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Taxing Hard-to-Tax Markets

Marcelo Arbex (University of Windsor)

Enlison Mattos (Sao Paulo School of Economics)

Laudo M. Ogura (Grand Valley State University)

Working paper 13 - 05

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Taxing Hard-to-Tax Markets

Marcelo Arbex* Enlinson Mattos† Laudo M. Ogura‡

July 27, 2013

Abstract

Tax enforcement costs constrain the government's ability to observe economic transactions, giving rise to hard-to-tax (HTT) markets. In these markets transactions are untaxed and consumers are better off than in taxed markets. This paper studies a novel approach to combat evasion in HTT markets: *consumer auditing*, which rewards consumers for requesting transaction receipts. We develop a Hotelling-type spatial model of sales taxation to analyze the welfare and distributional effects of the implementation of this policy. We find that *consumer auditing* allows for a lower tax rate and greater provision of the public good in the economy. We show that this policy not only can enhance welfare, but also equalize utilities of consumers across markets.

Keywords: taxation; hard-to-tax; tax evasion.

JEL Classification: H1, H21, H26.

1 Introduction

One of the main challenges in fighting tax evasion is the government's inability to detect non-compliance. To avoid taxation, individuals and businesses purposely misreport or conceal transactions. Moreover, the costs borne by the tax agency to enforce compliance imposes a limit on its reach, making it hard to tax all markets.

In this paper we develop a Hotelling-type spatial economy model of sales taxation where the cost of enforcement increases with the distance from the tax authority. In each market, there

**Corresponding autor:* Department of Economics, University of Windsor. Windsor, Canada. arbex@uwindsor.ca.; †São Paulo School of Economics, Getulio Vargas Foundation. São Paulo, Brazil. enlinson.mattos@fgv.br.; ‡Economics Department, GVSU. Grand Rapids, MI, USA. ogural@gvsu.edu. We are grateful to Klenio Barbosa, Daniel Monte and Christian Trudeau for helpful discussions. All errors are our own.

is a continuum of identical buyers and sellers, who cannot relocate. Based on the expected tax revenue from each location and the cost to audit it, the tax authority determines a maximum reach, i.e., a cut-off location of its enforcement policy. Markets within the government’s reach are audited directly (*government auditing*) and sellers pay taxes accordingly. Markets beyond the cut-off location are denoted as “hard-to-tax” (HTT) markets.¹

In HTT markets, the government does not directly enforce tax compliance and transactions would go unrecorded, rendering zero tax revenue. To overcome this problem, the government can offer a tax rebate to consumers as a reward for requesting sales receipts, a policy we hereafter call *consumer auditing*. This policy creates paper trail of transactions and triggers tax remittances from sellers to the tax authority. Consumers participation in the auditing process is, however, limited by private costs, i.e., monetary, leisure or psychological costs. The net tax revenue collected is used to provide a pure public good.

The optimal tax-enforcement policy balances the marginal social costs of taxation, including distortions created by tax enforcement, with the marginal benefits of providing the public good. We show that the size of the HTT economy increases with the government enforcement cost, but decreases with the private cost of auditing. A relatively higher cost of *consumer auditing* induces an optimal policy where a larger fraction of the economy is directly audited by the government and consumers participating in the auditing process are compensated with a higher tax rebate rate.

We also evaluate welfare and distributional effects of the introduction of *consumer auditing* in HTT markets. The government’s inability to audit transactions in HTT markets creates heterogeneity in an otherwise homogenous population. Consumers are ex-ante identical (same initial endowments), but are ex-post heterogenous as buyers in HTT markets are not burdened by sales tax and pay a lower price. As Alm, Martinez-Vazquez and Schneider (2004) point out, the beneficiaries of a HTT market are those who directly participate in the HTT activities. The implementation of the *consumer auditing* policy corrects this distortion to the extent that it reduces the tax rate and increases the provision of the public good in the whole economy,

¹Although there are well-known hard-to-tax markets, e.g., hotels and restaurants, household services and construction, there is no precise definition of HTT (Tanzi and Casanegra, 1989; Musgrave, 1990; Das-Gupta, 1994; Terkper, 2003; Alm, Martinez-Vazquez and Schneider, 2004).

equalizing utility levels across markets as all consumers are now subject to taxation.

Numerical results show that the economy's welfare depends on whether consumers participate in the auditing process and on the relative cost of *consumer auditing*. If the private auditing cost is relatively small, the *consumer auditing* policy is welfare enhancing. Although the economy as a whole is better off, there are winners and losers in this process. On one hand, buyers in markets audited directly by the government enjoy lower prices and thus more private and public consumption. On the other hand, consumers in the original HTT markets become worse off as prices increase with transactions being now taxed. In other words, consumers in markets farther away from the tax authority become worse off, but other consumers are better off, so that total welfare in the economy increases. Heterogeneity in the distribution of endowments over markets does not affect the main results. As in the homogenous case, the *consumer auditing* policy equalizes utilities of consumers of the same type across markets.

HTT markets are related to the existence of underground economies and tax evasion (Alm, Martinez-Vazquez and Schneider, 2004). In this sense, our paper builds on a large literature on tax evasion focused on interactions between buyers and sellers - e.g., Boadway, Marceau and Mongrain (2002), Chang and Lai (2004), Gordon and Li (2009), Marchese (2009), Huang and Ueng (2012) and Arbex and Mattos (2013).² Our main contribution resides in showing that while the existence of HTT markets is endogenously determined by the government's ineffective enforcement technology, a *consumer auditing* policy can reduce not only tax evasion but also distortions in the economy.

