Energy Price Processes
Used for Derivatives Pricing & Risk Management

In this first of three articles, we will describe the most commonly used process, Geometric Brownian Motion, and in the second and third pieces, we will introduce two processes that are gaining rapid acceptance for a wide range of applications involving commodity derivatives: Mean Reversion and Jump Diffusion. We will talk about the main uses of each of these processes and some of the pitfalls that practitioners should take into account when using these processes for pricing and managing the risk of various energy derivatives structures. By **Carlos Blanco, Sue Choi & David Soronow**, Financial Engineering Associates.

This is the first article in a three-part series that will focus on the main processes used to model commodity spot and forward prices. Stochastic processes form the basis of derivatives pricing and risk management models, as they allow us to model possible price evolution through time, and assign probabilities to possible future prices as a function of current spot and forward prices and a set of parameters that describe the possible variability of those prices over time, (hopefully matching empirical patterns).

Characteristics of Commodity Prices

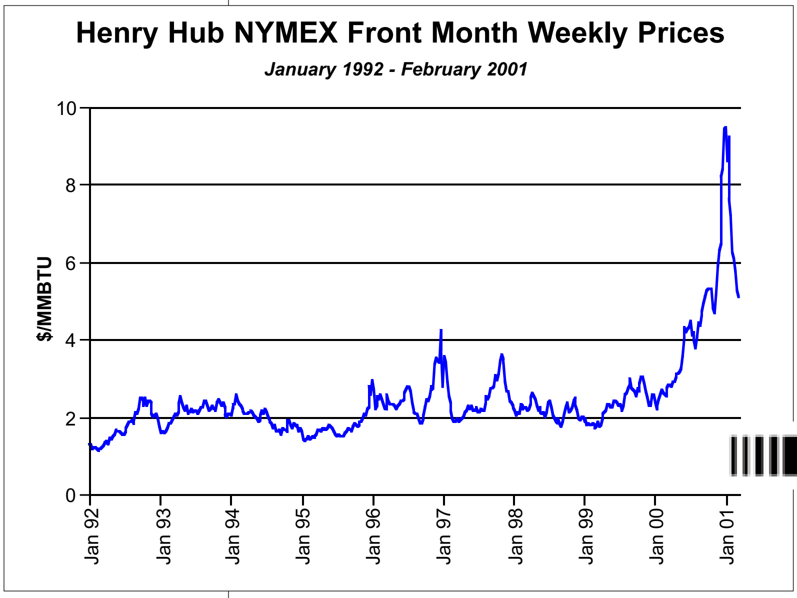
Before starting with the description of the mathematical models, it is important to keep in mind the actual behaviour of commodity prices that we are trying to model.

Commodity prices are somewhat different than other prices set in financial markets. Due to short term supply and demand imbalances, prices for short-term delivery of the commodity - or spot prices - tend to exhibit significantly different behaviour than prices for delivery of the commodity in the future, or forward prices.

Spot Prices: There are several important properties associated with the volatility of spot energy prices, principal among them being:

- ***Seasonal Effects:*** - In response to cyclical fluctuations in supply and demand mostly due to weather and climate changes, energy prices tend to exhibit strong seasonal patterns.
- ***Mean-Reversion:*** - Prices tend to fluctuate around and drift over time to values determined by the cost of production and the level of demand.
- ***Occasional Price Spikes:*** - Large changes in price attributed to major shocks (e.g. generation/transmission outages, unanticipated political events …).

Forward Prices: The forward markets provide the 'best guess' about the future spot price for different maturities. As we can see in the adjacent chart, there can be dramatic differences between prices for delivery of electricity in successive months.



Energy prices typically display seasonal variations in volatility, occasional price spikes, and a tendency to quickly revert to the average cost of production. Stochastic Processes used to model Commodity prices should capture the specific characteristics of the commodity.

In addition to seasonality and mean reversion, forward prices are characterised by exhibiting a different behaviour depending on the time to maturity. As contracts get closer to their maturity date, the volatility usually increases, (Samuelson's Hypothesis).

