Net Present Value and Other Investment Criteria

Capital budgeting decisions are based on the following criteria:

a) Net present value
b) The payback period
c) The discounted payback period
d) The average accounting return
e) The internal rate of return
g) The profitability index

A) The Net present value:

A manager that is trying to use the net present value criteria in making capital budgeting decisions is making decisions which maximizes the current price of the firm’s common stock.

Consider an investment in an asset (say price of a real estate) for a price of $2000.
Assume that the asset can be sold for $2500 one year from today.
Alternatively, the investor has an opportunity to earn an 8% rate of return on an investment of similar risk.

To decide whether to acquire the asset or not, we could use the future value concept, the present value concept or the rate of return concept.

Say we are using the future value concept.

If the investor invests the $2000 at 8% for one year, or he can invest the $2000 to acquire the real estate today.

If he selects the first option, then in one year, he will receive:

\[ FV = PV \times (1 + r) \]
\[ = 2000 \times (1 + 0.08) = 2160 \]

Clearly, the second alternative is better since the investor will have $2500 rather than $2160 one year from today. Therefore, the investor should acquire the real estate today for $2000.

The investor can also use the present value concept to evaluate the investment opportunity. He could say: at the 8% opportunity cost, what deposit would he have to make today in order to receive a return (future value) of $2500 on year from today. Using the present value concept:

\[ PV = \frac{FV}{(1 + r)} = \frac{2500}{(1+0.08)} = 2314.81 \]

That is greater than the cost of the real estate at the beginning (which is $2000). So it is better to acquire the real estate.

The present value approach to investment decision-making can also be described in terms of the difference between the present value of the investment ($2314.81) and the cost of the investment ($2000).
This difference is called the Net Present Value. So for our example: \( \text{NPV} = PV - \text{Cost} = $2314.81 - $2000 = $314.81 \).

Note that the investor could sell the real estate for $2314.81 immediately after purchasing it. That amount ($2314.81) represents the market value of the asset. Thus the net present value (NPV) is the difference between an investment's market value and its cost. An investment with a positive net present value provides a net benefit to the investor equal to the net present value; an investment with a negative net present value is unacceptable.

a) Estimating the Net Present Value:

Say a company is considering the acquisition of new construction equipment. The estimate for the cost to acquire the equipment is $10000. It is estimated that the company's cash revenues will increase by $9000 for each of the next two years. However cash expenses will increase by $4000 each year. (cost include among other costs, labor costs, income taxes. They do not include depreciation, which is not a cash expense). The equipment is expected to have a life of two years, and to have a salvage value of $3000. We assume an opportunity cost of 12% for this investment. The question is: Should the company invest in this equipment.

Remember that we care here about the incremental cash flows: that is the difference between any cash flow increases and any cash flow decreases (or revenue increases - cost increases).

The net cash inflow to the company is: $9000 - $4000 = $5000 per year for two years.

And at the end of the second year, the equipment will be sold for $3000.

So cash flows for the company are:

$5000 for the first year.

$5000 for the second year, plus the $3000.

At 12% what is the present values of the cash flows:

\[
\begin{align*}
\text{PV} &= \frac{5000}{(1.12)} = 4464.29 \\
\text{PV} &= \frac{5000}{(1.12)^2} + \frac{3000}{(1.12)^2} = 6377.55 \\
\text{The sum of both is:} &= 10841.84 \\
\text{So the NPV is positive in this case, and the company should accept the project.}
\end{align*}
\]

Algebraically, the net present value calculation can be summarized as:

\[
\text{NPV} = -C_0 + C_1(1+r) + C_2(1+r)^2
\]
where $C_0$ is the initial cost, $C_1$ is the first inflow, $C_2$ is the second inflow.

The more general formula for the net present value is:

$$NPV = -C_0 + C_1(1 + r) + C_2(1 + r)^2 + \cdots + C_t(1 + r)^t$$

where $t$ represents the last period during which a payment is received.

To summarize:

The Net Present Value rule: An investment should be accepted if the NPV is positive and rejected if it is negative.

---

B) The PayBack Rule:

The PayBack Period is the amount of time required to recover the initial investment for a capital budgeting project from the subsequent cash inflows produced by the project.

a) Defining the Rule:

The payback period rule specifies that an investment is acceptable if the sum of its undiscounted cash flows equals the initial investment before some specified cutoff time period.

Example:

Consider an investment with the following cash flows: an initial investment of $10000 and cash inflows of $2000, $5000, $3000 and $6000 at the end of years 1, 2, 3, and 4, respectively. What is the payback period. And according to the payback period rule, should this investment be accepted.

The initial investment is $10000, after three years the sum of the cash inflows equals $10000. So the payback period for this investment is three years.

Consequently this investment would be rejected if the cutoff time period were, for example two years.

Managers would accept the project if their cutoff date was greater than three years.

Note: Say the initial investment costs $13000 instead of $10000. What is the payback.

So we need another $3000 must be recovered in the fourth year, that has a cash inflow of $6000. To figure out the time, just divide $3000/$6000 = 0.5 years. Hence the payback period is 3.50 years.

