Part I

The Investment Setting and Asset Allocation Decision

1. What is an Investment

Individuals over the course of a lifetime will rarely have exactly the same amount of income as desired consumption. In some periods, they will have more income than desired consumption, at other time, the opposite will occur. The excess of income over consumption is saved. How these funds are employed is investment. An individual gives up current consumption in order to enjoy a greater amount of consumption in the future. So investment is defined as:

The current commitment of dollars for a period of time in order to derive future payments that will compensate the investor for the time the funds are committed, for the expected rate of inflation, and for the uncertainty of the future flow of funds. The required rate of return on one’s investment should compensate for the uncertainty of the return (called investment risk, so the investor receives a risk premium) and the expected rate of inflation (could be labelled as inflation risk that affects the purchasing power of a dollar).

2. Measures of Return and Risk

One important use of rates of return is in a comparison of the profitability of different investments. It is standard practice for investment managers to present the profitability of financial assets in either dollar terms or percentage terms. However, profitability in percentage terms enables investors to compare the rates of return on various assets of different levels of investments.

2.1. Measures of Historical Rates of Return

Students should be aware of the realized rates of return on an investment (say a mutual fund) or the historical rate of return. It is simply the rate of return that has been already been earned, as opposed to rates of return expected for the future. Realized rates of return are called ex-post rates of return. In contrast, ex-ante rates of return are rates of return that are expected to occur in the future. Investors may use historical rates of return to estimate future returns and risk of various securities - estimates that are needed to make portfolio investment decisions. When you plan your investment holdings, you can look at the historical
average rates of return, and those averages that can serve as the best estimate of what will happen in the future.

2.1.1. The holding Period Return (HPR)

Investors hold a security for a given period. The rate of return measured for this period is called the holding period return (HPR). It is defined as the total return from an investment, including all sources of income, for a given period of time. It is calculated as:

$$\text{HPR} = \frac{\text{Ending Value of Investment (including cash flow)}}{\text{Begining Value of Investment}}$$

A value greater than 1.0 indicates an increase in wealth; a value less than 1.0 indicates a decrease in wealth. The HPR cannot be negative. As an example, suppose you put $500 in an investment and at the end you have $575. Then the HPR is: $\text{HPR} = \frac{$575}{$500} = 1.15$

2.1.2. The Holding Period Yield (HPY)

It is the simple rate of return and defined as the total return from an investment for a given period of time stated as a percentage. It is calculated as:

$$\text{HPY} = \text{HPR} - 1$$

for example, the HPY from the above example is: $1.15 - 1 = 0.15$ or 15%. The holding period yield has some limitations since it does not accurately account for the timing of cash dividends that are paid more frequently than once a year.

2.1.3. The annual HPY

can be found by taking the nth root of the HPR to get the annual HPR then subtracting 1:

$$\text{Annual HPR} = \text{HPR}^{1/n}$$
$$\text{Annual HPY} = \text{HPR}^{1/n} - 1$$

where n is the number of years the investment is held.

For example, suppose the $500 were held for 3 years, after which time it had grown to $650. First, find the HPR: $\text{HPR} = \frac{$650}{$500} = 1.30$

Next find the annual HPR: $\text{HPR}^{1/3} = (1.30)^{1/3} = 1.0914$

Next, find the annual HPY: Annual HPY = $1.0914 - 1 = 0.0914$ or 9.14%.

In computing the annual HPY, we assume that the yield is constant for each year and the ending value of an investment can be a result of a price change or income, or both.
2.2. Computing Mean Historical Returns

To measure the rate of return on a portfolio of assets in a given year or to measure the return on a specific security (or a portfolio) across years, some averages must be calculated. There are two main methods for calculating averages of financial assets returns. Mean rate of return is defined as the average of an investment’s return over time. We find that for a single investment and for a portfolio.

2.2.1. Single Investment

a. The Arithmetic Mean (AM) - a measure of mean return equal to the sum of annual returns divided by the number of years. It is given by:

\[ AM = \frac{\sum_{i=1}^{N} HPY}{N} \]

where \( \sum HPY \) is the sum of the annual holding period yields. Where \( i \) goes from 1 to \( N \) and represents periods.

b. The Geometric Return (GM) - is an averaging method that compounds rates of return. That is, if \$1 is invested in Period 1, then it will be worth \$\left(1 + R_{1}\right)\) at the end of Period 1. The geometric method assumes that \$\left(1 + R_{1}\right)\) is invested in Period 2. At the end of Period 2, the investment will be worth the amount invested at the beginning of Period 2 times the value of a dollar invested in Period 2. That is, the investment at the end of Period 2 is worth \$\left(1 + R_{1}\right)\left(1 + R_{2}\right)\). Continuing this procedure over \( n \) periods, we get the value at the end of \( n \) periods. Therefore, the average (or mean) geometric rate of return is the \( n \)th root of the product of the annual holding period returns for \( n \) periods, minus one (1). It is calculated as:

\[ GM = \left(\prod_{i=1}^{N} HPR\right)^{1/N} - 1 \]

where \( \prod \) is the product of the annual holding period returns.