The paper proceeds as follows. Section 2 presents a model of HTT markets and *consumer auditing*. In Section 4 we illustrate our results numerically and introduce heterogeneous consumers. Section 5 concludes.

2 A Model of Hard-to-Tax Markets

The underlying framework builds on a spatial setting in the spirit of Hotelling's (1929) linear economy model. The economy is a continuous line of unity length, with markets being uniformly

²Practical policies adopted to reduce tax evasion such as tax reductions, prizes and monetary transfers are found in Brazil, China, Denmark, France, Italy, Puerto Rico and Sweden (see Eurofound, 2013).

distributed over the line and indexed by $x \in (0, 1]$. The tax authority is located at $x = 0$. In each market, there is a continuum of consumers, indexed by i , and a continuum of firms, indexed by j , where $i, j \in [0, 1]$. Firms produce a standard good with constant marginal cost and zero fixed cost. Markets are competitive. Consumers and firms cannot relocate and the price of the good in market x is p_x .

Sales are subject to a proportional tax τ . Firms should remit tax payments to the government, but they attempt to evade. For simplicity, concealment cost is zero. Thus, a firm remit tax payments only if audited, with a probability of detection $\theta \in [0, 1]$, which is exogenously determined by the technology of government audit. The tax authority incurs an access cost t per unit of distance to audit firms. That is, the tax authority incurs a cost tx to audit a transaction in market x . This cost imposes a limit on how far in the linear economy the government can directly audit transactions and thus collect taxes.³

In markets where it is not cost effective for the government to audit transactions, the tax authority has the option to delegate the tax auditing role to consumers. Taxes can still be collected if consumers request sales receipts, which create paper trail and force tax remittance from the sellers to the government. We denote this auditing mechanism *consumer auditing*. A consumer participation in the auditing process entails a disutility cost ϕ , which can be interpreted as a stigma cost (as in Moffitt, 1993) of requesting receipts or, depending on the technology employed, the waiting time for a receipt to be issued and time costs to send receipts to the tax authority. To reward consumer participation, the tax authority offers a fraction of the tax revenue collected, i.e., a tax rebate. Let $a \in \{0, 1\}$ denote whether consumers request sales receipts. When they do ($a = 1$), the tax authority collects tax revenue at the rate τ , but returns $\lambda\tau$ to consumers, where $\lambda \in [0, 1]$ is the tax rebate rate. Otherwise, if $a = 0$, tax revenue is zero.

To determine how far to go to collect taxes in this linear economy, the tax authority compares the net expected tax revenue of direct *government auditing* versus the net revenue from the alternative. Without *consumer auditing*, the government chooses to audit firms if and only if the

³The geographical notion of space helps to simplify the presentation of the model, in which case distance imposes a traveling cost for the government to audit transactions. However, distance in our model can be more generally interpreted as the difficulty to access transaction information in certain markets where it is easier for firms to conceal their activities.

expected tax revenue is at least equal to the cost of reaching those firms. When *consumer auditing* is considered, direct auditing is chosen if $(\theta\tau - tx) Q_x \geq (1 - \lambda)a\tau Q_x$, where $Q_x = \int_0^1 Q_{j,x} dj$ is the total amount of goods sold in market x and $Q_{j,x}$ is the amount produced by firm j in that market. Hence, there is a cut-off point \bar{x} such that only firms located at $x \in (0, \bar{x}]$ are audited directly by the tax authority:

$$\bar{x} = [\theta - (1 - \lambda)a] \frac{\tau}{t}. \quad (1)$$

In fact, \bar{x} divides the economy into two regions, region *I* and region *II* defined by $x \in (0, \bar{x}]$ and $x \in (\bar{x}, 1]$ respectively. The tax authority's inability to reach all markets is key in our setup and we denote markets in region *II* as hard-to-tax (HTT) markets. The effect of enforcement parameters on the cut-off point is straightforward: \bar{x} is increasing in the audit probability θ and decreasing in the auditing cost t . If consumers do not ask for receipts ($a = 0$), the HTT sector is larger. Figure 1 illustrates the linear economy, with the cut-off point \bar{x} separating the taxed and the HTT portions of the economy.

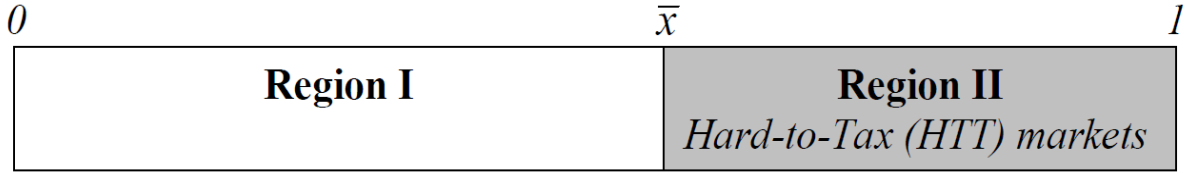


Figure 1: Hard-to-Tax Markets in a Linear Economy

Consumers and firms make optimal choices taking tax-enforcement policies and prices as given. Each consumer i at location x has a non-consumable endowment which is transformed into consumable goods by firms. Each firm produces an output $Q_{j,x}$ using a production technology with constant marginal return equal to one. Firm j located at x chooses how much to produce to maximize its expected profit $\Pi_{j,x}$, as follows:

