation. We assume that the efficiency of the flexible price equilibrium allocation holds through this study.

5.1 Optimal monetary policy

In order to determine the optimal policy in this context, we start by deriving a second order approximation to the representative household's utility losses due to the presence of domestic price and wage rigidities. As derived in the Appendix, the second-order approximation to the average welfare losses around the zero-inflation steady state, under the assumption of $\sigma = \eta = 1$, are given by

$$\mathbf{W} = \frac{1-\delta}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \left(\frac{1+\varphi}{1-\alpha}\right) \tilde{y}_t^2 + \frac{\epsilon_p}{\lambda_{p_H}} (\pi_{H,t})^2 + \frac{\epsilon_w(1-\alpha)}{\lambda_w} (\pi_{W,t})^2 \right\} + t.i.p.\,, \tag{45}$$

where $t.i.p.$ collects various terms that are independent of policy. Thus, the average period welfare loss is

$$\mathbf{L} = \frac{1-\delta}{2} \left[\left(\frac{1+\varphi}{1-\alpha}\right) var(\tilde{y}_t) + \frac{\epsilon_p}{\lambda_{p_H}} var(\pi_{H,t}) + \frac{\epsilon_w(1-\alpha)}{\lambda_w} var(\pi_{W,t}) \right]. \tag{46}$$

Note that the relative weight of each of the variances is a function of the underlying parameter values. The period welfare loss (46) is similar to that derived in Erceg et al. (2000) except for the presence of degree of openness (δ) and its dependence on domestic price inflation.

We are now ready to characterize optimal policy for our small open economy with sticky wages and domestic prices. The period welfare loss (46) implies that optimal policy should strike a balance in stabilizing domestic price inflation, wage inflation, and the output gap. Hence, the central bank will seek to minimize (45) subject to the sequence of equilibrium constraints given by (38), (40), and (42). The first-order conditions are:

$$(1-\delta)\left(\frac{1+\varphi}{1-\alpha}\right)\tilde{y}_t + \varsigma_{1t}\,\kappa_{p_H} + \varsigma_{2t}\,\kappa_w = 0, \tag{47}$$

$$(1-\delta)\frac{\epsilon_p}{\lambda_{p_H}}\pi_{H,t} - \varsigma_{1t} + \varsigma_{1t-1} - \varsigma_{3t} = 0, \tag{48}$$

$$(1-\delta)\frac{\epsilon_W(1-\alpha)}{\lambda_w}\pi_{W,t} - \varsigma_{2t} + \varsigma_{2t-1} + \varsigma_{3t} = 0, \tag{49}$$

$$-\varsigma_{3t} + \lambda_{p_H}\pi_{W,t} + \beta E_t\left\{\varsigma_{3t+1}\right\} = 0, \tag{50}$$

where ς_{1t}, ς_{2t}, and ς_{3t} are the Lagrange multipliers associated with the three period t constraints. The dynamical system describing the optimal monetary policy is thus composed of (47)-(50) together with constraints (38)-(42).

5.2 Evaluation of monetary policy rules

This section considers a number of simple monetary policy rules and presents some quantitative evaluation based on a calibrated version of the small open economy. The evaluation is based on the unconditional variances of major variables.

In this section, three different simple rules are studied. The first rule, which is referred to as a domestic inflation-based Taylor rule, requires that the domestic interest rate responds systematically to domestic inflation, whereas the second assumes that the domestic interest rate respond to CPI inflation. That rule is referred to as a CPI inflation-based Taylor rule. Other simple rule considered is to stabilize wage inflation. Formally, the domestic inflation-based Taylor rule (DIT, for short) is assumed to take this form:

$$r_t = \rho + \phi_\pi \pi_{H,t} + \phi_y \tilde{y}_t \tag{51}$$

The CPI inflation-based Taylor rule (CPIT, for short) is assumed to take the form

$$r_t = \rho + \phi_\pi \pi_{C,t} + \phi_y \tilde{y}_t \tag{52}$$

Finally, the wage inflation-based Taylor rule (WIT) is given by

$$r_t = \rho + \phi_\pi \pi_{W,t} + \phi_y \tilde{y}_t. \tag{53}$$

5.2.1 Impulse Response

Figure 2 displays the dynamic responses of main macro variables considered in the previous section to an exogenous domestic productivity shock under different policy regimes given the baseline calibration. For the sake of comparison, we also display the responses under the optimal rule.