\[ GM = \left(\prod_{i=1}^{N} (HPR_{1})(HPR_{2})......(HPR_{N})\right)^{1/N} - 1 \]

and \( i \) goes from 1 to \( N \) and represents periods. As an example, consider the following historical returns for an investment:
The arithmetic mean is:
\[ AM = \frac{\sum \text{HPY}}{N} = \frac{(0.10 + 0.25 + (-0.10))}{3} = 0.25/3 = 0.0833 = 8.33\% \]

The geometric mean is:
\[ GM = [(1.10) * (1.25) * (0.90)]^{1/3} - 1 = (1.2375)^{1/3} - 1 = 0.0736 - 1 = 0.0736 \]
or 7.36%.

For measuring long run performance, the geometric mean is considered more useful because it is a compound measure. The arithmetic mean is a good measure for the expected future rate but is biased upward. It is always at least as large as the geometric mean. If rates of return are the same from year to year the AM will equal the GM. The more variable returns are, the larger the difference between the AM and GM.

### 2.2.2. A Portfolio of Investments

The mean historical rate of return (HPY) for a portfolio, or collection of investments is given by a weighted average of individual HPYs in the portfolio. The weights are based on the beginning market value of the investments. Consider the following three stock portfolio:

<table>
<thead>
<tr>
<th>Year</th>
<th>Begining Value</th>
<th>Ending Value</th>
<th>HPR</th>
<th>HPY</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>100</td>
<td>110</td>
<td>1.10</td>
<td>0.10</td>
</tr>
<tr>
<td>2</td>
<td>110</td>
<td>137.5</td>
<td>1.25</td>
<td>0.25</td>
</tr>
<tr>
<td>3</td>
<td>137.5</td>
<td>123.8</td>
<td>0.90</td>
<td>-0.10</td>
</tr>
</tbody>
</table>

The HPR for the portfolio can be found by calculating the weighted average of the individual HPRs:

\[ \text{HPR}_\text{P} = (0.2) * (1.05) + (0.5) * (1.08) + (0.3) * (1.10) = 0.21 + 0.54 + 0.33 = 1.08 \]

The same can be done for the HPY:

\[ \text{HPY}_\text{P} = (0.2) * (0.05) + (0.5) * (0.08) + (0.3) * (0.10) = 0.01 + 0.04 + 0.03 = 0.08 = 8\% \]

While finding historical rates of return can be useful, we are more concerned with trying to predict future rates of return.
2.2.3. Calculating Expected Rates of Return and Risk

Students should be aware of the difference between certain and uncertain investments decisions. **Certainty**, is the situation in which the future value of the asset (or the rate of return) is known with a probability of 1. (A probability of 1 means that the asset’s future value or rate of return is certain). However, **uncertainty**, or risk, that is a situation in which there is more than one possible future value of the asset (or more than one possible rate of return). In this case, the asset value is a **random variable**. If investors know the probability of each random outcome, they face risk. If the probability of each outcome is unknown to investors, they face uncertainty.

Trying to predict the future involves risk and there is no guarantee that an investment will achieve a certain return. **Risk** - is defined as the uncertainty that an investment will earn its expected rate of return. To compute this, the investor assigns probability values to all possible returns. These probabilities range from zero (no chance) to one (complete certainty).

1. **The Expected Return** is given by:

   \[
   \text{Expected Return} = \sum_{i=1}^{n} (P_i) (R_i)
   \]

   Expected Return = \( \sum_{i=1}^{n} (P_i) (R_i) = (P_1)(R_1) + (P_2)(R_2) + \ldots + (P_n)(R_n) \)

   In words, the expected return from an individual investment is a weighted average of the possible outcomes. The weights are the probabilities associated with each potential outcome.

   For example, consider the following distribution of returns for an asset:

<table>
<thead>
<tr>
<th>Probability ( P_i )</th>
<th>0.20</th>
<th>0.50</th>
<th>0.30</th>
</tr>
</thead>
<tbody>
<tr>
<td>Return ( R_i )</td>
<td>-0.10</td>
<td>0.06</td>
<td>0.20</td>
</tr>
</tbody>
</table>

   The expected return is:

   \[
   \text{Expected Return} = \sum_{i=1}^{n} (P_i)(R_i) = (0.20)*(-0.10) + (0.50)*(0.06) + (0.30)*(0.20) = 0.07 \text{ or } 7%.
   \]

2. **Measuring the Risk of Expected Rates of Return**: it is the variance or a measure of variability that is equal to the sum of the probability of return times the squares of a return’s deviation from the mean. It is calculated as:

   \[
   \text{Variance} = \sum (P_i)(\text{possible return } - \text{Expected Return})^2 = \sum_{i=1}^{n} (P_i)(R_i - E(R_i))^2
   \]
Using the numbers from the previous example, the variance $\sigma^2$ is calculated as follows:

$$\sigma^2 = \sum_{i=1}^{n} (P_i)(R_i - E(R_i))^2$$

$$= (0.2)(-0.1 - 0.07)^2 + (0.05)(0.06 - 0.07)^2 + (0.3)(0.20 - 0.07)^2$$

$$= 0.00578 + 0.00005 + 0.00507 = 0.1090$$

### 2.2.4. Risk Aversion and Investment

Take the following two assets, A and B, starting at $100 each.