$$\Pi_{j,x}^I = p^I(1 - \theta\tau)Q_{j,x} - Q_{j,x}, \quad x \in (0, \bar{x}], \quad (2)$$

$$\Pi_{j,x}^{II} = p^{II}(1 - a\tau)Q_{j,x} - Q_{j,x}, \quad x \in (\bar{x}, 1]. \quad (3)$$

where $\Pi_{j,x}^I$ and $\Pi_{j,x}^{II}$ are the expected profits of firm j , and p^I and p^{II} are the prices in regions *I* and *II*, respectively. Profit maximization and perfect competition imply that equilibrium prices

in regions I and II are respectively $p^I = 1/(1 - \theta\tau)$ and $p^{II} = 1/(1 - a\tau)$.

Each consumer derives utility from the consumption of a private good (c) and a public good (g). This public good is financed by revenues from sales tax and is uniformly provided across regions. Utility is assumed to be additively separable, with linear utility from the private good and from the cost of auditing transactions, that is, $u_{i,x} = c_{i,x} - a\phi + v(g)$, where $v(g)$ is an increasing and concave function. Note that the cost $a\phi$ only applies to consumers in region II when they participate in the *consumer auditing* process. Hence, each consumer maximizes $u_{i,x}^I = c_{i,x}^I + v(g)$ in region I and $u_{i,x}^{II} = c_{i,x}^{II} + v(g) - a\phi$ in region II , where $a \in \{0, 1\}$.

In region II , *consumer auditing* affects consumption of the private good in two opposite ways. The price with receipt, $p_{a=1}^{II} = (1 - \tau)^{-1}$, is higher than without a receipt, $p_{a=0}^{II} = 1$, reducing consumption if a buyers request sales receipts. On the other hand, the tax rebate represents an additional source of income allowing buyers to increase consumption. The equilibrium consumption of the private good in regions I and II are respectively

$$c_{i,x}^I = \frac{L_{i,x}}{p^I} = (1 - \theta\tau)L_{i,x}, \quad x \in (0, \bar{x}], \quad (4)$$

$$c_{i,x}^{II} = \frac{L_{i,x}}{p^{II}} (1 + \lambda a \tau p^{II}) = [1 - (1 - \lambda)a\tau]L_{i,x}, \quad x \in (\bar{x}, 1]. \quad (5)$$

3 Optimal Tax Policies

In this section we consider the case where all consumers and firms are identical ($L_{i,x} = L$, $Q_{j,x} = Q$), with uniform distribution of consumers over $x \in (0, 1]$. Since there is a measure one of consumers and firms in each market, the aggregate endowment, production and consumption in each market x are respectively $L_x = L_{i,x}$, $Q_x = Q_{j,x}$, $c_x^I = c_{i,x}^I$ and $c_x^{II} = c_{i,x}^{II}$. The production technology implies that $Q_x = L_x$ for all $x \in (0, 1]$. In the case of homogenous buyers, it is possible to derive analytical solutions for the optimal policies and allocations.

The total utility of consumers in region I is determined as follows:

$$U^I = \int_0^{\bar{x}} [c_x^I + v(g)] dx = \bar{x} [(1 - \theta\tau)L + v(g)], \quad (6)$$

where \bar{x} is given by equation (1). The total utility in region II depends on whether consumers participate in tax auditing, i.e., $a = 0$ or $a = 1$, and it is given by

$$U_a^{II} = \int_{\bar{x}}^1 [c_x^{II} + v(g) - a\phi] dx = (1 - \bar{x}) [(1 - (1 - \lambda)a\tau)L + v(g) - a\phi]. \quad (7)$$

The government's balanced budget constraint is written as

$$\int_0^1 g dx = R^I(\tau, \lambda) + R^{II}(\tau, \lambda) = \int_0^{\bar{x}} (\theta\tau - tx) L dx + \int_{\bar{x}}^1 a(1 - \lambda)\tau L dx, \quad (8)$$

where $R^I(\tau, \lambda)$ and $R^{II}(\tau, \lambda)$ are the net tax revenue functions for regions I and II , respectively.

The planner's problem is to choose tax policy instruments τ and λ to maximize the economy's welfare, i.e., the sum of consumers' utilities. The government provides the amount g of the public good that balances its budget. Hence, the planner maximizes $(U^I + U_a^{II})$ subject to equation (8). Proposition 1 summarizes the results.

Proposition 1 *The optimal tax rate and tax rebate rate (τ, λ) satisfy the following conditions:*

$$\underbrace{v'(g) \left(\frac{\partial R^I}{\partial \tau} + \frac{\partial R^{II}}{\partial \tau} \right)}_{\text{Marginal Social Benefit of } \tau} = \underbrace{\frac{\partial \bar{x}}{\partial \tau} [c^{II} - a\phi - c^I] - \bar{x} \frac{\partial c^I}{\partial \tau} - (1 - \bar{x}) \frac{\partial c^{II}}{\partial \tau}}_{\text{Marginal Social Cost of } \tau}, \quad (9)$$

$$\underbrace{v'(g) \left(\frac{\partial R^I}{\partial \lambda} + \frac{\partial R^{II}}{\partial \lambda} \right)}_{\text{Marginal Social Benefit of } \lambda} = \underbrace{\frac{\partial \bar{x}}{\partial \lambda} [c^{II} - a\phi - c^I] - (1 - \bar{x}) \frac{\partial c^{II}}{\partial \lambda}}_{\text{Marginal Social Cost of } \lambda}. \quad (10)$$

