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Yuntong Wang (University of Windsor)

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Yuntong Wang*

Department of Economics
University of Windsor
Windsor, Ontario, Canada

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*Tel: (519) 253-3000 ext.2382; Fax: (519) 973-7096; E-mail: yuntong@uwindsor.ca

ABSTRACT: An airline lands in a number of airports in a region. An airport serves a number of airlines. Each airport charges a given amount of emission fees to those airlines using the airport. The total emission fees from all airports in the region must be shared among all airlines. We propose an axiomatic approach to this airline emission fees problem. We suggest a sharing rule called the *Decomposition rule* that is based on a few simple axioms. The Decomposition rule coincides with the Shapley value of the game associated with the problem and is shown in the core. Thus, no alliance of airlines can reduce their emission fees by forming an independent coalition. On the other hand, we also show that the Decomposition rule is split-proof. In other words, no airline has an incentive to split into two or more airlines.

JEL classification: C71, D61, D62.

Keywords: Airline emission fees; Shapley value; core; split-proofness.

1 Introduction

This paper considers the airline emission fees problem involving *a group of airlines* and *a group of airports*. We assume that each airline uses a number of airports and each airport serves a number of airlines. Each airport sets a given amount of emission fee and the total emission fees from all the airports must be shared among the airlines.

We propose a sharing rule for the problem. The rule is essentially based on the following Decomposition Principle (Young, 1994): *If a cost (fee) function decomposes into distinct cost elements, divide the cost of each element equally among those who use it. The charge to each user is his share of each cost element, summed over all elements.* For our emission fee problem, the Decomposition Principle implies that (1) Airlines which do not use an airport should not be charged for the fee associated with the airport, (2) All airlines which use a given airport should be charged equally, (3) The results of these allocations should be added together. We call the corresponding rule the *Decomposition rule*.

The Decomposition rule is characterized by the axioms of *Additivity*, *Anonymity*, *Zero Shares for Zero Fees*, and *Efficiency*. Additivity is a classical axiom (Shapley, 1953) in the cooperative game theory and has been a foundational axiom in the cost sharing literature (Moulin, 2002). Anonymity axiom says that airlines' names are irrelevant. Zero Shares for Zero Fees says that if an airport's fee is zero, it should be free to all airlines. Efficiency is simply the budget balance condition, i.e., the sum of all airlines' shares equals the total fees of all airports.

The Decomposition rule is intimately related to the *Shapley value* (Shapley, 1953) of a special game that is associated with the problem. We further show that the allocation given by the Decomposition rule is in the core of the game.

We emphasize the core property. It implies that there would be no coalition of airlines (usually called alliance) that can do better than that prescribed by the Decomposition rule and still cover the stand-alone fees (i.e., the total fees of the airports that the airlines in the coalition use). This property is important. Since it is legitimate for airlines to form alliances to provide better services, it would be equally legitimate for them to cooperate in sharing the emission fees. The Decomposition rule is immune to this coalitional manoeuvre.

Equally important is that we do not want an airline to break-up into two or more airlines either for the sake of emission fees sharing. We show that the Decomposition rule satisfies a weak version of the so-called *split-proof* property (Sprumont, 2005). That is, no airline has the incentive to break up into two or more airlines.

Finally, we point out that this paper is related to a number of papers in the cost sharing literature. Recently, there has been a renewed interest in the cost sharing problem on a fixed network. The problem can be traced back to the well-known *airport landing fee problem* (Littlechild and Owen, 1973). Wang (2013) recently extends the airport problem to a multi-airport model. On the other hand, Koster et al. (2001) have considered the core allocations for cost sharing problems on a tree network. Dong et al. (2012) have considered a polluted river network. Moulin and Laigret (2011) and Moulin and Hougaard (2012) have considered cost sharing problems on more general networks.

2 The Model

We use essentially the same model as in Wang (2011, 2013). For completeness, we repeat below. Let $M = \{1, ..., m\}$ be a set of airlines and $N = \{1, ..., n\}$ a set of airports, where m, n are two positive integers. Suppose that each airline uses a number of airports in the set N and each airport serves a number of airlines in the set M. Let $C = (c_1, ..., c_n)$ be the fees vector for the airports, where $c_j \geq 0$, $j \in N$ is the fee that airport j needs to allocate to the airlines.