Deficiencies of the payback rule:

a) The rule assumes that the cash inflows and outflows occur at a constant rate throughout the year. That is the timing of the cash inflows within the payback period is ignored. In contrast the NPV rule discounts these cash inflows.

b) All cash inflows after the cutoff time period are ignored, which the NPV rule discounts all cash inflows.

Advantages of the payback rule is that, it is simple to apply, easy to compute and understand. Mostly used for low-level, small investment decisions.
C) The Discounted PayBack Rule:

The discounted payback period rule specifies that a project is acceptable if the sum of the discounted cash flows equals the initial investment before a specified cut-off time period.

The rule is the same as in B, except that the value of money is considered since the cash flows are discounted.

Using the same data as in the previous example. and Say the discount rate is 10%.

Would the investment be acceptable if the cut-off period is 3 years.

If we find the present value of the future cash inflows, the sum for the first three years is $8204.36, and for the first four years is $12302.44. Therefore the project payback period is between three and four years, and it would be rejected based on the discounted payback rule.

The project requires 3.44 years, the fraction is calculated by using the amount, recovered in the fourth year: $1795.64 ($10000 - $8204.36) and divide it by the fourth year, present value cash flow: $6000/(1.1)^4:

Notice that using the NPV rule, the investment would be accepted, because:

($12302.44 - $10000) = $2302.44

Still some of the deficiencies applied to the payback rule, are the same for the discounted payback rule.

D) The Profitability Index rule:

The profitability index (PI) also known as the benefit/cost ratio is defined as the present value of the future cash flows divided by the initial investment. In one sense, the PI index comparable to the NPV rule, because it provides an indication of whether the present value of the future cash flows exceeds the initial investment.

If this ratio is greater than one, then the investment is desirable, because the present value of the future cash flows is greater than the initial outlay.

In an example, say the NPV is $12302.44 for an initial investment of $10000, the PI is simply 1.23 ($12302.44/$10000). This tell us that for every $ invested, $0.23 in NPV is created, that is, the value created per dollar invested.
E) The Internal Rate of Return:

The Internal Rate of Return (IRR) criteria is the most significant alternative to the net present value criterion. The IRR is the rate of return (or discount rate) which equates the present value of the cash inflows with the cash outlay, or cost of the investment. Or in other words, IRR is the rate that makes the NPV of an investment is zero.

Algebraically, the IRR is the solution for the discount rate \( r \) in the following equation:

\[
NPV = -C_0 + C_1(1+r) + C_2(1+r)^2 + \cdots + C_t(1+r)^t = 0
\]

or

\[
C_0 = C_1(1+r) + C_2(1+r)^2 + \cdots + C_t(1+r)^t
\]

\( C_0 \) is the initial cash outlay, or the initial investment for a particular capital budgeting project. \( C \)'s are the cash inflows. \( C_t \) is the cash inflow expected at the end of the year \( t \).

The above equation states that the IRR is the rate \( r \) such that the present value of the cash inflows is equal to the cash outlay.

Example:

Consider the following investment. \( C_0 = $10000 \) and the cash inflows are: \( C_1 = $2000; C_2 = $5000; C_3 = $3000; C_4 = $6000 \)

What is the internal rate of return for this investment. To calculate IRR, use the above equation, where:

\[
$10000 = $2000(1+r) + $5000(1+r)^2 + $3000(1+r)^3 + $6000(1+r)^4
\]

That is equivalent of solving for the yield to maturity of a level coupon bond. It can be solved either by trial and error or a calculator. Try three numbers, 5%, 10% and 20%, we will find that the present value at 10% is $12302, and at 20% the present value is $9768, so we expect the IRR to be between 10% and 20%. By trying other values, we will find that IRR is almost 18.9259%

What is the meaning of the IRR: It indicates that if an individual who invests $10000 in an investment receives a rate of return equal to 18.9259%. In other words, an individual would have to invest $10000 at 18.9259% in order to receive returns of $2000, $3000, $5000 and $6000 at the end of years 1, 2, 3 and 4 respectively.

The Rule is:

An investment project is acceptable if the IRR is greater than the rate of return the firm could earn on investments of equal risk: an investment project is unacceptable if the IRR is less than the rate of return required by the firm.

a) Problems with the IRR:

Two problems arise in calculating the IRR: the problem of nonconventional cash inflows and mutually
exclusive investments.

1) Nonconventional cash flows:

Define a cash inflow as having a positive sign and a cash outflow as having a negative sign. A conventional cash flow for capital budgeting project has a negative cash flow (an outlay) followed by positive cash flows (inflows). A nonconventional series of cash flows changes sign more than once.

Consider the following cash flows ( -$80, $500, -$500).

So we have two outflows and only one inflow.

\[ NPV = -80 + \frac{500}{1+r} - \frac{500}{(1+r)^2} \]

to find IRR, the above equation must be solved for \( r \) at which the NPV is zero.