Proof. See Appendix A.1. ■

According to equations (9) and (10), the tax authority sets the optimal tax instruments such that the marginal social benefit due to increased provision of the public good equals the marginal social costs caused by changes in the consumption of the private good and in the number of consumers that bear the tax auditing disutility cost (ϕ). For instance, consider an increase in the tax rate τ . The interpretation of equation (9) is as follows. On one hand, the additional tax revenue allows the government to provide more of the public good (LHS of (9)). On the other hand, there is an increase in the price p^I , which reduces consumption of the private good. In region II , if buyers participate in the tax audit process ($a = 1$), private consumption also falls as

the price increases with the higher tax rate. The higher tax rate expands region I as the cut-off \bar{x} increases, making more consumers subject to the higher price in region I . However, as region II shrinks, fewer consumers bear the disutility of tax auditing (ϕ). Analogous interpretation applies to a decrease in λ , which increases g , except that there is no effect on private consumption in region I and \bar{x} gets smaller, expanding the number of consumers who bear the auditing cost ϕ .

Table 1 presents equilibrium private consumption (c^{I*}, c^{II*}), provision of the public good (g^*) and size of the government audited economy (\bar{x}^*) with and without *consumer auditing* ($a = 0$ and $a = 1$, respectively).

Table 1 - Optimal Allocations and Cut-off Point

	c^{I*}	c^{II*}	g^*	\bar{x}^*
$a = 0$	$L - \sqrt{2g_0Lt}$	L	$v'(g_0) = 2$	$\sqrt{2g_0Lt}/Lt$
$a = 1$	$L - g_1 + (\phi^2/Lt) - \phi$	$L - g_1 + (\phi^2/Lt)$	$v'(g_1) = 1$	ϕ/Lt

g_0 and g_1 are the levels that satisfies the g^* conditions above for $a = 0$ and $a = 1$, respectively.

When $a = 0$, private consumption is lower for consumers in region I than in region II . This is due to the effect of taxation on the private good's price in region I , i.e., the tax increases price, reducing consumption. With *consumer auditing* ($a = 1$), the government collects more taxes and increases the provision of the public good, offsetting at least partially the decrease in consumption of the private good and the tax auditing disutility of consumers in region II . Notice that, in an economy without *consumer auditing*, the government's inability to tax HTT markets creates inequality and ex-ante identical agents become ex-post heterogenous. The introduction of the *consumer auditing* policy corrects this distortion, equalizing utility levels across regions ($u^I = u^{II}$). The following proposition summarizes this result.

Proposition 2 *The tax authority's inability to audit all markets gives rise to hard-to-tax (HTT) markets, where transactions are not taxed and consumers are better off than in taxed markets. The introduction of the consumer auditing policy corrects this distortion as the optimal tax-enforcement policy is such that the utility of consumers is equalized across markets.*

Proof. See Appendix A.2. ■

The fact that consumers enjoy more of the public good if everyone participates in the *consumer auditing* process can lead to a free-riding problem. Individually, a consumer might be tempted to not ask for receipts to avoid the corresponding disutility, while enjoying more of the public good. However, one can show that the gains in private consumption ($\lambda\tau L$) outweighs the cost of tax auditing (ϕ), so that an individual consumer is better off participating in the tax auditing process. The following proposition summarizes this result. We rule out the case of joint deviation as this would imply unrealistic coordination between a large number of consumers.

Proposition 3 *If the government introduces consumer auditing in HTT markets and all consumers request receipts ($a = 1$), an individual consumer would be worse off if he does not participate in the tax auditing process.*

Proof. See Appendix A.3. ■

4 Numerical Results

In this section we present results of a numerical analysis to obtain further insights and a quantitative sense of the implications of the model. First, we consider the case of homogenous consumers presented in the previous section and study how the private enforcement cost (ϕ) affects the optimal policies and the resulting welfare level. Next, we investigate the case of heterogeneous endowment and distributions of poor and rich consumers in the linear economy.

For the purpose of these exercises, we assume $v(g) = \alpha \ln(g)$, where $\alpha > 0$ is the relative weight of the utility derived from the public good. The baseline values, kept constant throughout the exercises, are as follows: $\alpha = 0.10$, $t = 0.25$ and $\theta = 0.80$. The value of α is set such that, in an economy without costly tax enforcement, ten percent of the economy's output is used to provide the public good. The government audit cost parameter $t = 0.25$ implies that a quarter of the production in the farthest market ($x = 1$) would be needed to cover the government audit cost. The relative size of this cost is considered in the numerical exercise by changing the value of ϕ , which is the consumer tax audit cost.

4.1 Homogenous Consumers

We consider first the homogenous consumers case. All consumers are identical at each location x and we normalize endowment to one ($L_x = L = 1$). Table 2 presents the resulting values of the economy's welfare ($W^* = U^{I*} + U_a^{II*}$), the cut-off point (\bar{x}^*), the provision of the public good (g^*) and the tax instruments (τ^* , λ^*) for different values of the consumer's disutility of tax auditing (ϕ).

