

Question 1

You are interested in forming a portfolio based on two securities with the following characteristics:

Parameter	A	B	
Expected Return	12%	18%	
Standard Deviation	25%	40%	
Correlation coefficient			0.8

- Calculate the expected return and standard deviation for the equally weighted portfolio.
- Would you choose to invest in this portfolio or invest in a single security (either A or B) and why.

Question 2

There are two securities with the following parameters:

Stock i	Expected Return	Standard Deviation of i
A	15%	40%
B	5%	0%

- What would be the correlation coefficient between A and B.
- If you constructed a portfolio with a standard deviation of 20%, what would be the weight in A and B.
- What would be the expected return on this portfolio.

Question 3

Bersshire Hathaway's highly successful CEO, warrrne Buffett, is one of the harshest critics of modern portfolio theory and diversification. Buffet has said, for example, that "[I]f you are a know-something investor, able to understand business economics, and find 5 to 10 sensibly priced companies that possess important long-term competitive advantages, conventional diversification makes no sense for you. It is apt simply to hurt your results and increase your risk".

Does Buffet avoid diversification altogether? Certainly his investment record is phenomenal, but even Buffett engages in diversification to some extent. For example, consider Bershire Hathaway's top three investment holdings at the end of 2002.

American Express Company	\$5.6 billion (32%)
The Coca-Cola Company	\$8.8 billion (51%)
The Gillette Company	\$2.9 billion (17%)
Total	\$17.3 billion

How much diversification do these three stocks provide? How much lower is this portfolio's standard deviation than that of a portfolio consisting only of Coca-Cola stock?

To answer these questions, assume that the historical mean returns, return standard deviations, and return correlations of these stocks are the best estimates of the future expected returns, return standard deviations, and return correlations. The following Table shows these historical statistics, based on monthly return data from 1990 through 2002.

	American Express	Coca-Cola	Gillette
Mean annual return	16.0%	16.1%	17.6%
Standard deviation	29.0%	24.7%	27.3%
Correlations	American Express/Coca-Cola	American Express/Gillette	Coca/Gillette
	0.361	0.317	0.548

Question 4

You have \$100,000 to invest in a portfolio containing Stock X, Stock Y, and a risk-free asset. You must invest all of your money. Your goal is to create a portfolio that has an expected return of 11.5% and that has only 70% of the risks of the overall market. If X has expected return of 30% and a beta of 1.60, Y has an expected return of 20% and a beta of 1.25, and the risk-free rate is 6%, how much money will you invest in stock X? How do you interpret your answer?

Question 5

1. If a portfolio has a positive weight for each asset, can the expected return on the portfolio be greater than the return on the asset in the portfolio that has the highest return? Can the expected return on the portfolio be less than the return on the asset in the portfolio with the lowest return? Explain.

2. Comment on the following quotation from a leading investment analyst.

Stocks that move perfectly with the market have a beta of 1. Betas get higher as volatility goes up and lower as it goes down. Thus, Southern Co, a utility whose share have traded close to \$12 for most of the past three years, has a low beta. At the other extreme, there is True North Networks, which has been as \$150 and as low as its current \$15.

3. What sort of investor rationally views the variance (or standard deviation) of an individual security's return as the security's proper measure of risk? What sort of investor rationally views the beta of a security as the security's proper measure of risk?

Answers (Note: these are brief answers to the questions, and therefore, more detailed answers should be provided to illustrate your calculations and explanations).

Question 1:

- a. Expected return = 15.00%
Standard deviation = 30.92 %.
- b. It would depend upon my risk aversion. It is possible that I would invest in individual securities if I were extremely risk averse (A) and risk loving (B).

Question 2:

- a. 0.00
- b. Weight on A = 1/2 and weight on B = 1/2.
- c. 10%.

