Electron Diffraction

Introduction

In 1924 de Broglie predicted that the wavelength of matter waves could be found by using the same relationship that held for light, namely,

\[ \lambda = \frac{h}{p} \]

where \( \lambda \) is the wavelength of a light wave, \( h \) is Planck's constant and \( p \) is the momentum of the photons. De Broglie proposed that matter, as well as light, has a dual character behaving in some circumstances like particles and in others like waves. He suggested that a particle of matter having a momentum \( p \) would have an associated wavelength \( \lambda \).

The first experimental evidence of the existence of matter waves was obtained by Davisson and Germer in 1927. They “reflected” slow electrons from a single crystal of nickel and applied de Broglie's relationship. The wavelength of the electrons was determined and compared with that calculated from Bragg's expression for X-ray diffraction. Excellent agreement was obtained.

Shortly thereafter Thomson's experiments with fast electrons supplied additional evidence about the behaviour of electrons. He photographically analyzed diffraction patterns produced by electrons passing through thin films of gold, aluminum and other materials. From measurements of the size of the electron diffraction rings on a fluorescent screen, the wavelength of the electrons was found and again agreed with that predicted by de Broglie's equation.

Since the ‘roaring twenties’, a whole new physics has sprung up around the wave-particle nature of light and very tiny objects like electrons and atoms. The wave properties of matter are no longer questioned. Recent experiments have succeeded in lowering the kinetic energy (i.e. temperature) of atoms so much that physicists joke about them diffracting around your fingers since their de Broglie wavelength is so large!

This experiment provides you with a ready made electron diffraction tube with which you will observe Thomson's rings by passing a beam though a thin sample of polycrystalline aluminum. Like Thomson, you will make measurements of the rings and using Bragg's Law confirm de Broglie's relationship.
Theory

Bragg’s Law simply states the condition that must be satisfied if waves reflecting off adjacent planes of atoms are to constructively interfere. It is written as:

\[ 2d \sin \theta = n\lambda \]  \hspace{1cm} (1)

where \( d \) is the separation of the planes, \( n \) is the order of the reflection, and \( \theta \) is the ordinary angle between the incident beam and the reflecting plane, which is the same angle between the reflected beam and the reflecting plane. (Note that this angle is not defined with respect to the surface normal, as in optics.)

**Figure 1.** Microscopic view of Bragg reflection from adjacent planes of atoms in the electron diffraction tube.

For a beam of electrons passing through a plane of atoms in a thin film, some of the electrons will be Bragg reflected from the original beam by an angle equal to \( 2\theta \), as shown in Figure 1.

Of course, in a crystal, there are many ways one can form planes of atoms. These different planes are categorized by three numbers known as Miller indices \((HKL)\). For a crystal such as aluminum, which has a face-centered cubic (FCC) structure (see Figure 2), the distance between two adjacent planes with Miller indices \((HKL)\) is:

\[ d = \frac{a}{\sqrt{H^2 + K^2 + L^2}} \]  \hspace{1cm} (2)

where \( a \) is the length of the edge of the unit cell in an FCC crystal (see Kittel). When the Miller indices are multiplied by the order of the reflection \( n \), any order \( n \) of Bragg reflection planes \((HKL)\) is considered to be first order Bragg reflection from planes \((hkl)\).

**Figure 2.** Face Centred Cubic (FCC) crystal structure.
Therefore,

\[ d = \frac{na}{\sqrt{h^2 + k^2 + l^2}} \]  \hspace{1cm} (3)

where \( nH = h, nK = k, \) and \( nL = l. \) Substituting into equation (1), we obtain:

\[ \lambda = \frac{2a \sin \theta}{\sqrt{h^2 + k^2 + l^2}} \]  \hspace{1cm} (4)

When the angle \( \theta \) is small, then (referring to Figure 1),

\[ \theta = \frac{r}{2D} \]  \hspace{1cm} (5)

and equation (4) becomes:

\[ \lambda = \frac{ar}{D \sqrt{h^2 + k^2 + l^2}} \]  \hspace{1cm} (6)

where \( D \) is the distance from the thin film target to the screen and \( r \) is the radius of the rings. The quantity \( a \) is known from X-ray diffraction measurements. \( D \) and \( r \) are obtained by direct measurements.

The observed diffraction pattern consists of rings of various radii produced by the constructive interference of electron waves reflected from the various families of planes within the randomly oriented crystals in the thin film target. The intensity of a reflection in the diffraction pattern is proportional to the square of the corresponding structure factor \( F, \) i.e.,

\[ I_{(hkl)} \propto \left| F_{(hkl)} \right|^2 \]  \hspace{1cm} (7)

For FCC structure,

\[ F_{(hkl)} \approx 1 + e^{-i\pi(h+k)} + e^{-i\pi(h+l)} + e^{-i\pi(k+l)} \]  \hspace{1cm} (8)

The structure factor actually takes into consideration the coordinates and differences in scattering power of the individual atoms, the Miller indices, and the addition of sine waves of different amplitudes and phase but of the same wavelength. When squared, the structure factor vanishes unless \( h, k, \) and \( l \) are all odd or even, in which case \( F \sim 4. \) Therefore rings will occur for which \( h, k, \) and \( l \) are all odd or even. At this point you should construct a table of the Miller indices that will produce rings, along with the sum of their squares in the second column, and a third column that has the square root of the second column. This will facilitate data analysis later. Do not go higher than (440).