<table>
<thead>
<tr>
<th>Return (A)</th>
<th>Probability</th>
<th>Return (B)</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>120</td>
<td>1</td>
<td>110</td>
<td>1/2</td>
</tr>
<tr>
<td>130</td>
<td>1/2</td>
<td>100</td>
<td>1/2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Expected Return ($)</th>
<th>120 or 20%</th>
<th>Expected Return ($)</th>
<th>120 or 20%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variance</td>
<td>0</td>
<td>Variance</td>
<td>100</td>
</tr>
</tbody>
</table>

the mean return on the two assets is identical. Both security A and B have the expected dollar return of $120 on a $100 investment. Variance is calculated as stated above. Which investments would you prefer? Empirical evidence suggests that investors would prefer security A over B, and those are called **risk averters**. **Risk averters**, other things being equal, are investors who dislike volatility or risk. They always prefer a certain investment over an uncertain investment as long as the expected returns on the two investments are identical. Thus, for risk averters to be convinced to buy security B, they would have to be compensated by a higher expected return. (you should know also that the difference between the expected rate of return on a risky asset and the riskless interest rate is known as the **risk premium**).

**Risk neutral investors** completely ignore an asset’s variance and make investment decisions based on only the asset’s expected return. **Risk seekers investors** are ones who like risk or variability in returns. These investors are willing to pay a higher price for an asset whose variance increases.

### 3. Standard Deviation ($\sigma$)

A measure of variability equal to the square root of variance, that is:

$$\sigma = (\sigma^2)^{1/2}$$

using the above example, we get:

$$\sigma = (\sigma^2)^{1/2} = (0.1090)^{1/2} = 0.10440.$$
4. **Coefficient of Variation**: it is a measure of relative variability that indicates risk per unit of return. In other words, the coefficient of variation accounts for differences in size of the return variable. It is defined as:

Coefficient of Variation = Standard Deviation of Returns/Expected Rate of Return

\[
\text{Coefficient of Variation} = \frac{\sigma_i}{E(R_i)}
\]

From our example, CV = 0.10440/0.07 = 1.49.

5. **Risk Measures for Historical Return**: the proper measure for historical returns is given by the variance of the holding period returns:

\[
\sigma^2 = \frac{\sum_{i=1}^{N} [HPY_i - E(HPY)]^2}{N}
\]

where \(HPY_i\) is the holding period yield for period \(i\); \(E(HPY)\) is the arithmetic mean for the HPY series; and \(N\) is the number of observations for HPY. The standard deviation is gain just the square root of the variance.

Students should be aware that sometimes, when we have sample data (data based on a sample of the whole population) rather than population data (data based on the whole population), we denote the mean by \(\overline{HPY} = \frac{\sum_{i=1}^{N} (HPY_i)}{N}\), and the sample variance is obtained by dividing by \(N - 1\) rather than \(N\). Specifically, \(\sigma^2 = \frac{\left(\sum_{i=1}^{N} [HPY_i - E(HPY)]^2\right)}{(N - 1)}\). Dividing by \(N - 1\), we obtain an unbiased formula for the variance (i.e., dividing by \(N - 1\) or \(N\) does not affect the analysis too much).

6. **The Covariance**: The expected rate of return and the variance provide us with information about the nature of the probability distribution associated with a single stock or for a portfolio of stocks. However, these numbers tell us nothing about the way the returns on securities *interrelate*. Suppose in some given month one stock produces a rate of return above its expected value. If we know in advance this going to happen, what does it do to our expectation for the rate of return on some other stock? When one stock produces a rate of return above its expected value, do other stocks have a propensity to do so as well? A statistic which provides us with some information about this question is the **covariance** between two stocks. It is an absolute measure of the extent which two assets move together over time, that is, how often they move up or down together. Covariance between \(R_A\) and \(R_B\) is defined as:

\[
\text{Cov} (R_A, R_B) = \frac{\sum_{t=1}^{N} (R_{A,t} - \overline{R_A})(R_{B,t} - \overline{R_B})}{N}
\]
where $\overline{R}_A$ and $\overline{R}_B$ are the means of A and B, respectively. (If we use the sample data, we should divide by $N - 1$, not $N$, and the same argument made about the variance holds here).

As a number, the covariance does not tell you much about the relationship between returns on the two stocks. If it is a positive number, it tells you that when one stock produces a return above its mean return, the other tends to do so as well.