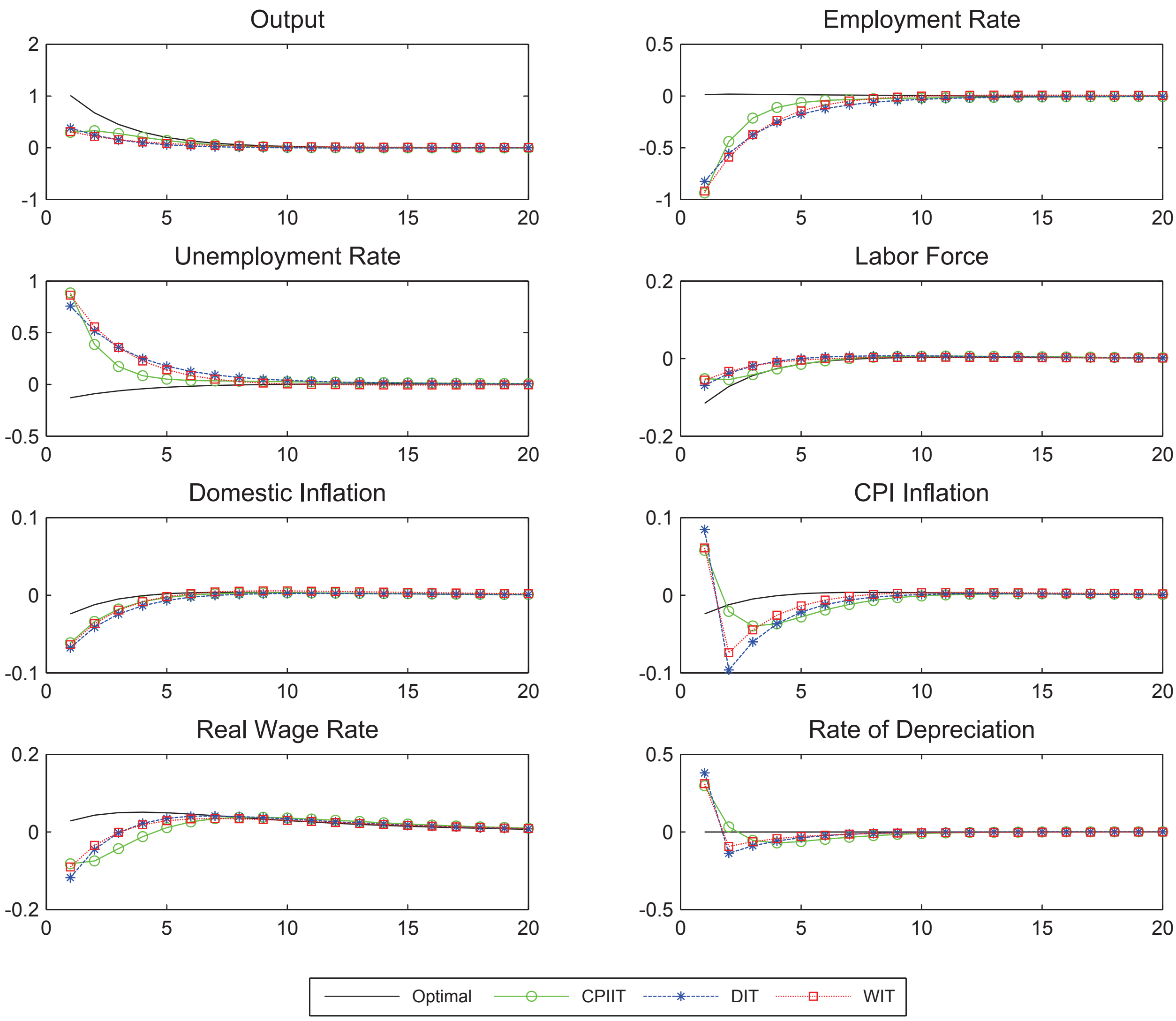

Figure 2: Impulse responses to a technological shock: Alternative Policy Rules

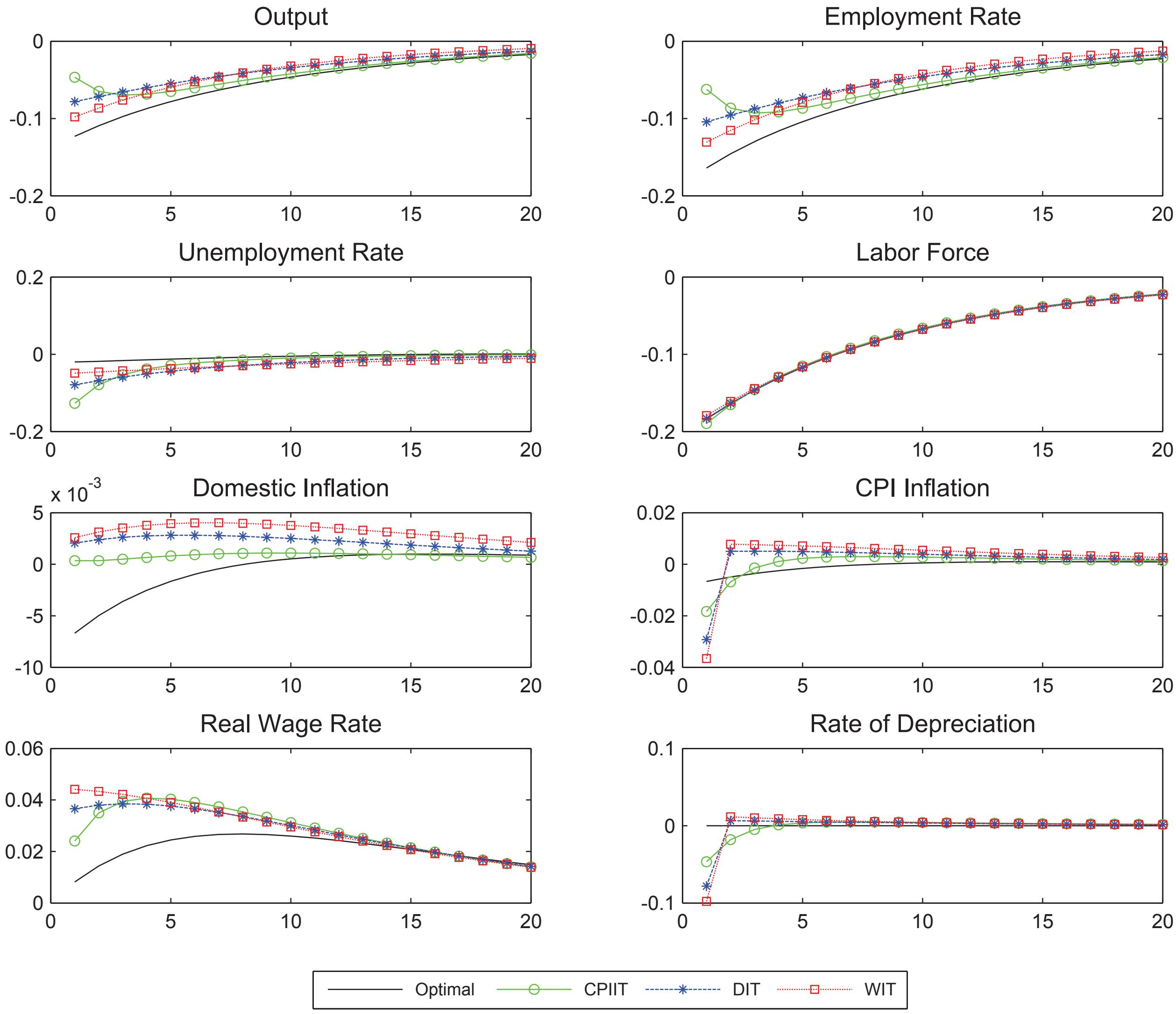

Figure 3: Impulse responses to labor supply shock: Alternative Policy Rules

We start by describing impulse responses under the optimal policy. Not surprisingly, we see that most variables remain stable to the shock under the optimal policy. However, output is more increasing, which implies that optimal policy is more accommodative of technological shock than any other alternative policies. The accommodative policy reaction, then, leads to largely unchanged unemployment rate. The response of CPI inflation is very limited by means of a more muted response of the rate of depreciation. Due to the presence of nominal wage rigidity and the muted response of CPI inflation, adjustment of real wage is considerably small. The stable movement real wage under optimal policy is able to help stabilizing employment.

Same figure displays the corresponding impulse responses under alternative policy rules. The main finding is that the CPI inflation-based Taylor rule is more accommodative of the productivity shock than any other policy rules, with output increasing more and employment remaining relatively stable. Therefore, the responses of the key variables are relatively more muted under the CPI inflation-based Taylor rule than other alternative