**Question 1:**
Suppose you have invested only in two stocks, A and B. You expect that returns on the stocks depend on the following three states of economy, which are equally likely to happen.

<table>
<thead>
<tr>
<th>State of Economy</th>
<th>Return on Stock A (%)</th>
<th>Return on Stock B (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bear</td>
<td>7.3%</td>
<td>-4.7%</td>
</tr>
<tr>
<td>Normal</td>
<td>11.5</td>
<td>5.4</td>
</tr>
<tr>
<td>Bull</td>
<td>16.6</td>
<td>24.3</td>
</tr>
</tbody>
</table>

1. Calculate the expected return of each stock.
2. Calculate the standard deviation of returns of each stock.
3. Calculate the covariance and correlation between the two stocks.

**Question 2:***
Consider the following set of returns for assets X and Y:

<table>
<thead>
<tr>
<th>Probability</th>
<th>0.2</th>
<th>0.2</th>
<th>0.2</th>
<th>0.2</th>
<th>0.2</th>
</tr>
</thead>
<tbody>
<tr>
<td>X_i</td>
<td>11%</td>
<td>9</td>
<td>25</td>
<td>7</td>
<td>-2</td>
</tr>
<tr>
<td>Y_i</td>
<td>-3%</td>
<td>15</td>
<td>2</td>
<td>20</td>
<td>6</td>
</tr>
</tbody>
</table>

a. Calculate the expected return on each asset; the variance (Standard Deviation) on each asset; and covariance between X and Y.
b. Suppose we invest half our assets in X and half in Y. Compute the portfolio return and risk.
c. For the following weights of \( w_x \) and \( w_y \), fill the table: mean and standard deviation of returns

<table>
<thead>
<tr>
<th>Percentage in X</th>
<th>100</th>
<th>75</th>
<th>50</th>
<th>25</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Percentage in Y</td>
<td>0</td>
<td>25</td>
<td>50</td>
<td>75</td>
<td>100</td>
</tr>
<tr>
<td>( E(R_p) )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \sqrt{Var(R_p)} )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

d. Suppose that you perform short selling: say you sell short 50% of your wealth in asset X (even though you do not already own shares of asset X) and buy 150% of asset Y. What do you think could happen to the portfolio expected return, and portfolio risk measures.
e. Can you find the minimum variance portfolio. That is the weight \( w_x \), that minimizes the portfolio variance.

**Question 3:**
You are interested in forming a portfolio based on two securities with the following characteristics:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expected Return</td>
<td>12%</td>
<td>18%</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>25%</td>
<td>40%</td>
</tr>
<tr>
<td>Correlation coefficient</td>
<td>0.8</td>
<td></td>
</tr>
</tbody>
</table>

a. Calculate the expected return and standard deviation for the equally weighted portfolio.
b. Would you choose to invest in this portfolio or invest in a single security (either A or B) and why.

**Question 4:**
Given the following information:

<table>
<thead>
<tr>
<th>Stock</th>
<th>Percentage of Portfolio</th>
<th>Beta</th>
<th>Expected Return</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>40 %</td>
<td>1.00</td>
<td>12%</td>
</tr>
<tr>
<td>2</td>
<td>25 %</td>
<td>0.75</td>
<td>11</td>
</tr>
<tr>
<td>3</td>
<td>35 %</td>
<td>1.30</td>
<td>15</td>
</tr>
</tbody>
</table>

a. Determine the expected return and beta for the above portfolio.
b. Given the preceding information, draw the security market line and show where the securities fit on the graph. Assume that the risk-free rate is 8% and that the expected return on the market portfolio is 12%. How would you interpret these findings.
Question 5:

1. Comment on the following quotation from a leading investment analyst.

 Stocks that move perfectly with the market have a beta of 1. Betas get higher as volatility goes up and lower as it goes down. Thus, Southern Co, a utility whose share have traded close to $12 for most of the past three years, has a low beta. At the other extreme, there is True North Networks, which has been as $150 and as low as its current $15.

2. Given the following situations, determine in each case whether or not the hypothesis of an efficient capital market (semistrong form) is contradicted.

   a) Through the introduction of a complex computer program into the analysis of past stock price changes, a brokerage firm is able to predict price movements well enough to earn a consistent 3% profit, adjusted for risk, above normal market returns.

   b) On the average, investors in the stock market this year are expected to earn a positive return (profit) on their investment. Some investors will earn considerably more than others.

   c) You have discovered that the square root of any given stock price multiplied by the day of the month provides an indication of the direction in price movement of that particular stock with a probability of 0.7.