If you are using a probability distribution about the possible returns on the two stocks, you should be using probabilities to find the covariance of expected returns, that is:

$$Cov(R_A, R_B) = \left( \sum_{t=1}^{N} P_t(R_{A,t} - \overline{R}_A)(R_{B,t} - \overline{R}_B) \right)$$

where $t$ represents states or periods.

The covariance number is an important one for us to know, because it is a critical input in determining the variance of a portfolio of stocks. As a number on its own, however, it does not describe very fully the nature of the joint distribution or the relationship between the two investments. We can, however, standardize the covariance and obtain a better descriptor called the correlation coefficient.

7. The Correlation Coefficient: The covariance number is unbounded. Theoretically, its range extends all the way from minus to plus infinity. We can bound it, however, by dividing it by the product of the standard deviations for the two investments. The resulting number is called the correlation coefficient, and it falls within the range -1 to +1. +1 is a characteristic of a perfect positive correlation and -1 is a perfect negative correlation. It is defined as:

$$\rho_{A,B} = \frac{Cov(R_A, R_B)}{\sigma(R_A)\sigma(R_B)}$$

If we square the correlation coefficient, we obtain a number called the coefficient of determination. This number tells us the fraction of the variability in the returns on the one investment that can be associated with variability in the returns on the other. Say the correlation coefficient is +0.90, we can say that approximately 81% of the variability in the returns on stock A can be associated with, or explained by, the returns on stock B.

As an example to find the covariance and correlation, say we have two stocks, A and B in the following table:

<table>
<thead>
<tr>
<th>Month</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stock A</td>
<td>0.04</td>
<td>-0.02</td>
<td>0.08</td>
<td>-0.04</td>
<td>0.04</td>
<td>0.02</td>
</tr>
<tr>
<td>Stock B</td>
<td>0.02</td>
<td>0.03</td>
<td>0.06</td>
<td>-0.04</td>
<td>0.08</td>
<td>0.03</td>
</tr>
</tbody>
</table>


Figure 2.1: TSE 300 and REITs index

So the covariance could be found by calculating the following:
\[
[(0.04 - 0.02)(0.02 - 0.30)] + [(-0.02 - 0.02)(0.03 - 0.03)] + [(0.08 - 0.02)(0.06 - 0.03)] + 
[(-0.04 - 0.02)(-0.04 - 0.03)] + [(0.04 - 0.02)(0.08 - 0.03)] = 0.0068.
\]
So the covariance = 0.0068/5. The correlation is left as an exercise.

**Example 2.1.** In this example, we present data on two series, the TSE 300 and Real Estate Investment Trust index (REITs) constructed by Toronto Stock Exchange. The latter index represents the performance of real state companies. It is simply an indicator of those companies traded on the TSE but sell real estate units to shareholders. Remember the investment assets made by these companies are commercial and residential properties.

We generate the logs of both series, and calculate the continuously rate of returns as: 
\[ r_t = \ln \left( \frac{index_t}{index_{t-1}} \right). \] 
If you like you could multiply by 100 to find the percentage returns. The graphs illustrate the behavior of both series. There is an upward trend in the TSE 300 series, while a downward trend in that of real estate. From a portfolio perspective, you will realize later that the combination of both series might represent a good opportunity for diversification, especially having a correlation coefficient of 0.338, which is less than 1.00.
Now we have the following matrices about covariances and correlations coefficients between the two series:

<table>
<thead>
<tr>
<th></th>
<th>Covariance</th>
<th>Correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>RTSE300</td>
<td>REITs</td>
</tr>
<tr>
<td>RTSE300</td>
<td>7.46E-05</td>
<td>3.07E-05</td>
</tr>
<tr>
<td>REITs</td>
<td>3.07E-05</td>
<td>0.00010</td>
</tr>
</tbody>
</table>

2.2.5. A Note on Continuous Compounding

Continuously compounded interest rates are used to such a great extent when options and other complex derivatives are being priced that it makes sense to get used to working with them now. Consider an amount A invested for n years at an interest rate of R per annum. If the rate is compounded once per annum, the terminal value of the investment is: $A(1 + \frac{R}{m})^n$

If it is compounded m times per annum, the terminal value of the investment is:

$$A(1 + \frac{R}{m})^{nm}$$

suppose that A = $100, R = 10% per annum, and n = 1, so that we are considering one year. When we compound once per annum (m = 1), this formula shows that the $100 grows to: $100 \times 1.1 = $110$
When we compound twice a year \((m = 2)\), the formula shows that the $100 grows to: 
\[ 100 \times 1.05 \times 1.05 = 110.25 \]

When we compound four times a year \((m = 4)\), the formula shows that the $100 grows to: 
\[ 100 \times 1.025^4 = 110.38 \]

The following table shows the effect of increasing the compounding frequency further (i.e., of increasing \(m\)).