policies. This is mainly due to the stabilization of the exchange rate and CPI inflation. The stable CPI inflation generated more muted response of real wage, which leads to relatively small change in unemployment rate and employment.

Figure 3 shows the response of the same variables to a labor supply shock under the optimal policy and the alternative policy rules. Notice that response of labor force is almost identical under different policy rules. The optimal policy stabilizes unemployment rate almost perfectly by fully accommodating the labor supply shock. Hence, the response of employment is very close to the labor force. Other variables, especially real wage, show relatively muted responses under optimal policy except domestic inflation. The responses of the same variables are similar under three different policy regimes except that the CPI inflation-based Taylor rules generates more muted responses of CPI and domestic inflation.

5.2.2 Second moments and welfare losses

Another interesting way to compare different policy regimes quantitatively is to calculate the standard deviations of key macroeconomic variables and the loss incurred by the economy from the shocks. Table 2 reports the main findings of this exercise. The left panel of table 2 contrasts the statistical properties of the main variables implied by the optimal policy with those generated under the alternative rules conditional on technology shock. The number confirms that key macroeconomic variables are relatively less volatile under the optimal policy even though output is more volatile. Especially, it is seen that the unemployment rate is very stable under the optimal policy. We can also see that CPI inflation–based Taylor rules generates relatively less volatile real wage and unemployment rate among the alternative rules.

The right panel of table 2 shows the relevant second moments conditional on labor supply shock. Under the optimal policy, unemployment rate is less volatile ever though output and employment are more volatile than those of alternative policies.

Table 2: Statistical Properties of Alternative Policy Regimes

	Technology Shock				Labor Supply Shock			
	Optimal	CPIIT	DIT	WIT	Optimal	CPIIT	DIT	WIT
Real Wage	0.0496	0.0871	0.1225	0.0836	0.0213	0.0275	0.0438	0.0419
CPI Inf	0.0200	0.0576	0.1140	0.0772	0.0069	0.0138	0.0250	0.0311
Output	0.8246	0.2945	0.3527	0.2516	0.1266	0.0512	0.0815	0.0882
Labor Force	0.0941	0.0512	0.0644	0.0449	0.1909	0.1437	0.1879	0.1623
Unemp Rate	0.1094	0.6046	0.7112	0.6948	0.0223	0.0904	0.0796	0.0450
Employment	0.0204	0.6462	0.7728	0.7395	0.1687	0.0682	0.1087	0.1176
Rate of Dep	0.0200	0.2584	0.2904	0.2017	0.0069	0.0510	0.0841	0.0857
Domestic Inf	0.0200	0.0447	0.0624	0.0521	0.0069	0.0004	0.0029	0.0032
Wage Inf	0.0038	0.0168	0.0317	0.0237	0.0016	0.0041	0.0074	0.0068