In order to capture energy markets reality - regardless of the price process being used - we should aim to incorporate, as the model's essential building blocks, the information contained in the forward price term structures(i.e. expected prices for delivery at different times), and the forward volatility term structure (i.e. expected variability of prices at different points in time). In the next two articles of this series, we will analyze the more sophisticated models, which also incorporate into the analysis the mean reversion and price spikes observed in many commodity prices.

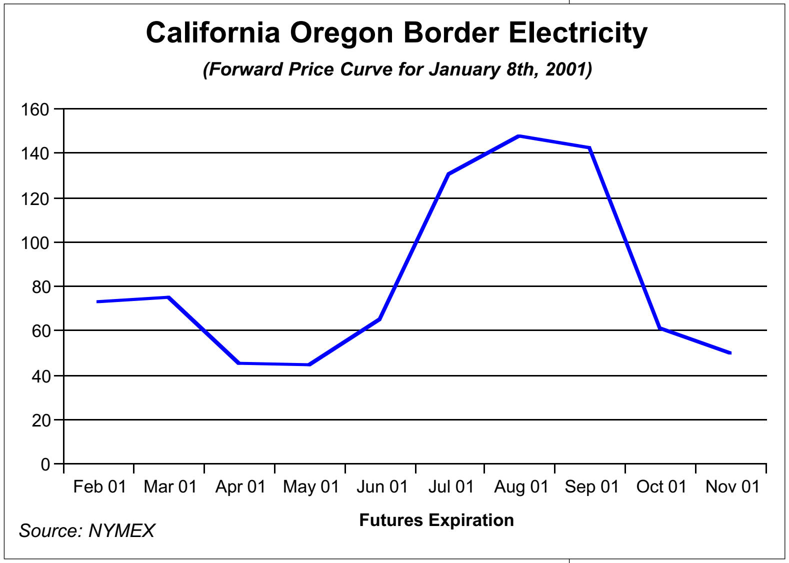
Derivatives pricing and Risk Management models are based on some type of assumption about the price evolution of the underlying commodity. Each of the processes that we are going to present have a set of advantages and disadvantages. The more simple ones may provide a simplistic view of reality, but allow us to characterise the multiple sources of risk in a very limited number of parameters, and therefore are easier to interpret and calibrate from market prices. The more complex processes provide the ability to incorporate more information about the possible price changes, but at the cost of having to estimate many more parameters and increasing the probability of model errors.

Brownian Motion, Random Walks & Black-Scholes

The most known price process is 'Brownian Motion', which takes the name from Scottish botanist Robert Brown, who in 1827 found that particles within water-suspended pollen grains followed a particular random movement that resembled a zigzag path. Even particles in pollen grains that had been stored for a century moved in the same way.

If we are using the spot price only, we need to add a drift to reflect the expected change in price to converge to the observed forward price. The commodity yield plays a large role in determining that drift.

When we are simulating forward prices, it is common to assume that the drift is zero, and use the current forward price as the starting point.



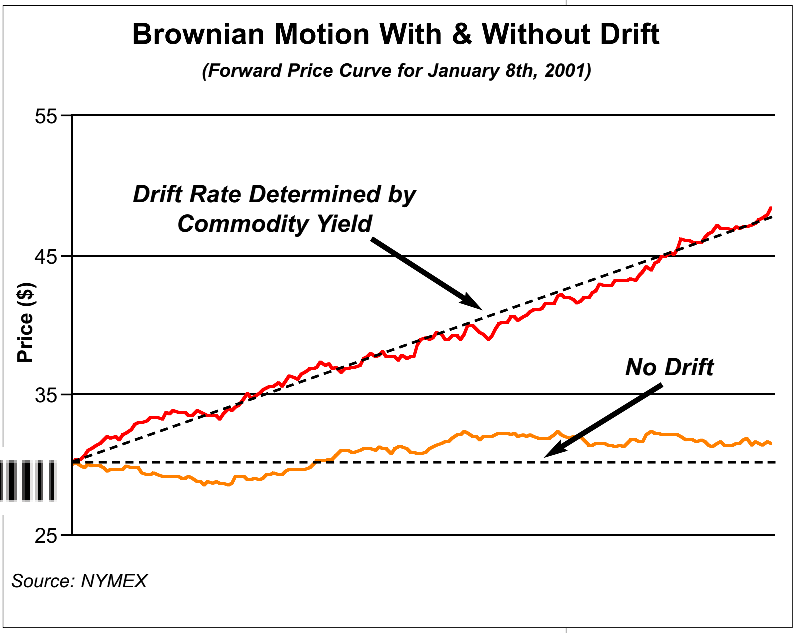
Main Stochastic Processes Used to Value Commodity Derivatives

Geometric Brownian Motion	Lognormal price diffusion
Mean-Reversion	Lognormal price diffusion with mean reversion
Mean Reverting Jump Diffusion	Same assumptions as the mean-reversion processes and adds a separate distribution for price jumps.