We find two values where the NPV is zero, 25% and 400%. There are two internal rates of return. That is an example of multiple rates of return problem. Since there is no rule of choosing either one, the IRR criteria should not be used, we better use the NPV criteria, in case of multiple rates of return problem.
b) Mutually exclusive investments:

An independent investment project is an investment whose acceptance or rejection does not affect, and is not affected by, the acceptance or rejection of any other projects. A set of mutually exclusive investment projects is a set of projects for which the acceptance of any one project in the set implies the rejection of all other projects in the set. For example, a company which is considering the purchase of a delivery truck may have under consideration numerous models produced by several manufacturers; since only one truck will be purchased, the alternatives under consideration comprise a set of mutually exclusive investments.

Mutually exclusive projects could differ in either scale or timing. In the former case, there are differences in the size of the initial outlay for the projects under consideration; in the latter case, the differences are generally between one project whose cash flows are concentrated in the early years of the project’s life and another project whose cash flows are concentrated in the later years. Only the scale problem will be discussed here.

Suppose we have the following two mutually exclusive projects, X and Y

\[
X: (-\$200, \$100, \$300, \$400) \\
Y: (-\$5000, \$4000, \$1000, \$2000)
\]

These investments are said to differ in scale because the cash flows for Y are substantially larger than those for X. Say the opportunity cost is 10%. Since they are mutually exclusive, we must determine first, whether each investment is acceptable and, second, in the event that both are acceptable, which is preferable.

At 10%, the NPV for X and Y are:

\[
\text{NPV}_X = \$439.37 \quad \text{and} \quad \text{NPV}_Y = \$965.44
\]

indicating that both are acceptable. Since both are acceptable, the IRRs are 87.20% and 22.81% respectively, for X and Y.

Note, that X has a higher IRR, but lower NPV than those of Y. Which one is preferable alternative. Using NPV, Y is preferable. But that could be misleading, since IRR is higher for X. So what do we do. We have to think in terms of a cross-over rate.

To understand the cross-over rate, we first think about a hypothetical investment (Y - X); this investment, when implemented along with investment X, will have the same cost and will generate the same cash flows as investment Y.

\[
(Y - X) \text{ requires that an additional } (-\$5000 - \$200) = \$4800 \text{ be invested in order to receive incremental cash flows of } \$3900, \$700, \text{ and } \$1600
\]

So, NPV \( (Y - X) = -\$4800 + \$3900/(1.10) + \$700/(1.10)^2 + \$1600/(1.10)^3 \)

\( (Y - X) \) has NPV of $526.07, and IRR of 17.7% (The cross-over rate). So it is acceptable.
We have determined that X is acceptable, and that (Y - X) is acceptable; viewed in this way, accepting project Y amounts to accepting project X plus another acceptable investment (Y - X). Therefore, Y is preferable alternative.

The internal rate of return for (Y - X) is the cross-over rate. In this example, and as shown in the figure, it indicates that we are indifferent between investments X and Y if the opportunity cost is equal to the 17.70% cross-over rate. If the opportunity cost is less than the cross-over rate, then investment Y is preferable; if the opportunity cost is greater than the cross-over rate, then investment X is preferable.
The Average Accounting Return

The average accounting return (AAR) is defined as average net income attributed to an investment divided by average book value of the asset. The average accounting return rule specifies that an investment is acceptable if its average accounting return exceeds a specified target level.

Example: Suppose that the cash inflow data of the preceding problems represent accounting net income rather than cash flows. What is the average accounting return. According to the average accounting return criteria, is the capital budgeting project acceptable.

Solution: The average net income is:

\[ \frac{($2000 + $5000 + $3000 + $6000)}{4} = $4000 \]

If the $10000 investment is depreciated to a value of zero, on a straight-line basis, over a four-year period, then the book value decreases by \( \frac{($10000 \times 4)}{4} = $2500 \) per year. The average book value is:

\[ \frac{($10000 + $7000 + $5000 + $2500 + $0)}{5} = $5000 \]

Then

\[ \text{AAR} = \frac{\text{Avg. net income}}{\text{Avg. book value}} = \frac{$4000}{$5000} = 80\% \]

the project is acceptable if the firm's target AAR is less than 80%.

The AAR rule has several serious deficiencies. First, the AAR method uses accounting income and book value data, which generally are not closely related to cash flows, the relevant data for financial decision making. Second, the AAR ignores the time value of money, income received in three years, for example, is treated as equivalent to income received in one year. Third, the target AAR must be arbitrarily specified because it is not a rate of return in the financial market sense.

The only redeeming features for the AAR rule are that it is easy to calculate and that the information needed for the calculation is almost always available. Nevertheless, these features do not compensate for the disadvantages of this method.
Chapter 8  
Making Capital Investment Decisions  
A) The Project Cash Flows: incremental cash flows

The major theme in this chapter is to identify the relevant cash flows for net present value analysis of a capital budgeting problem. The financial decisions must be based on cash flows rather than income.

a) Project cash flows: by undertaking a capital budgeting project, a firm changes current and future cash flows. The capital investment decision-making requires financial managers to determine whether the changes in cash flows which result from undertaking a particular project add value to the firm. The relevant cash flows for the analysis are the incremental cash flows associated with the project. The incremental cash flows consist of any and all changes in the firm’s cash flows that are a direct consequence of accepting a project. They can be defined as the difference between the firm’s cash flows, now and in the future, if the project is undertaken, and the firm’s cash flows if the project is not undertaken.