Table 2 - Homogeneous Consumers and HTT Markets

	ϕ	\bar{x}^*	τ^*	λ^*	W^*
$a = 0$	—	0.63	0.20	—	0.60
$a = 1$	0.02	0.08	0.15	0.33	0.65
	0.04	0.16	0.17	0.43	0.63
	0.06	0.24	0.19	0.51	0.62
	0.08	0.32	0.21	0.58	0.60
	0.10	0.40	0.23	0.64	0.59

Fixed parameters: $(\alpha, t, \theta, L) = (0.10, 0.25, 0.80, 1)$.

Table 2 shows that the economy's welfare in the presence of HTT markets depends on two main features of the economy: (i) whether consumers request sales receipts and (ii) the relative auditing costs (ϕ , t). If consumers do not participate in tax auditing ($a = 0$), the utility of consumers in the HTT region is bigger as the private good's price is lower and thus private consumption is higher ($u^I = 0.54$ and $u^{II} = 0.70$). As discussed in Section 3, the introduction of the *consumer auditing* policy ($a = 1$) equalizes utility across regions, for instance, for $\phi = 0.02$, $u^I = u^{II} = 0.65$. The fact that the government can now reach HTT markets increases tax revenue and the provision of the public good. Indeed, the amount of public good provided not only increases but also equals exactly the level that would have been provided if the government could costlessly ($t = 0$) enforce the tax code in all markets, i.e., $g = 0.10$. When ϕ is higher, the size of the economy that is audited by consumers ($1 - \bar{x}$) is smaller and the optimal tax rebate is higher to offset the greater disutility caused by tax auditing. Accordingly, the tax rate

also increases, leading to higher prices and lower private consumption of goods for all buyers. Consequently, total welfare decreases.

The effect of this policy on the cut-off point, i.e., the maximum reach of the tax authority, raises an interesting point. A smaller \bar{x}^* implies that some consumers who were in markets audited directed by the tax authority (region *I*) are now in region *II*, where they become responsible for tax enforcement. The fact that \bar{x}^* is different with and without the *consumer auditing* policy has implications for both the individuals' and the economy's welfare. In Figure 2, we illustrate the changes in consumers' utility levels across regions when $\phi = 0.02$.

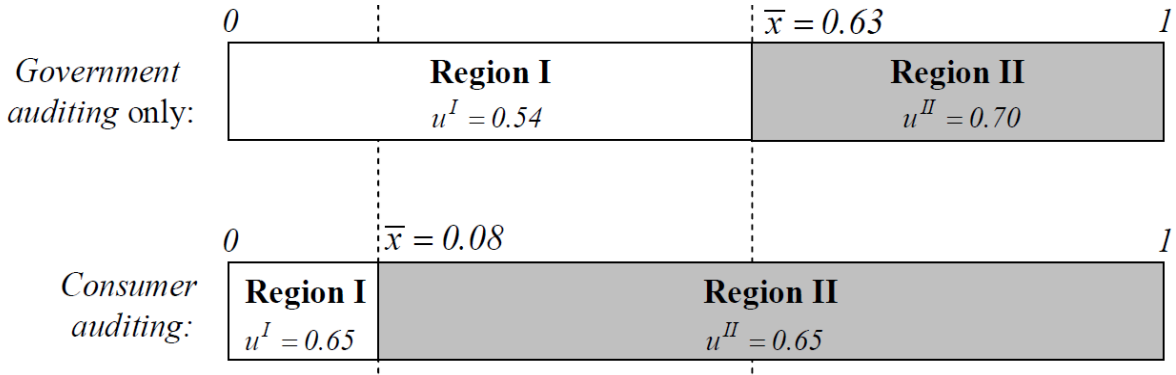


Figure 2: Consumers' utility levels across regions ($\phi = 0.02$)

In Figure 2, first notice that when *consumer auditing* is introduced and \bar{x}^* shrinks, there are consumers that remain in the same region, whether in region *I* or *II*. Consumers originally in region *I* who remain in markets audited by the government become better off with the new policy (u^I increases). The tax rate is lower, reducing the good's price, so that those consumers benefit from more private consumption as well as greater provision of the public good. In the numerical exercise, their utilities increase from $u^{I*} = 0.54$ to $u^{I*} = 0.65$.

Consumers who were in region *I* before the introduction of *consumer auditing* but who are now in markets in region *II* also benefit from this policy. They are the consumers located at $0.08 < x < 0.63$ in Figure 2. They now pay lower taxes and enjoy more of the public good, so their welfare levels improve despite the disutility from tax auditing (ϕ). As shown in Table 2 and in Figure 2, the utilities of these consumers also increase from $u^{I*} = 0.54$ to $u^{I*} = 0.65$.

However, the *consumer auditing* policy has a negative effect on those consumers originally in region *II*. The new policy forces sellers in this region to remit taxes, increasing the good's

price. A lower private consumption and the disutility cost of auditing leads to welfare losses for consumers despite the increased provision of the public good (their utilities decrease from $u^{II*} = 0.70$ to $u_P^{II*} = 0.65$ in the numerical exercise). Overall, as long as the disutility from tax auditing is not large enough, the aggregate welfare of the economy increases with the new policy despite the loss that some consumers suffer.