Since then, Brownian Motion has been used in multiple fields, including in finance, to model the behaviour of security prices. Over time, in the finance literature, Brownian motion came to be called 'Random Walk', in reference of the path of a drunk after leaving the bar on his way home.

The main properties of random walk process are:

- Price changes are independent of each other (no memory).
- Price changes have a constant mean and volatility.



Drifting Brownian Motion

Price Change Process = Drift effect (Non-random) + Volatility effect (Random)

For most commodities, it is possible to observe or build a forward price curve that provides the expected level of prices for different delivery dates. Within the forward price curve, the relation between different commodity prices is governed by the commodity yield. In order to incorporate the term structure of commodity yields, it is necessary to add a drift component to the Brownian motion.

■ For most commodities, it is possible to observe or build a forward price curve ■

model - which was based on the extension of Brownian motion contributed to the explosive growth in trading of derivatives.

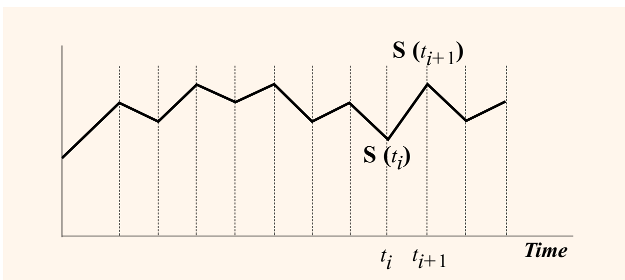
The original Black-Scholes model allowed only for pricing options on a non-dividend paying stock. Extensions to the Black-Scholes model, such as the Garman-Kohlhagen and Black(1976) model, allowed for pricing global commodity options, and options on futures respectively.

Geometric Brownian motion implies that returns have a lognormal distribution, meaning that the logarithmic returns, which are simply continuously compounded returns follow the normal (bell shaped) distribution. Consistent with reality, the lognormal distribution restricts prices from falling below zero (eg. the maximum negative return is 100%).

Price Evolution From t_i to t_{i+1}

$$S(t_{i+1}) = S(t_i) \exp\left[r - q - \tfrac{1}{2}\sigma^2\right](t_{i+1} - t_i) + \sigma\, \varepsilon_{i+1}\, S\, \varepsilon\sqrt{(t_{i+1} - t_i)}$$

Price at t_{i+1} | Price at t_i | Drift Component | Random Component



r = Risk free rate

q = Commodity yield

σ = Annualised standard deviation of returns

$(t_{i+1} - t_i)$ = Time step in years

ε_{i+1} is the random shock to price from t to $t+1$

exp = the base of the natural logarithm

Geometric Brownian Motion & Black-Scholes

In 1973, Fischer Black and Myron Scholes published their seminal paper on options pricing. The Black-Scholes option pricing

If S is the price, the change in price can be approximated by:

$$\Delta S = \mu S\Delta t + \sigma S \varepsilon\sqrt{\Delta t}$$

Over a short period of time, the logarithmic change in price is assumed to be normally distributed with:

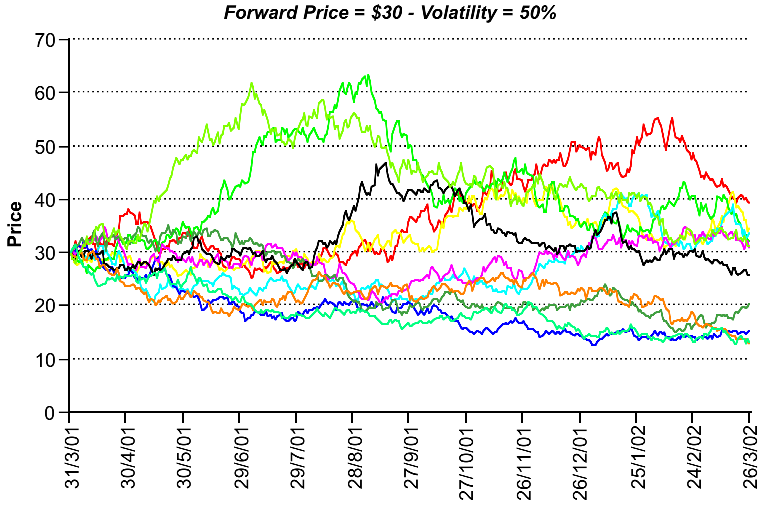
- Mean or Drift $\mu S\Delta t$

  with $\mu = r - q - \frac{1}{2}\sigma^2$

- Standard Deviation $\sigma S \varepsilon\sqrt{\Delta t}$

Geometric Brownian Motion - Sample Price Paths

Forward Price = $30 - Volatility = 50%



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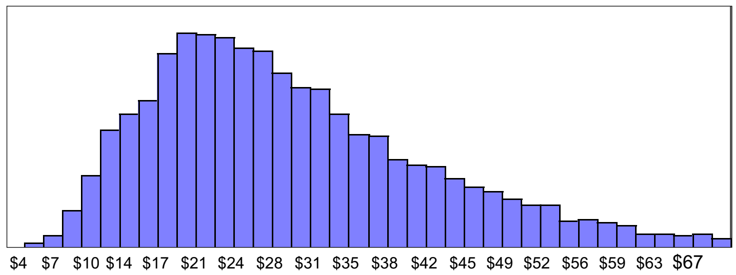
We can see randomly simulated price paths for daily steps during a year.

Price Histogram at Expiration - Geometric Brownian Motion

Forward Price = $30 - Volatility = 50%; 360 days



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The lognormal distribution allows for commodity prices between zero and infinity and has an upward bias (representing the fact that commodity prices can only drop 100% but can rise by more than 100%).

GBM Parameters: Volatility

In order to simulate possible future prices following GBM, we just need to know the current asset price, and the expected variability of the asset (volatility).

There are several methods to estimate 'expected' volatility.

Some traders prefer to use estimates based on historical prices ('historical volatilities'), while others prefer to use the volatilities implied by option market prices ('implied volatilities').

How to Calculate Historical Daily & Annual Volatility in Excel

1. Choose a particular price series. Prices could be hourly, daily, weekly, etc. Column A (rows 2–11) contains historical prices for asset x over a ten-day period.

2. Calculate the logarithmic price changes. The continuous returns are the natural logs of the price relatives. For example, cell B3's formula is **=LN(A3/A2)**. Column B (rows 3–11) shows the daily returns, denoted u.

	A	B
1	*X*	*u*
2	14.40	
3	14.20	-0.0140
4	14.25	0.0035
5	14.00	-0.0177
6	13.75	-0.0180
7	13.80	0.0036
8	13.60	-0.0146
9	13.75	0.0110
10	13.70	-0.0036
11	13.90	0.0145
12		
13	STDEV(u)	0.0126
14	SQRT(250)	15.8114
15	s*	0.1998

3. Use Excel's built-in STDEV (sample standard deviation) function to calculate the daily volatility (cell B13).

Under geometric Brownian Motion, volatilities are proportional to the square root of time. The process of converting volatilities between different time horizons is known as the square-root-of-time rule. This rule allows us to annualise hourly, daily, weekly, monthly or any other volatilities.

4. Using the square-root-of-time rule, we annualise to obtain s* (cell B15) assuming that time is measured in trading days and there are 250 trading days per year. The formulas are:

B14: =SQRT(250)
B15: =B13*B14

• Example 1: Suppose we have computed the daily volatility as being 1.26%. Then the annualized volatility would be equal to 19.98% (1.26% x $\sqrt{250}$).

The square root of time rule is commonly used in Risk Management to convert certain Value at Risk measures for different holding periods.

Introducing 'Seasonality' in the Simulation with the Forward Price & Volatility Curve

Prices and volatilities have a strong seasonal component that should be taken into account at the time of describing the possible evolution of prices through time.