For each airline $i \in M$, let $AP(i)$ be the set of airports that airline i uses. Denote AP such a mapping from M to 2^N. For each airport $j \in N$, let $AP^{-1}(j)$ be the set of airlines that use airport j. Denote AP^{-1} the inverse of AP. Assume that for each airport $j \in N$, $AP^{-1}(j) \neq \emptyset$. Denote $\mathcal{AP}$ the set of all possible mappings from M to 2^N. An airline emission fee problem is a list (M, N, AP, C) where $AP \in \mathcal{AP}$ and $C \in R^n_+$. A solution is a vector $x = (x_1, ..., x_m) \in R^m_+$ such that

$$\sum_{i \in M} x_i = \sum_{j \in N} c_j.$$

A rule is a mapping that assigns to each problem (M, N, AP, C) a solution

$x(M, N, AP, C)$. Through out the paper, we fix the sets M and N (except in Section 5). Thus, we simply call (AP, C) a problem and $x(AP, C)$ a solution.

In this paper, we propose and study the following rule which we call the *Decomposition rule*:

$$x_i(AP, C) = \sum_{j \in AP(i)} \frac{c_j}{|AP^{-1}(j)|}, i = 1, ..., m, \tag{1}$$

where $|AP^{-1}(j)|$ is the number of elements in the set $AP^{-1}(j)$.

Similar to Wang (2011, 2013), a characterization of the Decomposition rule (1) by the following axioms can be provided. The proof of the characterization (Theorem 1) can be found in the above mentioned paper.

Additivity: Fix an AP. For any $C^1 = (c_1^1, \ldots, c_n^1) \in R_+^n$ and $C^2 = (c_1^2, \ldots, c_n^2) \in R_+^n$, we have $x_i(AP, C^1 + C^2) = x_i(AP, C^1) + x_i(AP, C^2)$ for all $i \in M$.

Additivity is a classical axiom in the cooperative game theory (Shapley, 1953) and in the cost sharing literature (Moulin, 2002). In the context of airline emission fees problem, we can provide the following interpretation. If airports' fees are split into two parts, for example, the initial emission quotas and the future emission fees, and the allocation of each part of these fees is computed, then the sum of these two allocations would be equal to the allocation obtained by applying the rule to the unsplit total fees.

A permutation π of $M = \{1, ..., m\}$ is a one-to-one mapping from M to M, i.e., $\pi : M \to M$ and for all $i, j \in M$, $\pi(i) \neq \pi(j)$ if and only $i \neq j$.

Anonymity: For any permutation π of M and any $C \in R_+^n$,

$$\pi(x(AP, C)) = x(\pi(AP, C))$$

where $\pi(AP, C) = (\pi(AP), C)$, $\pi(AP)(i) = AP(\pi(i))$, and $\pi(x)_j = x_{\pi(j)}$ for $x \in R_+^n$.

In words, Anonymity requires that the fees allocated to the airlines do not depend on the airlines' names.

Zero Shares for Zero Fees: For any $i \in M$, if for all $j \in AP(i)$, $c_j = 0$, then $x_i(AP, C) = 0$.

In words, for any airline, if the fees of those airports it uses are all zeros, then the airline should pay zero fees as well. This rules out the *equal division* of the total fees.

Efficiency: $\sum_{i \in M} x_i(AP, C) = \sum_{j \in N} c_j$.

Efficiency is simply the budget balance condition, which holds by definition.

Theorem 1 *The Decomposition rule defined in (1) is the only rule that satisfies the axioms of Additivity, Anonymity, Zero Shares for Zero Fees, and Efficiency.*

Moreover, the Decomposition rule (1) coincides with the Shapley value (Shapley, 1953) of the following game $C(\cdot)$ that is associated with the problem (AP, C):

$$C(S) = \sum_{j \in AP(S)} c_j, \quad S \subseteq M, \tag{2}$$

where $AP(S) = \cup_{i \in S} AP(i)$, i.e., the set of airports that the airlines in S use.

The Shapley value of a game $C(\cdot)$ is defined by

$$\phi_i(C) = \sum_{S \subseteq M : i \in S} \frac{(|S|-1)!(m-|S|)!}{m!} [C(S) - C(S \setminus \{i\})], i = 1, ..., m. \tag{3}$$

Proposition 1 *The Shapley value of the game $C(\cdot)$ defined in (2) coincides with the Decomposition rule (1).*

The proof of the proposition can also be found in Wang (2011, 2013).

3 The Core Property and Split-Proofness

In this section, we first show that the game (2) generated by an emission fee problem (AP, C) is convex and, thus, the Shapley value of the game is a core allocation. Next, we show that the Decomposition rule is also split-proof (to be defined below).

