Question 1

Suppose you have invested only in two stocks, A and B. You expect that returns on the stocks depend on the following three states of economy, which are equally likely to happen.

State of Economy	Return on Stock A (%)	Return on Stock B (%)
Bear	7.3%	-4.7%
Normal	11.5	5.4
Bull	16.6	24.3

1. Calculate the expected return of each stock.
2. Calculate the standard deviation of returns of each stock.
3. Calculate the covariance and correlation between the two stocks.

Question 2

There are two stocks in the market, stock A and stock B. The price of stock A today is \$50. The price of stock A next year will be \$40 if the economy is in a recession, \$55 if the economy is normal, and \$60 if the economy is expanding. The attendant probabilities of recession, normal times, and expansion are 0.2, 0.6, and 0.2, respectively. Stock A pays no dividend.

Assume the CAPM is true and that $\beta_i = Cov(i, M)/\sigma_M^2$ where i is a risky asset and M is the market portfolio. Other information about the market includes: SD(R_M) = standard deviation of the market portfolio = 0.10; SD(R_B) = standard deviation of stock's B =0.12; $\overline{R_B}$ = expected return on stock B=0.09; Corr (R_A,R_M) = the correlation of stock A and the market = 0.8; Corr (R_B,R_M) = the correlation of stock B and the market = 0.2; Corr (R_A,R_B) = the correlation of stock A and stock B = 0.6;

a. If you are a typical risk-averse investor, which stock would you prefer? Why?

b. What are the expected return and standard deviation of a portfolio consisting of 60% of stock A and 40% of stock B?

c. What is the beta of the portfolio in part (b)?

Question 3

Miss Maple is considering two securities, A and B, and the relevant information is given below:

State of Economy	Probability	Return on Security A (%)	Return on Security B (%)
Bear	0.6	3.0%	6.5%
Bull	0.4	15.0%	6.5

1. Calculate the expected returns and standard deviations of the two securities.

2. Suppose Miss Maple invested \$2,500 in Security A and \$3,500 in security B. Calculate the expected return and standard deviation of her portfolio.

3. Suppose Miss Maple borrowed from her friend 40 shares of security B, which is currently sold at \$50, and sold all shares of the security. (She promised her friend she would pay her back in a year with the same number of shares of security B.). Then she bought security A with the proceeds obtained in the sales of security B shares and the cash of \$6,000 she owned. Calculate the expected return and standard deviation of the portfolio.

Question 4

Suppose the current risk-free is 7.6 percent. Potpourri Inc. stock has a beta of 1.7 and an expected return of 16.7 percent. (Assume the CAPM is true)

a. What is the risk premium on the market?

b. Magnolia Industries stock has a beta of 1.8. What is the expected return on the Magnolia stock?

c. Suppose you have invested $100,000 in a portfolio of Potpourri and Magnolia, and the beta of the portfolio is 1.77. How much did you invest in each stock? what is the expected return on the portfolio?

Question 5

Consider the following two stocks:

Stock i	Beta	Expected Return
A	1.4	25%
B	0.7	14%

Assume the CAPM holds. Based upon the CAPM, what is the return on the market? What is the risk-free rate?

Question 6

1. If a portfolio has a positive weight for each asset, can the expected return on the portfolio be greater than the return on the asset in the portfolio that has the highest return? Can the expected return on the portfolio be less than the return on the asset in the portfolio with the lowest return? Explain.

2. Comment on the following quotation from a leading investment analyst.

Stocks that move perfectly with the market have a beta of 1. Betas get higher as volatility goes up and lower as it goes down. Thus, Southern Co, a utility whose share have traded close to $12 for most of the past three years, has a low beta. At the other extreme, there is True North Networks, which has been as $150 and as low as its current $15.

3. Given the following situations, determine in each case whether or not the hypothesis of an efficient capital market (semistrong form) is contradicted.

a) Through the introduction of a complex computer program into the analysis of past stock price changes, a brokerage firm is able to predict price movements well enough to earn a consistent 3% profit, adjusted for risk, above normal market returns.

b) On the average, investors in the stock market this year are expected to earn a positive return (profit) on their investment. Some investors will earn considerably more than others.

c) You have discovered that the square root of any given stock price multiplied by the day of the month provides an indication of the direction in price movement of that particular stock with a probability of 0.7.

4. What sort of investor rationally views the variance (or standard deviation) of an individual security's return as the security's proper measure of risk? What sort of investor rationally views the beta of a security as the security's proper measure of risk?