We can replace the constant volatility parameter by a time dependent one (see figure opposite).

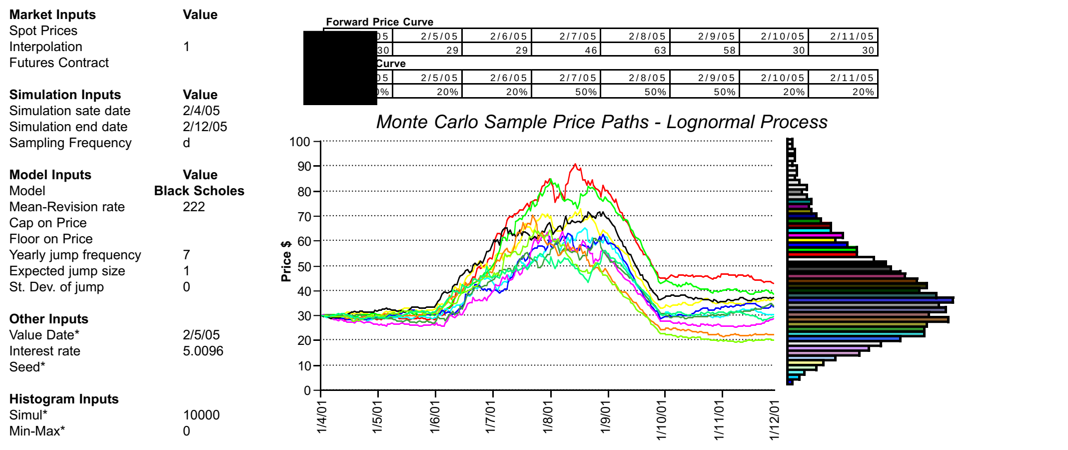
In this figure, we can observe simulated paths for a Electricity Forward Price Curve with strong price seasonality, and volatilities of 50% during each summer month (July, August, September) and 20% for all other months. We can observe the strong seasonal component in price variations given to seasonal price differences.

Pitfalls of Using Geometric Brownian Motion to Model Commodity Prices

1. Energy prices are not 'exactly' lognormally distributed.

Several empirical studies have shown that energy prices experience significant variations from lognormality. In the next two articles, we

FEA @Energy: Monte Carlo Simulation of Price Processes



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will introduce Mean Reversion and Jump Diffusion processes that better characterise the behaviour of certain commodities, especially electricity.

2. Extreme Price changes may be underestimated by the lognormal distribution

A direct consequence of the previous pitfall is that Geometric Brownian motion does not capture extreme price changes accurately. This pitfall is particularly important for Risk Management and the pricing of certain exotic options.

3. Volatilities are not known.

The only unknown parameter in the Geometric Brownian motion is the volatility of the underlying. The ideal volatility to use for modelling purposes would be the 'future volatility', but by definition, it is not possible to know the 'future volatility' until we know what has happened in the market, and by that time has become 'historical volatility'. Therefore, the volatility curve used as an input should be our "best estimate" of future volatility, and reflect our expectations regarding the variability of the asset price over the period of time under consideration.

4. Volatilities are not constant.

Any trader knows that volatilities change through time, and the assumption of constant volatilities may not be very realistic. More complex processes incorporate time varying volatility, and some Risk Management models assume that volatilities fluctuate just as asset prices do.

5. Beware of the model results for very high volatilities (e.g. above 300%).

Price paths generated with GBM with very high volatilities can be very different than what most traders have in mind at the time of using that process. The technical explanation is that when volatility is significantly large, the drift component starts to dominate the price evolution. For assets with very high volatilities and mean reversion (e.g. power), it is highly recommended to use other processes that better describe the evolution of the underlying, such as mean reversion or jump diffusion.

Conclusion

The development of general diffusion models contributed to the development of the options markets. Today, these models are still the most commonly used by market practitioners, largely due to the relative simplicity of estimating input parameters. However, as we will point out in the next articles, these models fail to capture many of the key characteristics of commodity prices. In particular, commodity prices tend to fluctuate around and drift over time to values determined by the cost of production, and often experience large changes in price due to shock events. The mean reversion model and jump diffusion model aim to modify the general diffusion price process in order to capture these additional market realities. ■

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