4.2 Heterogenous Consumers

In this section, we study how the heterogeneity of consumers across markets affects the optimal tax policies and the economy's welfare. At each location x there is a mass of consumers of size 1 and we allow for differences in terms of endowment and distribution of types along the linear economy. Consider two types of consumers, poor (P) and rich (R), and assume that the latter has an endowment $k \geq 1$ times larger than the former, i.e., $L_R = kL_P$. Let the distribution of poor and rich consumers be $\omega_P(x)$ and $\omega_R(x)$, respectively, where $\omega_P(x), \omega_R(x) \in [0, 1]$ and $\omega_P(x) + \omega_R(x) = 1 \forall x$. In the cases studied below, L_P is chosen so that the aggregate endowment of the economy is 1.

We consider three distinct cases regarding the distribution of types of consumers. First, consider a constant distribution of poor and rich consumers, for instance, $\omega_P(x) = \omega_R(x) = 1/2$. At each market x , the average income is the same, which resembles the homogenous consumers case studied before, with $L = (L_P + L_R)/2$. Second, assume that the proportion of rich consumers $\omega_R(x)$ is decreasing in x . That is, there are proportionally more rich consumers in markets that are accessible to the tax authority (low x markets). Last, consider the opposite case, i.e., $\omega_R(x)$ is increasing in x (there are proportionally more poor consumers in low x markets).

More specifically, for the second and third cases we assume $\omega_R(x) = 1 - x$ and $\omega_P(x) = x$ and $\omega_R(x) = x$ and $\omega_P(x) = 1 - x$, respectively. These two cases are intended to shed some light on the possible implications of the *consumer auditing* policy in different economies. For instance, in less developed countries where poorer people trade in peripheral markets (second case) and in developed economies where richer individuals might trade in peripheral markets (third case). Table 3 presents the numerical results for each case, where u_T^{S*} stands for the utility

of a consumer of type $T \in \{P, R\}$ that lives in region $S \in \{I, II\}$ and U_T^* is the aggregate utility of all consumers of type T .

The results for the first case (constant endowment distribution across x), shown in Table 3, are the same as in Table 2 for $\phi = 0.02$. In the absence of *consumer auditing* ($a = 0$), the cut-off points and the sales tax rate are not very different in the three cases, a result that is mainly driven by the fixed government's cost of auditing the economy. As mentioned before, residents of HTT markets are better off than their counterparts in the region audited by the government - due to the effect of taxation on the the private good's price. Government policies hit poor consumers harder if they live close to the tax authority because then they pay relatively higher prices and consume less. Both types are relatively worse off if there are more rich consumers farther way from the tax authority (third case) because the provision of public goods is the lowest and welfare falls accordingly.

Table 3 - Heterogeneous Consumers: HTT Markets

	Endowment distribution	\bar{x}^*	τ^*	λ^*	U_P^*	U_R^*	u_P^{I*}	u_P^{II*}	u_R^{I*}	u_R^{II*}
$a = 0$	$\omega_P(x) = \omega_R(x)$	0.63	0.20	—	0.15	0.45	0.26	0.37	0.82	1.03
	$\omega_P(x) = x, \omega_R(x) = 1 - x$	0.61	0.20	—	0.17	0.44	0.28	0.38	0.84	1.04
	$\omega_P(x) = 1 - x, \omega_R(x) = x$	0.66	0.22	—	0.13	0.46	0.25	0.36	0.80	1.02
$a = 1$	$\omega_P(x) = \omega_R(x)$	0.08	0.15	0.33	0.18	0.47	0.35	0.35	0.95	0.95
	$\omega_P(x) = x, \omega_R(x) = 1 - x$	0.06	0.15	0.35	0.18	0.47	0.35	0.35	0.95	0.95
	$\omega_P(x) = 1 - x, \omega_R(x) = x$	0.11	0.17	0.41	0.18	0.47	0.35	0.35	0.95	0.95

Fixed parameters: $(\alpha, t, \theta, \phi, k) = (0.10, 0.25, 0.80, 0.02, 2)$.

For $a = 1$, differences arise at the fourth decimal place between the utilities of same type consumers.

In all cases, when the *consumer auditing* policy is implemented ($a = 1$), the heterogeneity in utility levels is reduced within each group regardless of where consumers are located. In other words, for the parameters considered, the *consumer auditing* policy eliminates the utility inequality caused by the government's inability to audit all markets equally. The sales tax rate

(τ^*) is reduced and the provision of the public good increases, i.e., $g^* = 0.10$. Having access to a larger tax base, the government taxes the whole economy at a lower rate and audits directly only markets that are easy to access.

In our numerical exercise, for any endowment distribution, the economy is better off with the implementation of the *consumer auditing* policy. In fact, there are more consumers benefiting from this policy than otherwise. Welfare gains are mainly due the greater provision of the public good to the entire society and more private consumption for most consumers due to lower tax rates.⁴

5 Conclusion

Costly tax enforcement creates distortions in the economy. The inability of the tax authority to audit all transactions gives rise to hard-to-tax (HTT) markets. While some markets are audited and taxed, others might go untaxed. In this paper, we study the welfare and distributional effects of the adoption of a *consumer auditing* policy that rewards buyers for requesting transaction receipts in HTT markets. When the private cost of tax enforcement to consumers are relatively low, *consumer auditing* allows the sales tax rate to be lowered and the provision of the public good to be higher. The policy can be welfare enhancing and equalizes the utility of consumers across markets. However, there can be losers in this process. Consumers in the originally untaxed markets are negatively affected as they become burdened by the sales tax. Redistribution concerns may lead the social planner to weight consumers differently when designing and enforcing its optimal tax policy.