3. The semi-strong form of the Efficient Market Hypothesis asserts that all publicly available information is rapidly and correctly reflected in securities prices. This implies that investors cannot expect to derive above-average profits from purchases made after information has become public because security prices already reflect the information’s full effects.

   a. Identify and explain two examples of empirical evidence that tend to support the EMH implication stated above.

   b. Identify and explain two examples of empirical evidence that tend to refute the EMH implication stated above.
Answer 1:
1. \( E(R_A) = (7.3 + 11.5 + 16.6)/3 = 11.8\%
\[ E(R_B) = (4.7 + 5.4 + 24.3)/3 = 8.3\%
\]
2. \( \sigma^2_A = \{(0.073 - 0.118)^2 + (0.115 - 0.118)^2 + (0.166 - 0.118)^2\}/3 = 0.001446
\[ \sigma^2_B = \{(-0.047 - 0.083)^2 + (0.054 - 0.083)^2 + (0.243 - 0.083)^2\}/3 = 0.014447
\]
\[ \sigma_A = (0.001446)^{1/2} = 0.0380 = 3.80\%
\[ \sigma_B = (0.014447)^{1/2} = 0.1202 = 12.02\%
\]
3. \( \text{Cov}(R_A, R_B) = [(0.073-0.118)(-0.047-0.083) + (0.115-0.118)(0.054-0.083) + (0.166-0.118)(0.243-0.083)]/3 = [0.00585+0.000087+0.00768]/3 = 0.004539 \text{ and Cov}(R_A, R_B) = 0.004539/(0.0389 \times 0.1202) = 0.9397.
\]

Question 2:

a. To simplify matters, we have the expected value of X is 10%, and the expected value of Y is 8%. The variance of X is 0.0076, the variance of Y is 0.00708, and the covariance between X and Y is -0.0024.
\[ \text{Var}(X) = 0.2(0.11-10)^2 + 0.2(0.09 - 10)^2 + \ldots + 0.2(-0.02 - 0.10)^2 = 0.0076.
\]
\[ \text{Var}(Y) = 0.2(-0.03-0.08)^2 + \ldots + 0.2(0.06 - 0.08)^2 = 0.00708.
\]

Covariance between X and Y can be found as follows:
\[ \text{Cov}(X, Y) = \text{E}[(X - \text{E}(X))(Y - \text{E}(Y))] = \sum_{i=1}^{n} p_i (X_i - \text{E}(X))(Y_i - \text{E}(Y)) = 0.2(0.11 - 0.1)(-0.03-0.08) + 0.2(-0.03-0.08)(-0.02-0.10) + 0.2(-0.02-0.10)(0.06-0.08) = -0.0024.
\]

b. \( \text{E}(R_p) = aE(X) + bE(Y) = 0.5(0.10) + 0.5(0.08) = 9\%.
\]
\[ \text{Var}(R_p) = a^2\text{Var}(X) + b^2\text{Var}(Y) + 2ab\text{Cov}(X, Y) = (0.5)^2(0.0076) + (0.5)^2(0.00708) + 2(0.5)(0.5)(-0.0024) = 0.00247.
\]

or to say the standard deviation of the portfolio is \( \sqrt{\text{Var}(R_p)} = 0.0024 = 4.97\%.
\]

The advantage of portfolio diversification becomes clear in this example. With half our assets in X and half in Y, the expected return is halfway between that offered by X and by Y, but the portfolio risk is considerably less than either var(X) or var(Y).

c. The table will be:

<table>
<thead>
<tr>
<th>Percentage in X</th>
<th>100</th>
<th>75</th>
<th>50</th>
<th>25</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Percentage in Y</td>
<td>0</td>
<td>25</td>
<td>50</td>
<td>75</td>
<td>100</td>
</tr>
<tr>
<td>( E(R_p) )</td>
<td>10.0%</td>
<td>9.5</td>
<td>9.0</td>
<td>8.5</td>
<td>8.0</td>
</tr>
<tr>
<td>( \sqrt{\text{Var}(R_p)} )</td>
<td>8.72%</td>
<td>6.18</td>
<td>4.97</td>
<td>5.96</td>
<td>8.41</td>
</tr>
</tbody>
</table>
| d. \( \text{E}(R_p) = aE(X) + bE(Y) = -0.5(0.10) + 1.5(0.08) = 7.0\%.
\]
\[ \text{Var}(R_p) = a^2\text{Var}(X) + b^2\text{Var}(Y) + 2ab\text{Cov}(X, Y) = (-0.5)^2(0.0076) + (1.5)^2(0.00708) + 2(-0.5)(1.5)(-0.0024) = 0.02143
\]

or \( \sqrt{\text{Var}(R_p)} = 14.64\%.
\]

d. We know: \( \text{Var}(R_p) = a^2\text{Var}(X) + b^2\text{Var}(Y) + 2ab\text{Cov}(X, Y) = a^2\sigma_X^2 + b^2\sigma_Y^2 + 2ab\rho_{XY}\sigma_X\sigma_Y = a^2\sigma_X^2 + (1-a)\sigma_Y^2 + 2(a(1-a)\rho_{XY}\sigma_X\sigma_Y.
\]

Minimize the above equation by choosing a, that is set : \( d\text{Var}(R_p)/da = 0 \), we can find:
\[ a^* = (\sigma_Y^2 - \rho_{XY}\sigma_X\sigma_Y) / (\sigma_X^2 + \sigma_Y^2 - 2\rho_{XY}\sigma_X\sigma_Y)
\]

using the example above, we have \( \rho_{XY} = \text{Cov}(X,Y)/\sigma_X\sigma_Y = -0.0024/[(0.0872)(0.0841)] = -0.33
\]
so we can get: \( a^* = 0.487
\]