Some problems encounter the financial manager in the process of identifying incremental cash flows for a particular project. The problems are:

1) Sunk Costs: is the money the firm has already spent, or is committed to spend, regardless of whether it accepts the project or not. An example is the cost of feasibility study undertaken before the accept/reject decision is made is a sunk cost. This sunk cost is not an incremental cost, therefore it is not relevant to capital budgeting decision, and excluded from the analysis.

2) Opportunity cost: For the example the use of a previously bought land to build a factory on it, makes the firm loses the opportunity to use the land for some alternative project. An opportunity cost would be to sell the property. So the market value of the property is an opportunity cost which is an incremental cost to the project.

3) Side effects: An introduction of new product might have an influence on the sale of other products the firm sells. That is an example of a side effect, or called erosion. The resulting reduction in cash flow to other product lines is a direct consequence of introducing the new consideration.

4) Net working capital: It is known that net working capital is the difference between current assets and current liabilities. A capital budgeting project often requires an increase in net working capital during the life of the project. For example the introduction of a new product necessitates increases in cash balances, inventory and accounts receivables. Some of the financing is provided by increases in accounts payable. However, the increase in accounts payable is typically sufficient to finance the increase in current assets; hence, additional outlays are normally required to finance the increase in net working capital. This investment in net working capital is an incremental cash outflow.
5) Financing costs: Financing costs are not included in the computation of incremental cash flows.

6) Inflation: The discount rate is the nominal rate of return, it should include a factor that take care of the change in the average price level.

7) Government Intervention: Various levels of government offer incentives to promote certain types of capital investment including grants, investment tax credits and favorable rate for capital allowance (CCA) and subsidized loans. Since these change a project’s cash flows, they must be factored into capital budgeting analysis.

8) Other issues: First, cash flows are measured when they occur, not when the accrue in the accounting sense; second, since taxes are clearly cash outflows, we are concerned with after-tax incremental cash flows.

B) Pro Forma Financial Statements and Project cash flows

For a typical project, current cash outflows are followed by future cash inflows. Therefore, it is necessary to forecast or project these incremental cash inflows. Projected cash flows are derived from pro forma financial statements.

Example:

Take an example of a company that is considering the purchase of new equipment, expected to produce a better quality product at a lower cost.

Sales revenue is expected to increase by $40000 the first year, and 10 percent a year for each of the next four years. Operating costs will decrease by $20000 per year. The equipment costs $150000 and is expected to have a life of five years. The market value at the end of five years will be $60000. We assume a straight line depreciation and a 40% tax rate for income and capital gains. The company projects an investment of $15000 in working capital; that investment will be recovered at the end of the fifth year.

We know that:

- cash flows from assets has three components: operating cash flows, capital spending, and additions to net working capital.
- The capital spending for the new equipment is the amount spent on acquiring the asset, the $150000 outlay for purchase of the equipment.
- The addition to net working capital is the $15000 outlay for the investment in working capital. Therefore the total outlay at year 0 is $165000.
- Operating cash flow is defined in Chapter 2 as:

\[ \text{Earnings before interest and taxes} + \text{Depreciation - Taxes} \]
Note we do not deduct either interest or depreciation in computing operating cash flow. Interest is not deducted because it is not an operating expense. And we do not deduct depreciation because it is a non-cash expense.

<table>
<thead>
<tr>
<th></th>
<th>Year 1</th>
<th>Year 2</th>
<th>Year 3</th>
<th>Year 4</th>
<th>Year 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sales revenue</td>
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<td>$48000</td>
<td>$53240</td>
<td>$58564</td>
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<td>-Operating costs</td>
<td>-20000</td>
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<td>-20000</td>
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<td>-20000</td>
</tr>
<tr>
<td>-Depreciation</td>
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<td>18000</td>
<td>18000</td>
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<tr>
<td>EBIT</td>
<td>$42000</td>
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<td>$55240</td>
<td>$60564</td>
</tr>
<tr>
<td>Plus Depreciation</td>
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<td>18000</td>
<td>18000</td>
<td>18000</td>
<td>18000</td>
</tr>
<tr>
<td>-Tax (at 40%)</td>
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<td>18400</td>
<td>20160</td>
<td>22096</td>
<td>24225.60</td>
</tr>
<tr>
<td>Operating Cash flow</td>
<td>$43200</td>
<td>$45600</td>
<td>$48240</td>
<td>$51114</td>
<td>$54338.40</td>
</tr>
</tbody>
</table>

Notice that we are subtracting a decrease in costs. This actually increase the EBIT. (For year 1, EBIT = $42000 - (-20000) - 18000 = $43200).

For year 5, in addition to operating cash flows, we have cash flows related to the sale of asset and recovery of working capital.

The sale of the equipment at the end of year 5 results in an inflow to the company equal to $60000. Also the company has an inflow equal to $15000 as a recovery of the working capital.