**Experiment**

The apparatus is a Welch Scientific Electron Diffraction Tube (Model 2639) and Power Supply (2639A). The tube is evacuated to a pressure of 10-8 torr. It contains an indirectly heated cathode electron gun at one end. The gun is capable of producing a finely focused electron beam at energies up to 10 KeV. The other end of the tube has a phosphor screen. Located at a distance \( D = 180 \text{ mm} \) from the screen of the tube are 4 thin film samples. Two are made of polycrystalline aluminum, and the other two are samples of two dimensional hexagonal pyrolytic graphite, and polycrystalline hexagonal pyrolytic graphite. The graphite samples were very fragile and now lie in pieces on the bottom of the tube. The aluminum samples are still intact.
The electron diffraction tube is mounted inside a rugged housing which holds all of the power supplies needed for the tube. This unit has front panel controls for the intensity, focus, horizontal & vertical deflection, and energy of the electron beam. At the back of the unit is a jack for an external microammeter to measure the current of the electron beam.

Plug in the microammeter and turn on the tube. Allow it a minute to warm up. Be sure the current does not exceed 10 μA by adjusting the intensity knob. You can use up to μ25 A for a short time to see faint diffraction rings, but operation at high currents for extended periods of time will shorten the life of the electron gun.

Set the accelerating voltage of the beam to 7 KV. Use the focus and deflection knobs to locate the beam on the screen. Now steer the beam into the target samples and look for the thin film of aluminum. It is easy to recognize because the rings it produces are uniform in intensity all the way around. If there are bright speckles around the circumference, then you are aiming the beam through one of the few remaining samples of polycrystalline hexagonal pyrolytic graphite. If you get a diffraction pattern which consists not of rings, but a hexagonal pattern of bright dots almost a centimeter apart then you have found a rare remaining sample of two dimensional hexagonal pyrolytic graphite. We will only be concerned with aluminum for the purposes of this lab.

Keep the spot moving slowly when searching for a diffraction pattern. Turn down the intensity when the beam is not going through a sample to avoid damage to the screen, and turn up the intensity when you have found a pattern. Always work with a minimum current needed to give good results. Focus and align the beam for the best results and then adjust the acceleration voltage.

When you have found a diffraction pattern, bring a magnet close to the screen and observe the deflection and distortion of the pattern. This proves that it is caused by electrons and not photons. Be careful with the magnet, if the electron beam is deflected greatly, internal arcing might develop. You can also see the beam deflect slightly by rotating the cart on which the apparatus is mounted. (Why?)

Now measure the radius of each ring in the diffraction pattern. For each ring make several measurements across different diameters to get an average to correct for any distortions. Be sure to check the voltage while you are making measurements for any drift.

Apply the relationship derived earlier to calculate the wavelength of the electrons for all of the permitted rings. The lattice constant, a for aluminum is 4.05Å (Kittel). For the same voltage, calculate the de Broglie wavelength. Compare the two values at each ring. You might want to organize this information in a table as shown.

### Table 1. 7000 Volts.

<table>
<thead>
<tr>
<th>Reflection Plane</th>
<th>(h² + k² + l²)⁰.⁵</th>
<th>r_{average}</th>
<th>λ_{Bragg}</th>
<th>λ_{deBroglie}</th>
<th>% difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>111</td>
<td>1.732</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>200</td>
<td>2.00</td>
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<td></td>
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<td>220</td>
<td>2.828</td>
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<td></td>
<td></td>
<td></td>
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<tr>
<td>311</td>
<td>3.316</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>....more</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Since some of the allowed reflections cannot be observed because of their low intensity, check carefully for an obvious lack of agreement between λ_{Bragg} and λ_{de Broglie}. For example, if the choice of Miller indices for a particular measurement of the radius, r, of a ring pattern results in an unusual spread between the two wavelengths, then try other allowed combinations of the indices until good agreement is obtained.

Repeat the above procedure for at least two other voltage settings.
Questions

Derive the following expression for the de Broglie wavelength of an electron:

\[ \lambda = \sqrt{\frac{150}{V}} \]  \hspace{1cm} (9)

where \( \lambda \) is in Å, and \( V \) in volts. Would the above equation be valid for a 10 MeV beam of electrons? If not, why not?

If the electron beam is replaced by a beam of positive particles could you carry out the same analysis if the particles were positrons? What would be the result if we used protons?

Look at the electron diffraction tube closely. The walls are lined with graphite (a conductor). Why is this done? This inner lining and the screen are all connected to ground (earth), can you describe what would happen they were not connected to anything?

Why is the diffraction pattern a series of concentric rings? Would a single crystal produce the same result?

References

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