Proposition 2 *The game (2) is convex and thus, the Shapley value (3) is in the core of the game.*

Proof: We first show that the game (2) is convex. That is, for all $i \in M$, all $S, T \subset M \backslash \{i\}$ and $S \subset T$, we have

$$C(S \cup \{i\}) - C(S) \geq C(T \cup \{i\}) - C(T) \tag{4}$$

Suppose that $S \subset T \subseteq M$ and $i \notin T$. Let $H_S = AP(S \cup \{i\}) \setminus AP(S)$ and $H_T = AP(T \cup \{i\}) \setminus AP(T)$. Now we show that $H_S \supseteq H_T$.

Since

$$\begin{aligned} H_S &= AP(S \cup \{i\}) \setminus AP(S) \\ &= AP(S) \cup AP(i) \setminus AP(S) \\ &= AP(i) \setminus AP(S), \end{aligned}$$

$$\begin{aligned} H_T &= AP(T \cup \{i\}) \setminus AP(T) \\ &= AP(T) \cup AP(i) \setminus AP(T) \\ &= AP(i) \setminus AP(T), \end{aligned}$$

and

$$AP(S) \subseteq AP(T),$$

we have

$$H_S \supseteq H_T.$$

Therefore,

$$\begin{aligned} C(S \cup \{i\}) - C(S) &= \sum_{i \in AP(S \cup \{i\})} c_i - \sum_{i \in AP(S)} c_i \\ &= \sum_{i \in H_S} c_i \\ &\geq \sum_{i \in H_T} c_i \\ &= \sum_{i \in AP(T \cup \{i\})} c_i - \sum_{i \in AP(T)} c_i \\ &= C(T \cup \{i\}) - C(T). \end{aligned}$$

This shows that the game $C(\cdot)$ is a convex game. It is well-known that the Shapley value of a convex game is in the core of the game (Shapley, 1971). This completes the proof. Q.E.D.

An airline is split into two airlines if the two use the same set of airports. Formally, airline i is split into airlines i' and i'' if $AP(i') = AP(i'') = AP(i)$ and $M' = \{1, ..., i-1, i', i'', i+1, ..., m\}$. A rule is *split-proof* if it is not advantageous for an airline to split into two. Specifically, for any (M, N, AP, C), any $i \in M$ and M'. Define AP' as follows:

$$AP'(k) = AP(k), \forall k \in M \setminus \{i\};\ AP'(k) = AP(i), \text{if } k = i' \text{ or } i''.$$

We say x is split-proof if

$$x_{i'}(AP', C) + x_{i''}(AP', C) \geq x_i(AP, C). \tag{5}$$

Now we have the following result.

Proposition 3 *The Decomposition rule (1) is split-proof.*

Proof:

$$\begin{aligned}
x_{i'(}AP', C) + x_{i''}(AP', C) &= \sum_{l \in AP'(i')} \frac{c_l}{|(AP')^{-1}(l)|} + \sum_{l \in AP'(i'')} \frac{c_l}{|(AP')^{-1}(l)|} \\
&= \sum_{l \in AP'(i')} \frac{c_l}{|AP^{-1}(l)| + 1} + \sum_{l \in AP'(i'')} \frac{c_l}{|AP^{-1}(l)| + 1} \\
&\geq \sum_{l \in AP(i)} \frac{c_l}{|AP^{-1}(l)|} \\
&= x_i(AP, C).
\end{aligned}$$

This proves the proposition. Q.E.D.

4 Concluding Remarks

In this paper, we consider the airline emission fees problem. We model the problem as a cost sharing problem with multiple public goods (airports) and each agent (airline) demanding only a subset of the goods. We apply the results in the cost sharing literature and propose the Decomposition rule for the emission fees sharing problem. We also show that the Decomposition rule coincides with the Shapley value of the game associated with the problem and it has the core and split-proofness properties. The latter says that there is no advantage to split an airline into two or more airlines.

We point out that the emission fees problem can be combined with the *airport landing fees problem*. In particular, assume that each airport's cost contains two components, one is the emission fee and the other the landing fee. By Additivity, there is no difference either sharing these two fees separately or together. In practice, however, it might be convenient to combine the emission reduction fees with the landing fees.

The traditional economic approach to the air emission problem is often based on the airlines' marginal social costs. But this approach is often not practical because it is difficult to estimate these marginal social costs. An alternative approach is to create an emission trading market. This approach also has its drawbacks. For instance, how to allocate the initial emission quotas? The axiomatic approach proposed in this paper avoids some of these issues. More importantly, the normative (axiomatic) approach might be more appealing since the airline emission problem often involves international cooperation rather than competition, where equity or fairness is perhaps more important than economic efficiency.

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