The total cash flow for year 5 will be:
$54338.4 + $60000 + $15000 = $129338.4

Now use the NPV criteria to determine whether to accept the project or not. If the opportunity cost for this investment is 16%, then the present value of the cash inflows for years 1 through 5 is $191861 and the net present value of the investment is $26861. (That is net present value of the inflows minus the cost of the equipment and with the net working capital, which is $165000). Clearly, the NPV indicates the acceptance of the project.

Do not include this section in the lectures for financial management MBA.

C) Alternative Definitions of Operating Cash Flow:

According to the previous example, we calculated:

Operating Cash Flow = Earnings before interest and taxes + Depreciation - Taxes

For example, the operating cash flow for year 1 was:
$42000 + $18000 - $16800 = $43200

In this section we present alternative methods to computing operating cash flow. First define the following terms:
OCF = project operating cash flow
S = Sales (in dollars)
C = Operating costs
D = Depreciation
Tc = Corporate tax rate

Therefore:

\[ EBIT = (S - C - D) \] and taxes are equal to:

\[ EBIT \times T_c = (S - C - D) \times T_c \]

Using the above equations in the OCF formula, we find:

\[ OCF = EBIT + D - \text{Taxes} \]

The alternative methods are:

a) The Tax Shield Approach:

The tax shield approach emphasizes the manner in which depreciation affects cash flow. Since depreciation is a non-cash expense, its only impact on cash flow is through the effect on taxes.

The tax shield is derived from the following:

\[ OCF = (S - C - D) + D \times [(S - C - D) \times T_c] \]

This equation has two terms:

\[ [(S - C) \times (1 - T_c)] + (D \times T_c) \]

To say, that is the project’s after-tax income if depreciation were zero. In the absence of depreciation, operating cash flow would equal to net income after taxes.

\[(D \times T_c)\] is referred to as the depreciation tax shield. Since depreciation is non-cash expense, its only impact on operating cash flow calculations is its impact on taxes. The tax savings associated with the tax-deductibility of depreciation expense is equal to the \((D \times T_c)\)

For the previous example:

\[ OCF = [(S - C) \times (1 - T_c)] + (D \times T_c) = \$40000 \times (1 - 0.2) + (\$18000 \times 0.2) \]

\[ = \$32000 + \$3600 = \$35600 \]

Notice that the tax shield approach emphasizes the fact that the $18000 depreciation expense produces an $7200 tax saving for the...
b) The Bottom-Up Approach:
This approach accentuates the relationship between net income and operating cash flow. The calculation emphasizes the manner in which net income and depreciation determine OCF. Using the previous terms:

Net Income = EBIT - Taxes
= (S - C - D) \times [(S - C - D) \times (1 - T_c)]

Note that net income here ignores financing costs.

So:

OCF = EBIT + D - Taxes
= (EBIT - Taxes) + D
= ((S - C - D) \times (1 - T_c)) + D

So for the previous example:

OCF = Net income + Depreciation = $25200 + $18000 = $43200.


c) The Top-Down Approach

Using the same equations as before:

OCF = EBIT + D - Taxes
= (S - C - D) + D \times [(S - C - D) \times (1 - T_c)]

= (S - C) \times D + (S - C - D) \times (1 - T_c)

= Sales - Costs - Taxes

This method emphasizes the fact that operating cash flow is equal to sales minus expenses (including taxes, but excluding non-cash expenses such as depreciation).

For the example before:

OCF = $40000 - (-$20000) - $16800 = $43200.
Project Analysis and Evaluation
Chapter 9

The emphasis of this chapter is on the estimates of cash flows and their forecasts.

A) Evaluating NPV Estimates:

We know that a project is acceptable if its market value exceeds its cost. But most of the time, we cannot observe market value. Therefore we estimate it. Market value can be estimated using discounted cash flow analysis. This is a good sign, but we must remember that the numbers used to compute the NPV are estimates, and are subject to errors. A major source of error is in the forecast of future cash flows. Most of the cash flows forecasts are thought in terms of the expected value concept. For example, when we consider a cash flow of $5000 one year from now, this does not mean that the actual cash flow will be $5000 or even close to $5000. This is only an expected value of cash flows. Therefore the actual cash flow could be a great deal more or less than $5000.

Here we have to look at the Forecasting Risk: that is the risk that a financial manager makes an incorrect decision because of errors in forecasting cash flows. If forecasted cash flows are overly optimistic, a project may appear to have a positive NPV when, in fact, the net present value is negative. Some of the methods designed to identify sources of potential errors in cash flow forecasts follow.