To find the portfolio expected mean and variance, at the minimum variance level, we get:
\[ \text{E}(R_p) = aE(X) + bE(Y) = (0.487)(0.10) + (0.513)(0.08) = 8.974\%.
\]
\[ \text{Var}(R_p) = a^2\text{Var}(X) + b^2\text{Var}(Y) + 2ab\text{Cov}(X, Y) =
\]
\[ (0.487)^2(0.0076) + (0.513)^2(0.00708) + 2(0.487)(0.513)(-0.33)[(0.0872)(0.0841)] = 0.0024565 \text{ or } \sqrt{\text{Var}(R_p)} = 4.956\%
\]

Question 3:

a. Expected return = 15.00%; Standard deviation = 30.92% ; b. It would depend upon my risk aversion.

It is possible that I would invest in individual securities if I were extremely risk averse (A) and risk loving (B).

Solution 4:

a. The portfolio expected return is simply: \( (0.4 \times 12\%) + (0.25 \times 11\%) + (0.35 \times 15\%) = 12.8\%.
\]

Portifolio beta is : \( (0.4 \times 1) + (0.25 \times 0.75) + (0.35 \times 1.3) = 1.04
\]

Stock 3 should be above the line.

b. Stocks 1 and 2 seem to be right in line with the security market line, which suggests that they are earning a fair return, given their systematic risk. Stock 3, on the other hand, is earning more than a fair
return (above the security market line). We might be tempted to conclude that security 3 is undervalued. However, we may be seeing an illusion; it is possible to misspecify the security market line by using bad estimates in our data.

**Question 5:**

1. If we assume that the market has not stayed constant during the past three years, then the low volatility of Southern Co.’s stock price only indicates that the stock has a beta that is very near to zero. The high volatility of Texas Instruments’ stock price does not imply that the firm’s beta is high. Total volatility (the price fluctuation) is a function of both systematic and unsystematic risk. The beta only reflects the systematic risk. Observing price volatility does not indicate whether it was due to systematic factors, or firm specific factors. Thus, if you observe a high price volatility like that of TNN, you cannot claim that the beta of TNN’s stock is high. All you know is that the total risk of TNN is high.

2 a) The information which enables the brokerage firm to earn a consistent 3% abnormal profit is not costless. If the computer costs exceed the excess 3% profits from stocks, the firm is actually earning worse than normal returns. If the computer costs are less than the 3% profit, semi-strong capital market efficiency may be refuted. Also, brokerage fees may wipe out any trading profits.

b) The hypothesis of an efficient capital market is not contradicted. Except for very bad years, the average (and expected) return on the market is positive. This is considered a normal return. It is also a fair game. The fact that some investors enjoy higher returns than others is the result of the uncertainty in stock returns. Given any probability distribution, some observations will lie above the mean and some will lie below.

c) Semi-strong (as well as weak) capital market efficiency is contradicted. You have discovered a trading rule based on past, nearly costless, price information that enables you to forecast future prices with better-than-random accuracy. Thus, all relevant publicly available information has not been instantaneously incorporated into stock prices.

3. 

a) The semi-strong form of EMH receives support from numerous event studies that show that new information is rapidly assimilated into market prices. The market appears to correctly differentiate between events that have economic importance and those that do not. A list of examples include the following:

- Price impact of takeover announcements: announcements of takeovers have economic significance and are quickly reflected in market pricing with little further drift in cumulative abnormal returns (in the absence of further information).

- Price impact of dividends changes: various studies have concluded that dividend changes, which have economic significance, are quickly reflected in stock price changes. After the public announcements of a dividend change, stock prices generally adjust quickly, without further drift in CAR (cumulative abnormal return). These results support the semi-strong EMH.

Initial public offering (IPOs): under the premise that underwriters generally price IPOs below their true economic value, various studies have examined how quickly the market adjusts for the underpricing. Results indicate that the price adjustment takes place quickly (one day or less), which is essentially consistent with the semi-strong EMH.

Students may discuss also: Price impact of stock splits, unexpected world and economic events, and active manager underperformance.

b) Apparent pricing and performance anomalies have been documented in a growing number of studies. The ability to consistently predict stock price performance on the basis of publicly available information regarding company attributes or calendar effects is inconsistent with the semi-strong form of EMH.

Calendar effects: numerous studies have documented a seasonality to stock price performance. The January effect (in which small socks outperform large stocks in January) and various calendar effects appear to refute the semi-strong EMH, under which known seasonalities would be already reflected in market pricing.

Superior returns to small and/or neglected firms: small firms, often “neglected” by analysts, have been found to provide superior risk-adjusted returns relative to large stocks. Under the semi-strong EMH, public information regarding company attributes would already be reflected in pricing.

Exceptional track records: of certain firms and individuals appear attributable to superior fundamental analysis of public information (i.e., the Value Line enigma). These achievements seem to refute the semi-strong EMH.

Students might also discuss: superior returns of “value” stocks, extreme market moves (October 1987), and price reversals.