Answer 1:

1. E(R_A) $= (7.3 + 11.5 + 16.6)/3 = 11.8\%$

E(R_B) $= (-4.7 + 5.4 + 24.3)/3 = 8.3\%$

2. $\sigma_A^2 = \{(0.073 - 0.118)^2 + (0.115 - 0.118)^2 + (0.166 - 0.118)^2\}/3 = 0.001446$

$\sigma_A = (0.001446)^{1/2} = 0.0380 = 3.80\%$

$\sigma_B^2 = \{(-0.047 - 0.083)^2 + (0.054 - 0.083)^2 + (0.243 - 0.083)^2\}/3 = 0.014447$

$\sigma_B = (0.014447)^{1/2} = 0.1202 = 12.02\%$

3. Cov (R_A,R_B) $= [(0.073-0.118)(-0.047-0.083)+(0.115-0.118)(0.054-0.083)+(0.166-0.118(0.243-0.083)]/3 =$

$=[0.00585+0.000087+0.00768]/3=0.004539$

Corr (R_A,R_B) $= 0.004539/(0.0389 * 0.1202) = 0.9937.$

Answer 2:

The typical risk-averse investor seeks high returns and low risks. To assess the two stocks, find the risk and return profiles for each stock.

State of Economy	Pr.	R_A	$R_A - E(R_A)$	$(R_A - E(R_A))^2$	$P\times(R_A - E(R_A))^2$.
Recession	0.2	-0.2	-0.26	0.0676	0.01352
Normal	0.6	0.10	0.04	0.0016	0.00096
Expansion	0.2	0.20	0.14	0.0196	0.00392

* since security A pays no dividend, the return on A is simply $(P_1/P_0) - 1$.

E(R_A) $= 0.2(-0.20) + 0.6(0.10) + 0.2(0.20) = 0.06$

E(R_B) $= 0.09$ This is given in the problem.

Risk is calculated in table : Var(R_A) $= 0.0184$

Standard deviation is $(0.0184)^{1/2} = 0.1356$

$\beta_A = \{\text{Corr}(R_A, R_M)\sigma(R_A)\}/\sigma(R_M) = 0.8(0.1356)/0.10 = 1.085$

$\beta_B = \{\text{Corr}(R_B, R_M)\sigma(R_B)\}/\sigma(R_M) = 0.2(0.12)/0.10 = 0.24$

The return on stock B is higher than the return on stock A. The risk of stock B, as measured by its beta, is lower than the risk of A. Thus, a typical risk-averse investor will prefer stock B.

b. E(R_P) $= 0.6E(R_A) + 0.4E(R_B) = 0.6(0.06) + 0.40(0.09) = 0.0576$

$\sigma_P^2 = (0.6)^2\sigma_A^2 + (0.4)^2\sigma_B^2 + 2(0.6)(0.4)\text{Corr}(R_A, R_B)\sigma_A\sigma_B = 0.01361595$

$\sigma_P = 0.116687$

c. The beta of a portfolio is the weighted average of the betas of the components of the portfolio:

$\beta_P = (0.6)\beta_A + (0.4)\beta_B = (0.6)(1.085) + (0.4)(0.240) = 0.747$

Answer 3.

1. E(R_A) $= 0.6(0.03) + 0.4(0.15) = 0.078 = 7.80\%$

E(R_B) $= 0.6(0.065) + 0.4(0.065) = 6.5\%$

$\sigma_A^2 = 0.6(0.03 - 0.078)^2 + 0.4(0.15 - 0.078)^2 = 0.003456$

$\sigma_A = (0.003456)^{1/2} = 0.05878$

$\sigma_B^2 = \sigma_B = 0$

2. $W_A = \$2,500/\$6,000 = 0.417$

$W_B = 1 - 0.417 = 0.583$

E(R_P) $= 0.417(0.078) + 0.583(0.065) = 0.0704 = 7.04\%$

$\sigma_P^2 = W_A^2\sigma_A^2 = 0.0006$

$\sigma_P = (0.0006)^{1/2} = 0.0245 = 2.45\%$

3. Amount borrowed = -40*$50=$-2000

$W_A = \$8,000/\$6,000 = 4/3$

$W_B = 1 - W_A = -1/3$

E(R_P) $= (4/3)(0.078) + (-1/3)(0.065) = 0.0823 = 8.23\%$

$\sigma_P^2 = W_A^2\sigma_A^2 = (4/3)^2(0.003456) = 0.006144$

$\sigma_P = (0.006144)^{1/2} = 0.07838$

Question 4:

a. The risk premium $= R_m - R_f$

Potpourri stock return: $16.7 = 7.6 + 1.7[R_m - R_f]$, then $[R_m - R_f] = [16.7 - 7.6]/1.7 = 5.3529\%$

b. E(R_{Mag}) $= 7.6 + 1.8(5.353) = 17.2353\%$

c. $W_{Pot}\beta_{Pot} + W_{Mag}\beta_{Mag} = 1.77$

$1.7W_{Pot} + 1.8(1 - W_{Pot}) = 1.77$

$0.1W_{Pot} = 0.03$, then $W_{Pot} = 0.3$ and $W_{Mag} = 0.7$
Thus invest \$30,000 in Potpourri stock and \$70,000 in Magnolia.
$E(R_P) = 7.6 + 1.77(5.353) = 17.07\%$
Note: the other way to calculate $E(R_P) = 0.30(0.167) + 0.7(0.17235) = 17.07\%$

Question 5:

$0.25 = R_f + 1.4[R_m - R_f]$
$0.14 = R_f + 0.7[R_m - R_f]$
Substract the second equation from the first, we get: $0.11 = 0.4[R_m - R_f]$ or $[R_m - R_f] = 0.1571$
Put the above equation in any of the previous two ones for either asset A or Asset B, so we get:
$0.25 = R_f + 1.4[0.1571]$ and therefore $R_f = 3\%$
Use $[R_m - R_f] = 0.1571$ to find R_m. $R_m = 0.1571 + 0.03 = 18.71\%$.

Answer 6:

1. The expected return on any portfolio must be less than or equal to the return on the stock with the highest return. It cannot be greater than this stock's return because all stocks with lower returns will pull down the value of the weighted average return. Similarly, the expected return on any portfolio must be greater than or equal to the return of the asset with the lowest return. The portfolio return cannot be less than the lowest return in the portfolio because all higher earnings stocks will pull up the vlaue of the weighted average.

2. If we assume that the market has not stayed constand during the past three years, then the low volatility of Southern Co.'s stock price only indicates that the stock has a beta that is very near to zero. The high volatility of Texas Instruments' stock price does not imply that the firm's beta is high. Total volatility (the price fluctuation) is a function of both systematic and unsystematic risk. The beta only reflects the systematic risk. Observing price volatility does not indicate whether it was due to systematic factors, or firm specific factors. Thus, if you observe a high price volatility like that of TNN, you cannot claim that the beta of TNN's stock is high. All you know is that the total risk of TNN is high.

3 a) The information which enables the brokerage firm to earn a consistent 3% abnormal profit is not costless. If the computer costs exceed the excess 3% profits from stocks, the firm is actually earning worse than normal returns. If the computer costs are less than the 3% profit, semi-strong capital market efficiency may be refuted. Also, brokerage fees may wipe out any trading profits.

b) The hypothesis of an efficient capital market is not contradicted. Except for very bad years, the average (and expected) return on the market is positive. This is considered a normal return. It is also a fair game. The fact that some investors enjoy higher returns than others is the result of the uncertainty in stock returns. Given any probability distribution, some observations will lie above the mean and some will lie below.

c) Semi-strong (as well as weak) capital market efficiency is contradicted. You have discovered a trading rule based on past, nearly costless, price information that enables you to forecast future prices with better-than-random accuracy. Thus, all relevant publicly available information has not been instantaneously incorporated into stock prices.

4. A good answer might be something like the following:

A rational, risk-averse investor views the variance (or standard deviation) of her portfolio's return as the proper measure of the risk of her portfolio. If for some reason or another the investor can hold only one security, the variance of that security's return becomes the variance of the portfolio's return. Hence, the variance of the security's return is the security's proper measure of risk.

If an invidividual holds a diversified portfolio, she still views the variance (or standard deviation) of her portfolio's return as the proper measure of the risk of her portfolio. However, she is no longer interested in the variance of each individual security's return. Rather, she is interested in the contribution of an individual security to the variance of the portfolio.

Under the assumption of homogeneous expectations, all individuals hold the market portfolio. Thus, we measure risk as the contribution of an individual security to the variance of the market portfolio. This contribution, when standardized properly, is the beta of the security. While very few investors hold the market portfolio exactly, many hold reaonsably diversified portfolios. These portfolios are close enough to the market portfolio so that the beta of a security is likely to be a reasonable measure of its risk.