B) Scenario and Other “What IF” Analysis:

Scenario analysis, sensitivity analysis and simulation analysis are techniques used for evaluating the critical assumptions of net present value calculations.

a) Getting Started:

We start with an example: A firm is considering the production of a new product. The project requires an investment of $1000000 in plant and equipment, which has a ten-year life and no salvage value. For simplicity, we will assume that for tax purposes, fixed assets will be depreciated on a straight-line basis. The required rate of return is 16% and the tax rate is 34%. The firm has the following forecasts:

<table>
<thead>
<tr>
<th></th>
<th>Base Case</th>
<th>Lower Bound</th>
<th>Upper Bound</th>
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<tbody>
<tr>
<td>Unit Sales</td>
<td>10000</td>
<td>9000</td>
<td>11000</td>
</tr>
<tr>
<td>Price per Unit</td>
<td>$200</td>
<td>$190</td>
<td>$210</td>
</tr>
<tr>
<td>Variable Costs per Unit</td>
<td>$130</td>
<td>$125</td>
<td>$135</td>
</tr>
<tr>
<td>Fixed Costs per year</td>
<td>$425000</td>
<td>$410000</td>
<td>$440000</td>
</tr>
</tbody>
</table>
The Base Case forecasts represent the expected values for each forecasted item. The lower bound and upper bound are values such that the firm believes it is unlikely that actual value will be outside these bounds.

What is the net present value (NPV) of the project using the Base Case forecasts:
First compute the net income and operating cash flow:

<table>
<thead>
<tr>
<th>Item</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sales</td>
<td>$2000000</td>
</tr>
<tr>
<td>- Variable Costs</td>
<td>$1300000</td>
</tr>
<tr>
<td>- Fixed Costs</td>
<td>$425000</td>
</tr>
<tr>
<td>- Depreciation</td>
<td>$100000</td>
</tr>
<tr>
<td>EBIT</td>
<td>$175000</td>
</tr>
<tr>
<td>- Taxes (34%)</td>
<td>$59500</td>
</tr>
<tr>
<td>Net Income</td>
<td>$115500</td>
</tr>
</tbody>
</table>

EBIT = $175000
+ Depreciation = $100000
- Taxes = $59500
Operating Cash Flow = $215500

Next compute the present value of an annuity of $215500 per year for ten years, discounted at the 16% required rate of return:

\[
PV = 215500 \times PVIFA(16\%, 10) = 215500 \times 4.833227 = 1041560
\]

The Net Present Value (NPV) = $1041560 - $100000 = $41560

b) Scenario Analysis:
Since the NPV is positive, we might conclude that the project is acceptable. However, we must analyze this result more carefully, in light of the lower and upper bound forecasts. Scenario Analysis examines the effect on all variables, and consequently on NPV, of a hypothetical sequence of events.

Example: For our scenario analysis, we examine both the worst case and the best case as included in the table:

<table>
<thead>
<tr>
<th></th>
<th>Worst Case</th>
<th>Best Case</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unit Sales</td>
<td>9000</td>
<td>11000</td>
</tr>
<tr>
<td>Price per Unit</td>
<td>$190</td>
<td>$210</td>
</tr>
<tr>
<td>Variable Costs per Unit</td>
<td>$135</td>
<td>$125</td>
</tr>
<tr>
<td>Fixed Costs per year</td>
<td>$440000</td>
<td>$410000</td>
</tr>
</tbody>
</table>
Project Analysis:

The worst case scenario indicates the least favorable outcome for each of the four forecasted variables. While the best scenario indicates the most favorable outcome for each of the variables. For example, the worst case scenario will use the lower bound for unit sales and price per unit but the upper bound for variable and fixed costs. The best case scenario will use the upper bound for sales and price; the lower bound for variable and fixed costs. For each scenario, we now compute the net income, cash flow, net present value and internal rate of return for the proposed project.

The results are presented in the following:

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Net Income</th>
<th>Cash Flow</th>
<th>NPV</th>
<th>IRR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base Case</td>
<td>$115000</td>
<td>$215500</td>
<td>$41650</td>
<td>17.11%</td>
</tr>
<tr>
<td>Worst Case</td>
<td>-29700</td>
<td>70300</td>
<td>-660224</td>
<td>-5.94</td>
</tr>
<tr>
<td>Best Case</td>
<td>280500</td>
<td>380500</td>
<td>839043</td>
<td>36.33</td>
</tr>
</tbody>
</table>

Since some of the NPV are negative, we need further scenarios to determine the NPV and IRR, and there is many possibilities here. We can not exclude the other scenarios, all depends on the market studies of the firm, the product, competition, marketing strategies, and so forth.

c) Sensitivity Analysis:

It is designed to determine the sensitivity of one variable to changes in another variable. For our purposes we are interested in the sensitivity of NPV to variation in the level of each of the four forecasted variables.

For example, we might attempt to determine the change in NPV which results if actual sales are at the lower bound of our forecasted range, while all other variables are held at their base case levels.

Sensitivity analysis is a variant of scenario analysis in which each scenario represents a change in only one variable, rather than a number of variables.

Example:
Assume that all variables, except unit sales take on their base case levels. What is the effect on net present value and internal rate of return if unit sales are at their lower bound.

The answer is the following:

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Unit Sales</th>
<th>Net Income</th>
<th>Cash Flow</th>
<th>NPV</th>
<th>IRR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base Case</td>
<td>10000</td>
<td>$215500</td>
<td>$41650</td>
<td>17.11%</td>
<td></td>
</tr>
<tr>
<td>Worst Case</td>
<td>9000</td>
<td>169300</td>
<td>-181735</td>
<td>10.93</td>
<td></td>
</tr>
<tr>
<td>Best Case</td>
<td>11000</td>
<td>261000</td>
<td>264856</td>
<td>22.82</td>
<td></td>
</tr>
</tbody>
</table>
In the above example we include other cases, the Base Case, and Best Case. Remember, for the Best, worst and Base cases, we have the same variables, Price per unit, variable and fixed costs, as in the base case.