The effect of increasing the compounding frequency on the value of $100 at the end of one year when the interest rate is 10% per annum

<table>
<thead>
<tr>
<th>Compounding frequency</th>
<th>Value of $100 at end of one year</th>
</tr>
</thead>
<tbody>
<tr>
<td>Annually ((m = 1))</td>
<td>110.00</td>
</tr>
<tr>
<td>Semiannually ((m = 2))</td>
<td>110.25</td>
</tr>
<tr>
<td>Quarterly ((m = 4))</td>
<td>110.38</td>
</tr>
<tr>
<td>Monthly ((m = 12))</td>
<td>110.47</td>
</tr>
<tr>
<td>Weekly ((m = 52))</td>
<td>110.51</td>
</tr>
<tr>
<td>Daily ((m = 365))</td>
<td>110.52</td>
</tr>
</tbody>
</table>

The limit as \(m\) tends to infinity is known as continuous compounding. With continuous compounding, it can be shown that an amount \(A\) invested for \(n\) years at rate \(R\) grows to:

\[ Ae^{Rn} \]

where \(e\) is the mathematical constant, 2.71828. In the example, \(A = 100\), \(n = 1\), and \(R = 0.1\), so that the value to which \(A\) grows with continuous compounding is:

\[ Ae^{Rn} = 100e^{0.1} = 110.52 \]

This is (to two decimal places) the same as the value using daily compounding. For most practical purposes, continuous compounding can be thought of as being equivalently to daily compounding. Compounding a sum of money at a continuously compounded rate \(R\) for \(n\) years involves multiplying it by \(e^{Rn}\). Discounting it at a continuously compounded rate \(R\) for \(n\) years involves multiplying by \(e^{-Rn}\).

Let \(R_t\) denote the simple (say monthly) monthly return on an investment. The continuously compounded monthly return, \(r_t\) is defined as:

\[ r_t = \ln(1 + R_t) = \ln(P_t/P_{t-1}) \]
where \( \ln(\ ) \) is the natural log function. To see why \( r_t \) is called the continuously compounded return, take exponential of both sides to give the following:

\[
e^{r_t} = 1 + R_t = P_t/P_{t-1}
\]

Rearranging we get:

\[
P_t = P_{t-1}e^{r_t}
\]

so that \( r_t \) is the continuously compounded growth rate in prices between months \( t-1 \) and \( t \). This is to be contrasted with \( R_t \) which is the simple growth rate in prices between months \( t-1 \) and \( t \) without any compounding. Furthermore, since \( \ln(x/y) = \ln(x) - \ln(y) \), it follows that:

\[
r_t = \ln(P_t/P_{t-1}) = \ln(P_t) - \ln(P_{t-1}) = p_t - p_{t-1}
\]

where \( p_t = \ln(P_t) \). Hence, the continuously compounded monthly return, \( r_t \) can be computed simply by taking the first difference of the natural logarithm of monthly prices.

### 2.2.6. Logarithms

The Plumber’s Helper for Financial Research. When we consider security returns, especially in the context of the distribution from which they came, logarithms are helpful for several reasons. For one thing, logarithms reduce the impact of extreme values that might distort the true distribution. Takeovers rumors, for instance, sometimes cause huge price swings, both up and down, in the value of a particular security. A stock that has not moved for ninety-nine days but doubles in one day will show an average daily gain of 1 percent over this one hundred-day period. If we take the logarithm of daily returns over this one hundred-day period, the average daily logreturn is 0.69 percent. This lower figure lessens the impact of the one big return. A logreturn is the logarithm of a return.

Logarithms also make other statistical tools more appropriate. The theory behind linear regression, for instance, assumes normally distributed random variables. Analysts frequently use linear regression of security returns on market returns to estimate beta. Logarithms reduce the effect of extraordinary deviations from normality. Any time analysts are working on stock return distributions, it is good practice to take the raw returns, convert them to return relatives, and then take the natural logarithm of the return relatives. Then you treat the logreturns as you would any other value for calculating statistics such as variance or correlation.
Further, log-returns have the nice property that they can be interpreted as continuously compounded returns—so that the frequency of compounding of the return does not matter and thus returns across assets can more easily be compared. That makes the continuously compounded returns to be time-additive. For example, suppose that a weekly returns series is required and daily log returns have been calculated for 5 days, numbered 1 to 5 representing the returns on Monday through Friday. It is valid to simply add up the 5 daily returns to obtain the return for the whole week:

- **Monday return:** \( r_1 = \ln\left(\frac{p_1}{p_0}\right) = \ln p_1 - \ln p_0 \)
- **Tuesday return:** \( r_2 = \ln\left(\frac{p_2}{p_1}\right) = \ln p_2 - \ln p_1 \)
- **Wednesday return:** \( r_3 = \ln\left(\frac{p_3}{p_2}\right) = \ln p_3 - \ln p_2 \)
- **Thursday return:** \( r_4 = \ln\left(\frac{p_4}{p_3}\right) = \ln p_4 - \ln p_3 \)
- **Friday return:** \( r_5 = \ln\left(\frac{p_5}{p_4}\right) = \ln p_5 - \ln p_4 \)