⁴For the parameters used, we can still observe welfare gains, though smaller, if the government implements the *consumer auditing* policy but it is Pareto constrained, i.e., the policy is such that consumers in HTT markets are just as well-off with or without *consumer auditing*.

Appendix

A.1 Proof of Proposition 1

Proof. Here we show the derivation of the optimality conditions for the tax policy instruments (τ and λ). The social planner maximizes the economy's welfare function $W = U^I + U^{II}$, subject to the balanced budget constraint (8), with U^I and U^{II} given by (6) and (7), respectively. Hence, the derivatives with respect to τ and λ are

$$\frac{\partial W}{\partial \tau} = \bar{x} \frac{\partial c^I}{\partial \tau} + (1 - \bar{x}) \frac{\partial c^{II}}{\partial \tau} + [c^I - c^{II} + \phi(a)] \frac{\partial \bar{x}}{\partial \tau} + v'(g) \left(\frac{\partial R^I}{\partial \tau} + \frac{\partial R^{II}}{\partial \tau} \right), \quad (11)$$

$$\frac{\partial W}{\partial \lambda} = (1 - \bar{x}) \frac{\partial c^{II}}{\partial \lambda} + [c^I - c^{II} + \phi(a)] \frac{\partial \bar{x}}{\partial \lambda} + v'(g) \left(\frac{\partial R^I}{\partial \lambda} + \frac{\partial R^{II}}{\partial \lambda} \right). \quad (12)$$

These derivatives equal zero for interior solutions of τ and λ , so that the optimality conditions (9) and (10) are obtained. Corner solutions are possible depending on the parameters of the model, but we discard them since they are not interesting or plausible in reality. Thereafter, we assume that W is well-behaved so that the second order conditions for local maximization of W are satisfied at the optimal τ and λ given by (9) and (10). ■

A.2 Proof of Proposition 2

Proof. Here, we show the derivation of the results in Table 1. Assume that all consumers request transaction receipts in region II , i.e. $a = 1$. Define:

- (i) $C^I = \int_{x=0}^{\bar{x}} (1 - \theta\tau) L dx$ as aggregate utility from private consumption in region I ,
- (ii) $C^{II} = \int_{x=\bar{x}}^{x=1} (1 - (1 - \lambda)a\tau) L dx$ as the counterpart of C^I for region II ,
- (iii) $V^I = \int_{x=0}^{\bar{x}} v(g) dx$ as the aggregate utility derived from g in region I , and
- (iv) $V^{II} = \int_{x=\bar{x}}^{x=1} v(g) dx$ as the counterpart of V^I in region II ,

where \bar{x} is given by equation (1).

The objective function of the planner can be written as the following Lagrangean expression:

$$\mathcal{L} = C^I + C^{II} - (1 - \bar{x}) \phi + V^I + V^{II} + \mu \left(\int_0^{\bar{x}} (\theta\tau - tx) L dx + \int_{\bar{x}}^1 (1 - \lambda) a\tau L dx - g \right),$$

where μ is the Lagrangean multiplier for the balanced budget constraint (8). Evaluating at $a = 1$ and replacing \bar{x} according to (1), we obtain

$$\begin{aligned} \mathcal{L} = & -L \left(\frac{\theta^2 \tau^2 - 2\theta\tau^2 + 2\theta\tau^2\lambda - t + \tau t + \tau^2 - 2\tau^2\lambda - \tau\lambda t + \tau^2\lambda^2}{t} \right) \\ & - \left(1 - \frac{\tau(\theta - 1 + \lambda)}{t} \right) \phi + v(g) \\ & + \mu \left(-\frac{1}{2} \frac{L\tau^2(\theta - 1 + \lambda)^2}{t} + \frac{\theta\tau^2 L(\theta - 1 + \lambda)}{t} + (1 - \lambda)\tau L \left(1 - \frac{\tau(\theta - 1 + \lambda)}{t} \right) - g \right). \end{aligned}$$

Interior solutions for optimal τ and λ require

$$\begin{aligned}
\tau : \quad & -L \left(\frac{2\theta^2\tau - 4\theta\tau + 4\theta\tau\lambda + t + 2\tau - 4\tau\lambda - \lambda t + 2\tau\lambda^2}{t} \right) + \left(\frac{\theta - 1 + \lambda}{t} \right) \phi \\
& + \mu \left(\frac{-L\tau(\theta - 1 + \lambda)^2}{t} + \frac{2\theta\tau L(\theta - 1 + \lambda)}{t} \right. \\
& \left. + (1 - \lambda)L \left(1 - \frac{\tau(\theta - 1 + \lambda)}{t} \right) - \frac{(1 - \lambda)\tau L(\theta - 1 + \lambda)}{t} \right) = 0, \\
\lambda : \quad & -L \left(\frac{2\theta\tau^2 - 2\tau^2 + 2\tau^2\lambda}{t} \right) + \frac{\phi\tau}{t} \\
& + \mu \left(\frac{-L\tau^2(\theta - 1 + \lambda)}{t} + \frac{\theta\tau^2 L}{t} - \tau L \left(1 - \frac{\tau(\theta - 1 + \lambda)}{t} \right) - \frac{1 - \lambda\tau^2 L}{t} \right) = 0.
\end{aligned}$$