Now we happen if we change other variables:

What is the effect on NPV and IRR if variable costs are at their lower bound.

Sensitivity analysis of variable costs is performed:

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Variable Costs</th>
<th>Cash Flow</th>
<th>NPV</th>
<th>IRR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base Case</td>
<td>$130</td>
<td>$215500</td>
<td>$41650</td>
<td>17.11%</td>
</tr>
<tr>
<td>Worst Case</td>
<td>135</td>
<td>182500</td>
<td>-117936</td>
<td>12.76</td>
</tr>
<tr>
<td>Best Case</td>
<td>125</td>
<td>248500</td>
<td>201057</td>
<td>21.22</td>
</tr>
</tbody>
</table>

These results are derived by varying only variable costs. In the worst case scenario, variable costs are $135 per unit and the best case scenario, variable costs are $125 per unit. For both scenarios, unit sales, price and fixed costs take on their base case values.

Another scenario: What is the effect on NPV and IRR if fixed costs are at their lower bound:

Sensitivity analysis of fixed costs is performed in the following table:

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Fixed Costs</th>
<th>Cash Flow</th>
<th>NPV</th>
<th>IRR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base Case</td>
<td>$425000</td>
<td>$215500</td>
<td>$41650</td>
<td>17.11%</td>
</tr>
<tr>
<td>Worst Case</td>
<td>440000</td>
<td>205600</td>
<td>-6288</td>
<td>15.83</td>
</tr>
<tr>
<td>Best Case</td>
<td>410000</td>
<td>225400</td>
<td>89409</td>
<td>18.36</td>
</tr>
</tbody>
</table>

Now, what conclusions we can derive:

The most important conclusion we find is that net present value is more sensitive to changes in the unit sales forecast than either the variable cost or the fixed cost forecasts. When sales are at the lower bound, net present value is -$181735; when variable costs and fixed costs are at the lower bound, net present values are -$117936 and -$6288. That reject the fact that sales are typically more difficult to forecast than are costs.

Another conclusion derived is that for the worst case in the three sensitivity analysis, NPV is negative when the variables are at their lower bound forecasts. This is not encouraging, and further evaluation of the project is required.

Note: Other forms of analysis could be done, like Decision trees and Simulation Analysis (or Monte Carlo Analysis). That requires computer sophistication and skills.

C) Break-Even Analysis:
We have discovered that the unit sales forecast is the crucial variable is determining whether the project is likely to have a positive net present value. Consequently, the relationship between sales volume and profitability warrants additional attention. In this section, we develop several forms of break-even analysis; in general, the goal of break-even analysis is to identify the level of sales below which the project is unacceptable.

Before doing so, we need to know the difference between Fixed and Variable Costs

Variable Costs: are costs that vary with production. They are frequently assumed to be proportional to the level of production. (Exp: labor cost). Total variable cost (VC) are equal to variable cost per unit (v) times the number of units of output (Q).

\[ VC = v \times Q \]

Fixed Cost: do not change over a specified time interval and must be paid regardless of production levels. Rent is an example of fixed cost.

Total Costs are variable and fixed costs during a period of time:

\[ TC = VC + FC \]

or

\[ TC = v \times Q + FC \]

a) Accounting Break-even:

The accounting break-even point is the sales level at which a project’s net income is zero.

The general formula for finding the break-even point can be derived from the following:

we know:

\[ Net \ Income = (S - VC - FC - D) \times (1 - t) \]

where:

\[ S = Sales, \ VC = total \ variable \ costs, \ FC = Fixed \ Costs, \ D = depreciation \ and \ t = tax \ rate. \]

If we let \( P \) = Selling price per unit, \( v \) = variable cost per unit, and \( Q \) = total units sold, then:

\[ S = P \times Q \ and \ VC = v \times Q \]

Net income can be written as:

\[ Net \ Income = [(P \times Q) - (v \times Q) - FC - D] \times (1 - t) = 0 \]

Since accounting break-even is the sales level for which net income is zero:

then set the above equation = 0

\[ [(P \times v + FC) - D] \times (1 - t) = 0 \]

we get:

\[ (P \times v + FC) - D = 0 \]
The left hand side is equal also to EBIT, so this statement indicates that the value of Q for which net income is zero is the same value for Q for which EBIT is zero. So if we solve the problem, we get:

\[ Q = \frac{FC + D}{(P - v)} \]

D) Operating Cash Flow, Sales, Volume and Break-Even:

We consider further the break-even concept but from different perspectives. We consider further the cash break-even and financial break-even points. Assume here for simplicity that the firm does not pay income tax.