**Return over the week:** \( \ln p_5 - \ln p_0 = \ln\left(\frac{p_5}{p_0}\right) \)

### 2.2.7. Psychic return

It is something that people frequently experience. It comes from an individual disposition about something. Why, for instance, do so many people in the investment business wear Rolex watches? Why do people like designer jeans that may have no better quality than pants that cost one-third as much? A $20 watch will keep time to within a second a month. Is the Rolex more accurate? Are the designer jeans three times as comfortable? The answer, of course, is that people get utility from the watch and the jeans. They feel good about themselves when they wear these things. These benefits are called *psychic return* and are very real.

### Part II

**Determinants of rates of return**

#### 2.3. The real-risk free rate

*The Real risk free rate* is the basic interest rate if there were no uncertainty at all about the future. It is also called the *pure time value of money.*

Factors that affect this rate include the following:

1. A subjective factor that affects the RFR is the time preference individuals have for current consumption over future consumption.
2. An objective factor that affects the RFR is the set of investment opportunities in the country. This set in turn is affected by real growth.
3. Real growth is a function of:
   a. The long run growth of the labor force
   b. The long run growth in the number of hours worked
   c. The long run growth in the productivity of the labor force

2.4. Factors influencing the nominal risk free rate of return

The nominal risk-free rate of interest is one stated in money, not real, terms. Therefore the nominal rate is affected by all those factors influencing RFR, and also by other factors, especially conditions in the capital markets and expected inflation.

1. Conditions in the capital market
   a. The cost of funds at any given time is the interest rate that sets the supply of funds equal to the demand for funds.
   b. This rate can be affected by changes in monetary policy that affect the supply of funds available, among other factors.

2. Expected inflation
   a. If the price level is expected to change over the life of an investment, then an investor will try to be compensated for the loss of his purchasing power.
   b. In order to compensate an investor for the loss of purchasing power, the nominal rate can be found by the following:

\[ \text{Nominal RFR} = (1 + \text{Real RFR})(1 + \text{Rate of Inflation}) - 1 \]

If the real risk free rate is 4%, then the nominal risk free rate would be:
\[ \text{Nominal RFR} = (1 + 0.04)(1 + 0.10) - 1 = (1.04)(1.10) - 1 = 1.1440 - 1 = 0.1440, \text{ or } 14.4\% \]

c. One can turn this equation around to solve for the real risk free rate, given the nominal risk free rate and the rate of inflation

\[ 1 + \text{Real RFR} = \frac{(1 + \text{Nominal Risk Free Rate})}{(1 + \text{Rate of Inflation})} - 1 \]

For example, suppose the nominal risk free rate were 10% and the rate of inflation were 6%, then
\[ \text{Real RFR} = \frac{1.1}{1.06} - 1 = 0.0377 \text{ or } 3.77\% \]

3. Common effects: all the factors to this point are assumed to affect all investments equally.
2.5. Risk Premium

That is the increase in the required rate of return over the risk-free rate to compensate the investor for any uncertainty.

1. Business risk: The uncertainty of income flow stemming from the nature of the firm's business operations. Investors demand a risk premium based on the uncertainty caused by the basic business of the firm.

2. Financial risk: The uncertainty introduced by the method of financing. Borrowing requires fixed payments which must be paid ahead of payments to stockholders. The use of debt increases uncertainty of stockholders incomes and causes an increase in the stock's risk premium.

3. Liquidity risk: The uncertainty associated with the secondary market for the security. It is the ability to buy or sell an asset quickly without a substantial change in price, and it has two dimensions:
   a. How quickly can the asset be bought or sold;
   b. How close to its current price can it be bought or sold.

4. Exchange rate risk: The uncertainty of returns associated with investing in a currency other than one's home currency. Changes in exchange rates affect the investors return when converting an investment back into the “home” currency.

5. Country risk, or political risk: The uncertainty of returns associated with changes in the political or economic environment of a country. Individuals who invest in countries that have unstable political-economic systems must include a country risk-premium when determining their required rate of return.

6. These factors determine the firm’s fundamental risk, which in turn determines its risk premium:

\[ \text{Risk Premium} = f(\text{Business Risk, Financial Risk, Liquidity Risk, Exchange Rate Risk, Country Risk}) \]

2.6. Risk Premium and Portfolio Theory

A different view of risk was developed by Markowitz, Sharpe and others.

1. Investors should use a measure of risk that is based on the external market.

2. Under certain assumptions, the correct measure of risk is the comovement of the security with the market, i.e., its systematic risk.

3. The component of a security’s total risk that is not related to the market is called unsystematic risk, and can be diversified away.