Using the equations above together with the government budget constraint (8), we obtain the following results:

$$\begin{aligned}
\mu &= 1, \\
\tau &= \frac{2Lt\phi - \phi^2 + 2gtL}{2L^2\theta t}, \\
\lambda &= -\frac{-\theta\phi^2 + 2\theta gtL - 2Lt\phi + \phi^2 - 2gtL}{2Lt\phi - \phi^2 + 2gtL}.
\end{aligned}$$

The expressions in Table 1 can be obtained using the results above. To get private consumption (c^{I*} and c^{II*}), use functions (4) and (5). To get \bar{x}^* , use equation (1). The optimal provision of the public good g^* can be obtained from the derivative of \mathcal{L} with respect to g , which yields $v'(g^*) = \mu$. Results for $a = 0$ can be obtained in an analogous way. ■

A.3 Proof of Proposition 3

Proof. In order for consumers to participate in the tax auditing process, the tax rebate for requesting receipts has to be larger than the disutility from the tax auditing, i.e., $\lambda\tau L \geq \phi$, conditional on all consumers in region *II* requesting receipts. The inequality above assumes that an individual consumer's attempt to deviate has no observable effect on the equilibrium price of the private good or on the provision of the public good due to the negligible size of each individual consumer's demand. Substituting τ and λ obtained from (9), we must show that

$$\frac{1}{2} \frac{(1 - \theta)}{L\theta t} (2Lt\phi - \phi^2 + 2gtL) \geq 0, \tag{13}$$

which requires that

$$2Lt\phi - \phi^2 + 2gtL \geq 0. \tag{14}$$

Since $\bar{x}_{a=1} = \phi/(Lt) \leq 1$ (see Table 1), we must have $0 \leq \phi \leq Lt$. Now, multiplying all sides of that inequality by $-\phi$ and adding $2Lt\phi + 2Lt\phi$, we get

$$2Lt(\phi + g) \geq -\phi^2 + 2Lt(\phi + g) \geq Lt(\phi + 2g). \tag{15}$$

Hence, because $-\phi^2 + 2Lt(\phi + g) \geq Lt(\phi + 2g) \geq 0$, (14) is satisfied, which verifies that each individual consumer is better off by participating in the the tax auditing process when others do so. ■

References

- [1] Alm, James; Jorge Martinez-Vazquez and Friedrich Schneider, 2004. “Sizing” the Problem of the Hard-to-Tax, In: James Alm, Jorge Martinez-Vazquez, and Sally Wallace, Eds., Contributions to Economic Analysis, vol. 268, pages 11-75.
- [2] Arbex, Marcelo A. and Enlinson Mattos, 2013. “Tax evasion: Is This a Government Fight, or Can Anyone Join?,” Working Papers 1302, University of Windsor, Department of Economics.
- [3] Boadway, Robin; Nicolas Marceau and Steeve Mongrain, 2007. “Redistributive Taxation under Ethical Behaviour,” Scandinavian Journal of Economics, vol. 109(3), pages 505-529, 09.
- [4] Chang, Juin-jen and Ching-chong Lai, 2004. “Collaborative Tax Evasion and Social Norms: Why Deterrence Does Not Work,” Oxford Economic Papers, Oxford University Press, vol. 56(2), pages 344-368.
- [5] Das-Gupta, Arindam, 1994. “A Theory of Hard-to-Tax Groups.” Public Finance, 49 (Supplement), pages 28-39.
- [6] Eurofound, 2013. Tackling Undeclared Work in 27 European Union Member States and Norway: Approaches and measures since 2008, Eurofound, Dublin.
- [7] Gordon, Roger and Wei Li, 2009. “Tax Structures in Developing Countries: Many puzzles and a possible explanation.” Journal of Public Economics, vol. 93(7-8), pages 855-866.
- [8] Hotelling, Harold, 1929. “Stability in Competition.” Economic Journal, vol. 39, pages 41-57.
- [9] Huang, Che-Chiang and K.L. Glen Ueng, 2012. “On the Non-equivalence of Specific and Ad Valorem Taxation in Competitive Markets.” Mimeo. 12th Public Economic Theory meeting, Taipei, Taiwan.
- [10] Marchese, Carla, 2009. “Rewarding the Consumer for Curbing the Evasion of Commodity Taxes?,” FinanzArchiv: Public Finance Analysis, vol. 65(4), pages 383-402.
- [11] Moffitt, Robert, 1983. “An Economic Model of Welfare Stigma.” The American Economic Review, vol. 73(5), pages 1023-1035
- [12] Musgrave, Richard A., 1981. “Income Taxation of the Hard-to-Tax Groups.” In Richard A. Musgrave, Ed., Fiscal Reform in Bolivia. Cambridge, MA: Cambridge University Press.
- [13] Paschoal, Bruno V.L., 2012. Punição, Recompensa, Persuasão e Ajuda : Estratégias Regulatórias a partir do Caso Nota Fiscal Paulista. Available at <http://hdl.handle.net/10438/10380>.

- [14] Tanzi, Vito and Milka Casanegra de Janscher, 1989. "The Use of Presumptive Income Taxation in Modern Tax Systems." Proceedings of the 42nd Congress of the International Institute of Public Finance, Athens 1986, Aldo Chinicone and Ken Messere, Eds.
- [15] Terkper, Seth, 2003. "Managing Small and Medium-Size Taxpayers in Developing Economies." Tax Notes International, January 13, 2003, pages 211-234.