Accounting break-even and cash flow:

Remember, at the break even point, Net income is zero, and also EBIT is zero.

so:

\[ OCF = EBIT + D - T = 0 + $4000 - 0 = $4000 \]

If sales remain at the break-even point, the IRR is zero, the project will return a cash flow of $4000 per year for ten years.

a) Sales Volume and Operating Cash flow:

We know that:

\[ OCF = EBIT + D - Taxes = (S - C - D) + D - ((S - C - D) \times T) \]

If we ignore taxes, so:

\[ OCF = EBIT + D = (S - C - D) + D \]

Also we defined

\[ EBIT = ((P - v) \times Q) - FC - D \]

So operating cash flow will be:

\[ OCF = (S - C - D) + D = ((P - v) \times Q) - FC + D = ((P - v) \times Q) - FC \]

Or:

\[ OCF = ((P - v) \times Q) - FC \]

If we solve for Q we get:

\[ Q = \frac{FC + OCF}{P - v} \]

For the accounting break-even point, we know that OCF = D, then:

\[ Q = \frac{FC + D}{P - v} \]

b) The Cash break-even point:

That is the level of sales for which the operating cash flow is zero. That is easy to find from the above equation, just set OCF = 0

\[ Q = \frac{FC}{P - v} \]
A project that has sales equal to the cash break-even level throughout the life of the project has an internal rate of return - 100%. That is the original investment produces no cash inflow, so the investment represents a total loss of the original investment.

c) Financial break-even point:

It is the level of sales such that the net present value of the project is zero.

We need first to find the OCF for a project and then use the OCF in the formula $Q = \left( F + D \right) / (P - v)$ to find $Q$.

Say the investment costs $40000, for ten years, and the required return is 18%. Net present value is zero where:

$40000 = OCF \times PVIFA (18\%, \, 10)$

We can find that $OCF = 8900.59$

using $Q = \left( F + D \right) / (P - v) = 4360:24$
A) Operating Leverage

The level of fixed costs for a capital budgeting project has an important impact on the break-even concepts developed in the previous section. The precise nature of this relationship is defined in the following.

a) The Basic Idea

Operating Leverage is the extent to which a capital budgeting project, or a firm as a whole, relies on fixed production costs. A firm or a project with relatively high level of fixed costs has high operating leverage. Generally, a high level of fixed costs is also associated with a relatively large investment in plant and equipment; such a situation is said to be capital-intensive.

b) Measuring Operating Leverage

The degree of operating leverage (DOL) is defined as follows:

\[
DOL = \frac{\text{Percentage Change in OCF}}{\text{Percentage Change in Q}}
\]

This is equivalent to:

\[
DOL = 1 + \frac{FC}{OCF}
\]

Example: Suppose sales for the project under consideration increase from 2400 to 3000. What is the effect on operating cash flow. What is the degree of operating leverage.

Solution: Remember, OCF is equal to $4000 at the accounting break-even point of 2400 units. To find the DOL, the percentage change in OCF and Q are measured using the break-even point as the base quantity. Operating Cash Flow for an output of 3000 units can be computed as follows:

\[
OCF = (P-v)Q - FC = ($5.95 - $3.45)Q - $2000 = $5500
\]

The percentage change in operating cash flow is:

\[
\frac{($5500 - $4000)}{$4000} = .375 = 37.5\%
\]

The percentage change in output is:

\[
\frac{(3000 - 2400)}{2400} = .25 = 25\%
\]

Therefore, the degree of operating leverage is (37.5/25%) = 1.5.

The alternative calculation of DOL provides the same result:

\[
DOL = 1 + \frac{FC}{OCF} = 1 + \frac{$2000}{$4000} = 1.5
\]

Example: Suppose that sales for the project under consideration by the firm increase from 2400 units to 3600 units. What is the effect on operating cash flow. What is the degree of operating leverage.

Solution: Using OCF = $4000 when Q = 2400, for Q = 3600 units:

\[
OCF = (P-v)Q - FC = ($5.95 - $3.45)Q - $2000 = $7000
\]

The percentage change in operating cash flow is:

\[
\frac{($7000 - $4000)}{$4000} = .75 = 75\%
\]
The percentage change in output is: \((3600 - 2400)/2400 = 50\%\)

Therefore the degree of operating leverage is \((75\%/50\%) = 1.5\). This example illustrates the fact that, for a given level of output \((Q = 2400)\), the degree of operating leverage remains constant regardless of the size of the increase in \(Q\). This conclusion can be verified by observing the alternative formula for the calculation of DOL; for a given level of \(Q\), both FC and OCF remain constant regardless of the size of the increase in \(Q\).

Example: Suppose that sales increase from 3000 to 3600 units. What is the degree of operating leverage.

Solution: When \(Q = 3000\), \(OCF = \$5500\), When \(Q = 3600\), \(OCF = \$7000\). The percentage change in operating cash flow is: 27.27\% and the percentage change in output is: \((3600 - 3000)/3600 = 20\%\).

Then the DOL is \((27.27\%/20\%) = 1.3636\).

This example illustrates the fact that the DOL changes when the level of \(Q\) changes. When we consider changes in \(Q\) from a base of 2400 units, DOL = 1.5; for changes in \(Q\) from a base of 3000 units, DOL decreases to 1.3636. This is representative of the general result that DOL decreases when the base level of \(Q\) increases.