Question 3:

The Table shows that for Coca-Cola's stock during this period, the mean annual return was 16.1% and the annualized standard deviation of the return was 24.7%. In contrast, a portfolio consisting of 32% American Express stock, 51% Coca-Cola stock and 17% Gillette stock has an expected return of $0.32(0.16) + 0.51(0.161) + 0.17(0.176) = 0.163$, or 16.3%.

The Portfolio's expected standard deviation, based on these weights and the statistics provided in the Table was:

$$\sigma_P = [w_1^2(\sigma_1)^2 + w_2^2(\sigma_2)^2 + w_3^2(\sigma_3)^2 + 2w_1w_2\rho_{12}\sigma_1\sigma_2 + 2w_1w_3\rho_{13}\sigma_1\sigma_3 + 2w_2w_3\rho_{23}\sigma_2\sigma_3]^{1/2}$$

or

$$\sigma_P = [0.32^2(0.290)^2 + 0.51^2(0.247)^2 + 0.17^2(0.273)^2 + 2(0.32)(0.51)(0.361)(0.290)(0.247) + 2(0.32)(0.17)(0.317)(0.290)(0.273) + 2(0.17)(0.51)(0.548)(0.273)(0.247)]^{1/2} = 0.210$$

or 21.0%

The standard deviation of a portfolio with these three stocks is only $21.0/24.7 = 85.0\%$ of the standard deviation of a portfolio composed exclusively of Coca-Cola stock. Therefore, Berkshire Hathway actually achieved substantial diversification in the sense of risk reduction, even considering only its top three holdings.

Question 4:

$$E(R_P) = 0.115 = w_x(0.3) + w_y(0.2) + (1 - w_x - w_y)(0.06)$$

$$\beta_P = 0.7 = w_x(1.6) + w_y(1.25) + (1 - w_x - w_y)(0)$$

solving these two equations in two unknowns gives $w_x = -0.3848684$
 $w_y = 1.05263$
 $w_r = 0.3322368$
 and the amount of stock X to sell short = $0.3848684(\$100,000) = \$38,486.84$

Question 5:

1. The expected return on any portfolio must be less than or equal to the return on the stock with the highest return. It cannot be greater than this stock's return because all stocks with lower returns will pull down the value of the weighted average return. Similarly, the expected return on any portfolio must be greater than or equal to the return of the asset with the lowest return. The portfolio return cannot be less than the lowest return in the portfolio because all higher earnings stocks will pull up the value of the weighted average.

2. If we assume that the market has not stayed constant during the past three years, then the low volatility of Southern Co.'s stock price only indicates that the

stock has a beta that is very near to zero. The high volatility of Texas Instruments' stock price does not imply that the firm's beta is high. Total volatility (the price fluctuation) is a function of both systematic and unsystematic risk. The beta only reflects the systematic risk. Observing price volatility does not indicate whether it was due to systematic factors, or firm specific factors. Thus, if you observe a high price volatility like that of TNN, you cannot claim that the beta of TNN's stock is high. All you know is that the total risk of TNN is high.

3. A good answer might be something like the following:

A rational, risk-averse investor views the variance (or standard deviation) of her portfolio's return as the proper measure of the risk of her portfolio. If for some reason or another the investor can hold only one security, the variance of that security's return becomes the variance of the portfolio's return. Hence, the variance of the security's return is the security's proper measure of risk.

If an individual holds a diversified portfolio, she still views the variance (or standard deviation) of her portfolio's return as the proper measure of the risk of her portfolio. However, she is no longer interested in the variance of each individual security's return. Rather, she is interested in the contribution of an individual security to the variance of the portfolio.

Under the assumption of homogeneous expectations, all individuals hold the market portfolio. Thus, we measure risk as the contribution of an individual security to the variance of the market portfolio. This contribution, when standardized properly, is the beta of the security. While very few investors hold the market portfolio exactly, many hold reasonably diversified portfolios. These portfolios are close enough to the market portfolio so that the beta of a security is likely to be a reasonable measure of its risk.