4. Therefore the risk premium for an asset is a function of its systematic risk only.

\[ \text{Risk Premium} = f(\text{Systematic Market Risk}) \]
Note: Studies of the relationship between market measures of risk and accounting variables that determine fundamental measures of risk find that the two risk concepts are closely related. This is to be expected in a well-functioning market. Therefore, the risk premium can be viewed by either of the specifications given above.

2.7. Relationship between Risk and Return

A. Security market line: the risk-return combinations available to investors in the market for all risky securities.

B. Movement along the SML

A security’s position on the SML can change if any source of risk changes.

C. Changes in the Slope of the SML

1. The risk premium for any risky asset is given by: \( \text{RP}_i = E(R_i) - \text{NRFR} \)

   \( \text{RP}_i \) = risk premium for asset \( i \);

   \( E(R_i) \) = expected return for asset \( i \);

   \( \text{NRFR} \) is the nominal return on a risk-free asset.

2. One portfolio on the SML is the market portfolio that contains all risky assets. Its risk premium is:

   \( \text{RP}_m = E(R_m) - \text{NRFR} \)

   where “\( m \)” is the market portfolio.

3. The market risk premium is not constant over time, because the slope of the SML changes. This change in \( \text{RP}_m \) is represented by the SML pivoting on the RFR along the vertical axis.

Summary of changes in the required rate of return

1. A movement along the SML reflects changes in the risk characteristics of a specific investment.

2. A change in the slope of the SML will occur due to a change in investor attitudes toward risk. This will affect all risky investments, but not all in the same manner or degree.

3. A shift in the SML results from a change in market conditions. This will affect all investments.

Summary: before investing, a person should have a clear-cut goals in mind. One should also be fully aware of the constraints that exist, as well as of the risk factors that are present. In this context, we outline the procedure any investor should follow in setting up an investment program.
2.8. The Asset Allocation Decision

Questions to be answered: are

1. What is asset allocation?
2. What are the four steps in the portfolio management process?
3. What is the role of asset allocation in investment planning?
4. Why is a policy statement important to the planning process?
5. What objectives and constraints should be detailed in a policy statement?
6. How and why do investment goals change over a person’s lifetime and circumstances?
7. Why do asset allocation strategies differ across national boundaries?

Asset Allocation is the process of deciding how to distribute an investor’s wealth among different countries and asset classes for investment purposes. An investment class is comprised of securities that have similar characteristics, attributes, and risk/return relationships.

2.9. Individual Investor Life Cycle

2.9.1. The Preliminaries

1. Life Insurance: Term life - Provides only death benefit; Universal and Variable Life - Provides both death benefits and a saving plan. Other forms of insurance include Health insurance to pay medical bills, disability insurance to provide an income, automobile and liability insurance to pay for damage to your home or automobile.

2. Cash Reserve: A safety cushion against unforeseen needs or events. Should be about six-month living expense reserve or having liquid investments that could be easily converted to cash without loss of value.

2.9.2. Individual Investor Life Cycle

It has the following four phases:

1. Accumulation phase: it is from early to middle years, where individuals are acquiring assets to meet short-term needs. Individuals in the accumulation phase are willing to make moderately high-risk investments in the hopes of making above-average nominal returns over time.
2. Consolidation phase: where earnings exceed expenses, but individuals are past the career midpoint. Individuals in this phase have some concerns about capital preservation, they do not have very large risks that may put their current nest egg in jeopardy.
3. **Spending phase**: starts at retirement where expenses are covered by social security and pensions and other investments.

4. **Gifting phase**: may be concurrent with spending phase. Individuals set up trusts and provide gifts for family and friends.

**Life Cycle investment goals** include the following:

a. Near-term, high priority: shorter-term financial objectives: that is accumulating funds to make a house down payment, buy a new car, or take a trip. Parents with teenage children might have goals to accumulate funds to help pay college expenses. High risk investments are not suitable for these objectives.

b. Long-term, high priority: typically include retirement planning. High risk investments can be used to help meet these objectives.

c. Lower priority goals: Vacations, redecorating, etc.

2.10. The Portfolio Management Process

2.10.1. Policy Statement:

1. Policy statement specifies the types of risks the investor is willing to take and the investors goals and constraints.

2. All investment decisions should be consistent with the policy statement.

3. The policy statement should be reviewed and updated from time to time.

The advantages about having a policy statement that it makes the investor understand the needs, objectives and constraints and forces him or her to confront the risks inherent in investing. In other words too, it guides all investment decisions and discipline the investment process and protects the investor from inappropriate actions by the manager.

2.10.2. Examination of conditions:

1. Financial and economic conditions should be analyzed and forecast of future conditions made.

2. These forecasts need to be updated periodically because of changing conditions in the markets.

   This helps in determining strategies in meeting the investor’s goals.

2.10.3. Portfolio Construction:

1. The portfolio should be constructed in accordance with the risk and return objectives stated in the policy.

2. The other constraints must also come into play here.
2.10.4. Feedback Loop and Continual Monitoring:

1. One must continually examine financial and economic conditions.
   2. When necessary, the policy statement should be revised.
   3. Performance should be measured and compared to a benchmark.