note: Standard deviations expressed in percent

Table 3 reports the variance of output gap, domestic price inflation and wage inflation, and the welfare losses associated with the three alternative policy rules (DIT, CPIT and WIT). We display the effects of changing the inverse of the Frisch elasticity of labor supply (as implied by changes in φ). The top panel reports statistics corresponding to the benchmark calibration of the elasticity of labor supply, namely, $\varphi = 5$. Relative to that benchmark, second panel assumes a lower inverse of the Frisch elasticity of labor supply ($\varphi = 1$), while the third panel reports results for a higher inverse of the Frisch elasticity of labor supply ($\varphi = 10$). The main findings of this exercise are consistent with the quantitative evaluation of the standard deviation conducted in table 2, that CPI inflation-based Taylor rule generates relatively small welfare losses. Under all the calibrations considered, the ranking among alternative policy rules is not affected.

From the analysis, we can see that stabilizing unemployment rate is important to reduce the welfare loss incurred by both technology and labor supply shocks. The conventional simple interest rate rules, however, do not respond to unemployment rate. Therefore, introducing the unemployment rate as an another argument into the Taylor-rule type interest rate rule will be welfare-enhancing.

Table 3: Contribution to welfare losses

	Technology Shock				Labor Supply Shock			
	Optimal	CPIT	DIT	WIT	Optimal	CPIT	DIT	WIT
				$\varphi = 5$				
$Var(\tilde{y})$	0.0006	0.0036	0.0045	0.0041	0.0028	2.0E-5	0.0005	0.0008
$Var(\pi_H)$	2.2E-5	0.0005	0.0011	0.0008	0.0001	0.0018	0.0033	0.0036
$Var(\pi_W)$	0.0003	0.4310	0.3870	0.4750	0.0012	0.3330	0.1350	0.0503
Loss	0.0857	1.6900	1.9400	2.0000	0.4200	1.5200	2.0300	2.3200
				$\varphi = 1$				
$Var(\tilde{y})$	0.0024	0.4360	0.3890	0.4980	0.0089	0.2440	0.1260	0.0420
$Var(\pi_H)$	0.0004	0.0034	0.0043	0.0039	0.0015	0.0001	0.0007	0.0012
$Var(\pi_W)$	0.0002	0.0002	0.0006	0.0004	0.0012	0.0024	0.0046	0.0052
Loss	0.1510	1.5600	1.7300	1.8300	0.7020	1.5200	2.1700	2.2700
				$\varphi = 10$				
$Var(\tilde{y})$	9.6E-5	0.4070	0.3840	0.4710	0.0004	0.2710	0.1380	0.0568
$Var(\pi_H)$	0.0006	0.0036	0.0046	0.0042	0.0029	4.0E-5	0.0005	0.0011
$Var(\pi_W)$	6.3E-6	0.0006	0.0012	0.0009	4.5E-5	0.0014	0.0031	0.0035
Loss	0.0862	1.6500	1.9800	2.0400	0.3920	1.2200	1.6000	1.6300

note: Entries are percentage units of natural output

5.3 Optimal simple rules

Despite the previous section studies alternative simple interest rate rules in responses to shocks, in practice most to the central banks implement simple feedback interest rate rules. For this reason, we study the optimal operational interest rate rules. Such a rule is obtained by searching, within the class of Taylor-type rules, for the parameters that minimize the unconditional period utility given by

$$(1-\delta)\left[\left(\frac{1+\varphi}{1-\alpha}\right)var(\tilde{y}_t)+\frac{\epsilon_p}{\lambda_{p_H}}var(\pi_{H,t})+\frac{\epsilon_w(1-\alpha)}{\lambda_w}var(\pi_{W,t})\right].$$