By constructing a policy statement, we help answering the following concerns:

- What are the real risks of an adverse financial outcome, especially in the short-run?
- What probable emotional reactions will I have to an adverse financial outcome?
- How knowledgeable am I about investments and markets?
- What other capital or income sources do I have? how important is this particular portfolio to my overall financial position?
- What, if any, legal restrictions may affect my investment needs?
- What, if any, unanticipated consequences of interim fluctuations in portfolio value might affect my investment policy?

2.11. Input to the Policy Statement

2.11.1. Investment Objectives

Investors must state goals in terms of both risk and return

- **Capital preservation**: that is minimize the risk of a real loss. They seek to maintain the purchasing power of their investment. In other words, the return needs to be no less than the rate of inflation.

- **Capital appreciation**: an aggressive strategy for portfolio growth where capital gains provide real growth over time for future needs. That is buying assets at a low price and selling them later at a higher price. Generally, longer-term investors seeking to build a retirement or college education fund may have this goal.

- **Current income**: emphasis on income over capital gains and generate spendable funds. Retirees may favor this objective for part of their portfolio to help generate spendable funds.

- **Total return**: it is a balance between capital appreciation and current income with moderate risk exposure.
2.11.2. Investment Constraints

- **Liquidity needs:** investors should be thinking about their liquidity needs.

- **Time horizon:** that is longer time horizon favors risk acceptability and short time horizon favors less risky investments because losses are harder to overcome in a short time frame.

- **Tax concerns:** interest and dividends are taxed at investor’s marginal tax rate. Interest on municipal bonds exempt from federal income tax and from state of issue. Interest on federal securities exempt from state income tax and contributions to an RRSP may qualify as deductible from taxable income.

- **Legal and regulatory factors:** like some investments prohibit insider trading.

- **Unique needs and Preferences:** such as personal preferences.

2.12. The Importance of Asset Allocation

An investment strategy is based on four decisions:

1. What asset classes to consider for investment.
2. What normal or policy weights to assign to each eligible class.
3. The allowable allocation ranges based on policy weights.
4. What specific securities to purchase for the portfolio

Most (85% to 95% of the overall investment return is due to the first two decisions, not the selection of individual investments.

3. Statement Policy

In this part, we present an example of policy statement that includes some objectives and constraints. For example, we take the following policy of All Souls Congregational Church that describes its endowment fund. In the following, we describe its purpose and philosophy, responsibilities, objectives and constraints (Sources: *Association for Investment Management and Research AIMR, Investment Policy* (Tokyo, Japan: Seminar Proceedings, April, 1994, 18-20).

3.1. General Purpose and Philosophy

There are two primary purposes to the All Souls Congregational Church endowment fund:
1. To provide regular source of funds for maintenance for the church.

2. To provide stability within the operating budget when short-term funding are inadequate.

The philosophy of the fund is that it should be managed as to preserve the purchasing power of the assets through time and provide for a consistent flow of income to the church. Also, future generations should benefit from the endowment at the same level as the current generation.

3.2. Responsibilities

The responsibilities of the fund is in the hands of the Board of Trustees, Investment advisory committee, and Treasury. The board of trustees set explicit investment policies consistent with the objectives of the fund; define appropriate long-range objectives; and ensure that the Investment advisory committee follows the established rules. The Investment advisory committee may periodically advice the board on investment policy and meet at least quarterly to discuss the actual management of the fund. The treasurer has the responsibility for any subsidiary or fund accounting. The treasurer and the Investment advisory board will jointly determine the calculation of return allocations to any such subsidiary funds.

3.3. Objectives

The fund should be managed on a total return basis. The long-term annual rate of return objective, including any distribution to the general fund, is 11% annually.

3.4. Constraints

3.4.1. Eligible Asset classes

The Investment advisory committee may use any of the following asset classes:


3. Corporate bonds in companies whose debt is rate at least BB by Standard & Poor’s, and convertible preferred stock.

4. Foreign fixed income securities, and foreign equity securities, including those traded via American Depositary Receipts (ADRs).
5. Mutual funds, and closed-end investment companies.

The following types of transactions are expressly prohibited: Short sales; commodities; Venture capital; Purchase of securities on margin and direct real estate investment.

3.4.2. Asset Allocation

Because of the long-term horizon, the fund should be predominately invested in equity securities. An allocation of less than 50% of the fund assets to equities must be approved by the Board of Trustees. As much as 15% of the entire portfolio may be invested in foreign securities. In general, no more than 5% of the fund assets should be held in the equity securities of any one company.

3.4.3. Income

In general, the annual allocation in support of the general operating fund of the church will be no more than 6% of the average of the year-end value of the endowment fund, less the value of the subsidiary funds, over the past three calendar years.

3.5. Reporting

The Investment advisory committee will report verbally and in writing to the congregation at each annual meeting of the church and report verbally to the Board of Trustees at least twice a year.