We consider the following specification of the interest rate rules:

$$r_t=\rho+\phi_y y_t+\phi_{\pi_H}\pi_{H,t}+\phi_{\pi_C}\pi_{C,t}+\phi_{\pi_W}\pi_{W,t}+\phi_u u_t,$$

where we added the unemployment rate as argument relative to the alternative policy rules considered in the previous section.

Table 4: Optimal Simple Rules

	Technology Shock						Labor Supply Shock					
	ϕ_y	ϕ_{π_H}	ϕ_{π_C}	ϕ_{π_W}	ϕ_u	Loss	ϕ_y	ϕ_{π_H}	ϕ_{π_C}	ϕ_{π_W}	ϕ_u	Loss
(a)	-0.04	1.47				6.75	-0.007	1.54				5.82
(b)	-0.62	1.14			-1.84	0.40	-0.093	1.11			-0.17	0.08
(c)	-0.07		1.95			2.60	-0.027		1.11			4.82
(d)	-0.37		1.14		-0.66	0.65	-0.050		1.10		-0.12	0.26
(e)	-0.03			1.01		5.26	-0.031			1.90		25.8
(f)	-0.47			1.12	-0.91	0.45	-0.011			1.12	-0.25	0.14

Table 4 reports the optimized coefficients and the corresponding welfare loss of the simple interest rate rules specified above. Row (a) shows the optimized coefficients and the resulting welfare loss for a specification corresponding to the domestic inflation-based Taylor rule, while row (b) shows corresponding results if the interest rate is allowed to respond to the unemployment rate. Row (c) and (d) shows the same results for the cases of CPI inflation-based Taylor rules (with or without unemployment rate as an arguments in the interest rate rules). The same results for the wage inflation-based Taylor rules are shown in row (e) and (f), respectively.

Notice that for all cases, the inflation coefficients are positive and above one, whereas the output coefficients are negative and small. This results are consistent with the findings of Galí (2011b). The negativity of output coefficient is opposed to the conventional Taylor rule. When interest rate responds to unemployment rate, the output coefficients increase significantly in absolute values, while the inflation coefficients change slightly. The unemployment coefficients are negative and relatively larger than output coefficients in absolute value. The welfare losses are reduce significantly once the interest rate is allowed to respond to the unemployment rate. This result points to the desirability of unemployment stabilization in monetary policy, which is in line with the findings of Blanchard and Galí (2010) and Faia (2009).

The optimized simple rule for the specification of CPI inflation-based Taylor generates relatively small welfare lose when unemployment is not allowed in the policy. When unemployment rate is augmented, the optimized CPI inflation-based Taylor rule is not the best welfare loss-minimizing rule. The merit of CPI inflation-based Taylor rule is that it reduces unemployment fluctuation by stabilizing real wage. Once unemployment rate is controlled, stabilizing power of CPI inflation-based Taylor rule is diminished.

6 Conclusion

In this paper, we extend Galí's (2011a,b) New Keynesian model with unemployment to a small open economy. Within this framework, we study the optimal monetary policy rule and compare the performances of alternative policy rules. We also compute optimized simple rules within a class of the conventional Taylor rule. The main findings regarding the issue of monetary policy design can be summarized as follows. First, the optimal policy is to seek to minimize variance of domestic price inflation, wage inflation, and the output gap when both domestic price and wage are sticky. Second, a policy that responds to an unemployment rate is welfare enhancing. Last, controlling CPI inflation induces relatively small welfare losses.

Our study, however, has some obvious limitations that may indicate possible directions for future work. First, as pointed out by Galí (2011b), the only source of unemployment is the positive wage markup from noncompetitive labor market. However, as shown in the text, the wage markup is easily fixed by simple fiscal policy (employment subsidy). Therefore, introducing certain forms of real frictions into the labor market would improve the model's performance.

The cyclical movements of CPI inflation and real wage rate implied in our model are not consistent with patterns observed in the data. This is manily due to the assumption of complete exchange rate pass-through of nominal exchange rate to prices of imported goods. The phenomenon of imperfect pass-through is well known and documented. Therefore, it may be possible to fix this anomaly by incorporating the assumption of imperfect pass-through.

There is a growing number of papers that incorporate a traded-goods and a non-traded-goods sector into the context of a New Keynesian, small open economy model to study different issues (e.g. Kam, 2007: Kuralbayeva, 2011). Therefore, it would be interesting to study how the presence of non-traded goods into domestic goods market affect the major findings in the paper.

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Appendix

In this appendix we derive a second-order approximation to the utility of the representative household around an efficient steady state. As has been discussed in the main text, we restrict our study to the special case of $\sigma = \eta = 1$. Frequent use is made of the following fact:

$$\frac{X_t - X}{X} = x_t + \frac{1}{2}x_t^2 ,$$

where x_t is the log deviation from steady state for the variable X_t. The second-order Taylor approximation of the household $i's$ period t utility, $U_t(i)$, around a steady state and intergrating across households yields

$$\int_0^1 (U_t(i) - U)di \;\simeq\; U_C C\left[c_t + \left(\frac{1}{2} + \frac{C}{2}\frac{U_{CC}}{U_C}\right)c_t^2\right] + U_N N\left[n_t + \left(\frac{1}{2} + \frac{N}{2}\frac{U_{NN}}{U_N}\right)N_t^2\right] + t.i.p. ,$$

where $t.i.p.$ stands for terms independent of policy.

Using the fact $\frac{C}{2}\frac{U_{CC}}{U_C} = -\frac{1}{2}$ and $\frac{1}{2} + \frac{N}{2}\frac{U_{NN}}{U_N} = \frac{1+\varphi}{2}$ and the market clearing condition $c_t = (1-\delta)y_t + \delta y_t^*$, we have

$$\int_0^1 (U_t(i) - U)di \;\simeq\; U_C C(1-\delta)y_t + U_N N\left[\int_0^1 n_t(i)di + \frac{1+\varphi}{2}\int_0^1 n_t^2(i)di\right] + t.i.p. ,$$

Define aggregate employment as $N_t = \int_0^1 N_t(i)di$, or, in terms of log deviations from the steady state and up to a second-order approximation,

$$n_t + \frac{1}{2}n_t^2 \simeq \int_0^1 \tilde{n}_t(i)di + \frac{1}{2}\int_0^1 \tilde{n}_t(i)^2 di .$$

Note also that

$$\begin{aligned}\int_0^1 n_t(i)^2 di &= \int_0^1 (n_t(i) - n_t + n_t)^2 \, di \\ &= \tilde{n}_t^2 - 2n_t\epsilon_w \int_0^1 (w_t(i) - w_t)\, di + \epsilon_w^2 \int_0^1 (w_t(i) - w_t)^2 \, di \\ &= n_t^2 + \epsilon_w^2 var_i \{w_t(i)\} ,\end{aligned}$$

where we have used the labor demand function $n_t(i) - n_t = -\epsilon_w (w_t(i) - w_t)$, and the fact that $\int_0^1 (w_t(i) - w_t)\, di = 0$ and that $\int_0^1 (w_t(i) - w_t)^2 \, di = var_i \{w_t(i)\}$ is of second order.

The next step is to derive a relationship between aggregate employment and output:

$$\begin{aligned}N_t &= \int_0^1 \int_0^1 N_t(z,i)didz = \int_0^1 N_t(z)\int_0^1 \frac{N_t(z,i)}{N_t(z)}didz \\ &= \Delta_{w,t}\int_0^1 N_t(z)dz = \Delta_{w,t}\left(\frac{Y_t}{A_t}\right)^{\frac{1}{1-\alpha}}\int_0^1 \left(\frac{Y_t(z)}{Y_t}\right)^{\frac{1}{1-\alpha}} dz \\ &= \Delta_{w,t}\Delta_{p_H,t}\int_0^1 \left(\frac{Y_t(z)}{Y_t}\right)^{\frac{1}{1-\alpha}} dz ,\end{aligned}$$

where $\Delta_{w,t} = \int_0^1 \left(\frac{w_t(i)}{w_t}\right)^{-\epsilon_w} di$ and $\Delta_{p_H,t} = \int_0^1 \left(\frac{p_{H,t}(z)}{P_{H,t}}\right)^{-\epsilon_p} dz$.

Thus, the following second-order approximation of the relation between (log) aggregate output and (log) aggregate employment holds:

$$n_t = \frac{1}{1-\alpha}(\tilde{y}_t - a_t) + d_{w,t} + d_{p_H,t},$$

where $d_{w,t} = \log \int_0^1 \left(\frac{w_t(i)}{w_t}\right)^{-\epsilon_w} di$ and $d_{p_H,t} = \log \int_0^1 \left(\frac{p_{H,t}(z)}{P_{H,t}}\right)^{-\epsilon_p} dz$.

Lemma 1. $\quad d_{p_H,t} = \frac{\epsilon_p(1-\alpha+\alpha\epsilon_p)}{2(1-\alpha)^2} var_z \{p_{H,t}(z)\}$.

Proof. See Galí and Monacelli (2005). □

Lemma 2. $\quad d_{w,t} = \frac{\epsilon_w}{2} var_i \{w_t(i)\}$.

Proof. See Erceg et al. (2000). □

Now, one-period aggregate welfare can be written as

$$\int_0^1 \frac{U_t(i) - U}{U_c C} di = \\ -\frac{1-\delta}{2}\left[\left(\frac{1+\varphi}{1-\alpha}\right)\tilde{y}_t^2 + \frac{\epsilon_p(1-\alpha+\alpha\epsilon_p)}{(1-\alpha)} var_z\{p_{H,t}(z)\} + \epsilon_w(1-\alpha)\left[1+\varphi\epsilon_w\right] var_i\{w_t(i)\}\right] + t.i.p. \, ,$$

where $t.i.p.$ stands for terms independent of policy.

Lemma 3.

$$\sum_{t=0}^{\infty} \beta^t var_z \{p_{H,t}(z)\} = \frac{\theta_{p_H}}{(1-\beta\theta_{p_H})(1-\theta_{p_H})} \sum_{t=0}^{\infty} \beta^t \pi_{H,t}^2 \, ,$$
$$\sum_{t=0}^{\infty} \beta^t var_i \{w_t(i)\} = \frac{\theta_w}{(1-\beta\theta_w)(1-\theta_w)} \sum_{t=0}^{\infty} \beta^t \pi_{w,t}^2 \, .$$

Proof. See Woodford (2003, Chapter 6). □

Collecting the previous results, we can write the second-order approximation to the small open economy's aggregate welfare function as follows:

$$\mathbf{W} = -\frac{1-\delta}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \left(\frac{1+\varphi}{1-\alpha}\right)\tilde{y}_t^2 + \frac{\epsilon_p}{\lambda_{p_H}} (\pi_{H,t})^2 + \frac{\epsilon_w(1-\alpha)}{\lambda_w} (\pi_{w,t}^R)^2 \right\} + t.i.p. \, ,$$

where $\lambda_{p_H} = \frac{(1-\theta_{p_H})(1-\beta\theta_{p_H})}{\theta_{p_H}(1-\alpha+\alpha\epsilon_p)}(1-\alpha)$ and $\lambda_w = \frac{(1-\theta_w)(1-\beta\theta_w)}{\theta_w(1+\epsilon_w\varphi)}$.